Neutron Radius of ^{208}Pb from Sub-Coulomb Pickup*

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The (d,t) and (p,d) reactions on ^{208}Pb have been studied at several energies below the Coulomb barrier. Relatively simple and parameter-free analysis yields the absolute normalization of the asymptotic tail for the neutron states near the Fermi surface. A wide range of Woods-Saxon potentials that match the binding energy and the asymptotic tail yield a rather unique value of the rms radius for the neutron excess, namely 5.99 ± 0.10 fm. This is very close to the radius deduced from Coulomb-energy differences.

The experimental measurement of the radius of the neutron distribution in nuclei is in a rather inconclusive state. A number of experimental techniques have been applied to this question, but none of them provides clear-cut quantitative answers. While the rms radius of the proton (or charge) distribution is known to a fraction of a percent, there could easily be 10% uncertainties in the neutron radius.

In recent years considerable attention has been devoted to the interpretation of Coulomb energy differences. The results, when properly corrected for exchange, spin-orbit, and various continuum effects, indicate that the radius of the neutron excess is substantially smaller than can be explained by any reasonable model.[1] This difference varies from ~20% in the lighter mirror nuclei to 6% in ^{208}Pb, but it appears to be a rather general feature. Several theoretical analyses[2] have failed to arrive at a simple general explanation for this phenomenon, except perhaps for the suggestion by Negele[3] and Okamoto[4] that this may be a reflection of a possible violation of charge independence in the nucleon-nucleon force.

Other methods of determining neutron radii, such as the analysis of π^+ and π^- total cross sections,[5] the cutoff in K-mesonic x rays,[6] and the analysis of proton elastic scattering[7] generally are consistent with the smaller neutron-excess radius, though the uncertainties are generally too large to clearly distinguish what amounts to a 2% difference in ^{208}Pb, since these techniques are sensitive to the total neutron distribution.

The purpose of this note is to report on a new experimental determination of the neutron-excess distribution in ^{208}Pb. The technique involves the neutron-pickup reactions (p,d) and (d,t). Some of the ambiguities associated with the analysis of such data in the framework of the distorted-wave Born approximation disappear if the incident and outgoing energies are well below the Coulomb barrier.[8] While perhaps it may be argued that the (p,d) reaction is better understood than the (d,t), the Q value for ^{208}Pb(d,t) is -1.1 MeV as compared with -5.2 MeV for the (p,d) reaction. Therefore, the (d,t) reaction can satisfy the sub-Coulomb condition better in both channels than the (p,d) reaction does.

We have measured cross sections for the reaction ^{208}Pb$(d,t)^{207}$Pb to five hole states between 8 and 11.5 MeV at 120°, 135°, and 150° by use of a split-pole spectrograph and the deuteron beam from the Argonne tandem accelerator. In addition, the reaction ^{208}Pb(p,d_0) was investigated at 11.0 MeV. Absolute cross sections accurate to 10% were determined by reference to elastic-scattering measurements at 60°. These results are summarized in Table I.

It seems quite clear that the cross section in this reaction is proportional to the square of the wave function in the asymptotic tail. The behavior of this tail with radius is of course determined by the binding energy; the extraction of the absolute value by means of a standard distorted-wave analysis was performed with the computer code DWUCK.[9] At the lower energies ($E_d < 9$ MeV) these numbers are independent of the distorting potentials; at the higher energies a small distortion needs to be included, but any reasonable potential gives practically the same answer.

Finite-range and polarization effects have been ignored. Such effects have been estimated for the low-energy (d,p) reaction on ^{209}Bi to cause a slight increase ($\lesssim 15\%$) in the calculated cross

TABLE I. Experimental cross sections in μb/sr for the (d,t) and (p,d) reactions leading to neutron-hole states in ^{207}Pb. The values of $|\psi|^2$ are neutron densities at 15 fm, in units of 10^{-8} neutrons/cm^3, needed to fit the cross sections.

Hole-state configuration		$3p_{1/2}$		$3p_{3/2}$		$2f_{5/2}$		$2f_{7/2}$		$1i_{13/2}$	
Excitation energy (keV)		0		897		570		2339		1633	
E_{beam} (MeV)	θ	σ	$\lvert\psi\rvert^2$	σ	$\lvert\psi\rvert^2$	σ	$\lvert\psi\rvert^2$	σ	$\lvert\psi\rvert^2$	σ	$\lvert\psi\rvert^2$
7.98[a]	150°	31.4	2.02	3.8	1.03	4.2	0.407				
8.97[a]	150°	205	1.99	51.6	1.06	44.4	0.378				
8.97[a]	120°	127	1.80	32.7	1.08	28.0	0.406				
9.47[a]	150°	377	1.87	124.	1.07	117.	0.410				
9.96[a]	150°	611	1.90	278.	1.07	222.	0.385			4.0	0.205
10.96[a]	135°							26.1	0.130	20.0	0.176
11.45[a]	150°							61.7	0.137	47.0	0.187
10.98[b]	150°	1.8	1.86								
Average			1.92		1.06		0.397		0.134		0.189

[a] (d,t) reaction. [b] (p,d) reaction.

sections.[10] For the (d,t) reaction finite-range effects seem to be a sensitive function of the triton wave function.[11] From the fact that the values of $|\psi|^2$ extracted from the (p,d) reaction and the (d,t) reaction to the ground state are in good agreement when zero-range calculations are used (with normalization factors of 1.53 and 3.33, respectively), we conclude that our neglect of these effects is justified.

Assuming, for the moment, that the hole states in ^{207}Pb are indeed good single-hole states with absolute spectroscopic factors $S=2j+1$, we need to know how the magnitude of the asymptotic tail is related to $\langle r^2\rangle^{1/2}$ or some other bulk radius of the wave function. Exploring Woods-Saxon potentials of different diffuseness and adjusting the radius and depth to give the experimental separation energy and the observed value for the asymptotic tail may serve as a guide in this. The root-mean-square radii of all such wave functions seem to be reasonably constant, but the sensitivity had to be explored in more detail.

First of all, we need to make some remarks concerning the spin-orbit potential. In order for a spin-orbit potential to fit the parameters (energy and cross section) of a spin-orbit doublet, of which we have two, its shape must be different from the usual ones. If the spin-orbit potential is taken to be of the Thomas (derivative) shape, then a fit to the data requires that the mean radius $r_0A^{1/3}$ be at least ~0.5 fm smaller than the radius of the central well. Alternatively, the spin-orbit potential may be absorbed in the central well itself. A further difficulty arises in that the strength of the spin-orbit well required to fit the $2f_{5/2}$-$2f_{7/2}$ separation is about 50% smaller, for any shape, than that required for the $3p$ splitting.

These difficulties in finding a unique potential are not surprising, but they raise some problems in calculating a unique mean-square radius for the neutron excess. In particular, difficulties arise in estimating the parameters for the $1h_{9/2}$ orbit for which no pickup data could be obtained at these low bombarding energies.

To study the ambiguities, we have adopted a simplified model. We use a Woods-Saxon potential with no spin-orbit term, and adjust parameters separately for each orbit. With three parameters (V, r_0, and a) to be determined and two experimental quantities (binding energy and asymptotic tail), the family of possible potentials obtained for each orbit is represented by a line in three-dimensional parameter space. Because of spin-orbit and l-dependent effects, these lines

TABLE II. Mean-square radii.

State	$\langle r^2\rangle^{1/2}$ [a] (fm)
$3p_{1/2}$	6.10 ± 0.10
$3p_{3/2}$	5.97 ± 0.09
$2f_{5/2}$	5.92 ± 0.05
$2f_{7/2}$	5.76 ± 0.05
$1i_{13/2}$	6.20 ± 0.15
$1h_{9/2}$	5.90 ± 0.15 [b]
Average [c]	5.99 ± 0.10

[a] The uncertainties reflect both experimental errors and some of the uncertainties in the wave functions used to estimate $\langle r^2\rangle$ from the magnitude of the tail.

[b] Not directly from data, but estimated by extrapolating from the $p_{1/2}$ and $f_{5/2}$ states using a variety of Woods-Saxon potentials.

[c] Weighted by the number of particles in each orbit.

will not be expected to intersect; but they will put constraints on the data. Potentials with $r_0 \approx 1.21$ fm and $a \approx 0.8$ fm seem to be most nearly consistent with the data, though these values are somewhat different from those found by other workers.[8, 12, 13]

The values of $\langle r^2\rangle^{1/2}$ are given in Table II. The range of variation associated with reasonably shaped potentials ($0.45 \leq a \leq 1.0$ fm) is included in the uncertainty quoted. Inclusion of reasonable spin-orbit potentials does not alter this range. The net uncertainty in the mean-square radii is small but is still considerably larger than the uncertainty associated with the measurements. The parameters of the $1h_{9/2}$ orbits were estimated by various extrapolations from the parameters of the $3p_{1/2}$ and $2f_{5/2}$ orbits. All these orbits have odd l and $j = l - \frac{1}{2}$, and all belong to the same oscillator quantum number $2n + l = 7$.

The values of $\langle r^2\rangle^{1/2}$ for the observed orbits are remarkably insensitive to the uncertainties in cross sections; a 10% uncertainty in the measurement leads to 0.8% uncertainty in $\langle r^2\rangle^{1/2}$. Similarly, a slight change in the absolute spectroscopic factor would not matter much, but a factor-of-2 reduction[14] would increase the value of $\langle r^2\rangle^{1/2}$ by about 5% and thus would bring this radius into accord with the theoretical expectation. The justification for such a drastic and uniform renormalization of spectroscopic factors is difficult to assess.

The final value shown in the third line of Table III is somewhat larger than the mean-square radius deduced from Coulomb-energy differences,

TABLE III. Summary of root-mean-square radii[a] in ^{208}Pb.

	$\langle r^2\rangle^{1/2}$ (fm)
Protons (charge)	5.51
Neutron excess (from Coulomb energy difference)	5.95
Neutron excess (from present work)	6.04 ± 0.10
Neutron excess (predicted from Woods-Saxon well)	6.28

[a] For purposes of comparison with the charge radius, the finite size of the nucleon is folded into the radii.

but the difference is within the uncertainties. The fact that it is substantially smaller than the value of $\langle r^2\rangle^{1/2}$ that one would predict from the charge radius suggests that perhaps the "anomaly" is indeed in the radius of the neutron excess.

Finally, we may use our data to reconstruct the neutron halo, which in its more faint, wispy tail has the excess orbits as its primary constituents. A rough estimate indicates that less than 20% of the neutron density at a radius of 10 fm is from orbits other than the ones studied here and this percentage drops to 3% at 15 fm. Our results in terms of a neutron density, obtained by normalizing Hankel functions to our data and grafting this density onto one from a Woods-Saxon calculation,[1] in which the finite size of the nucleon is folded in, are shown in Fig. 1. Also shown

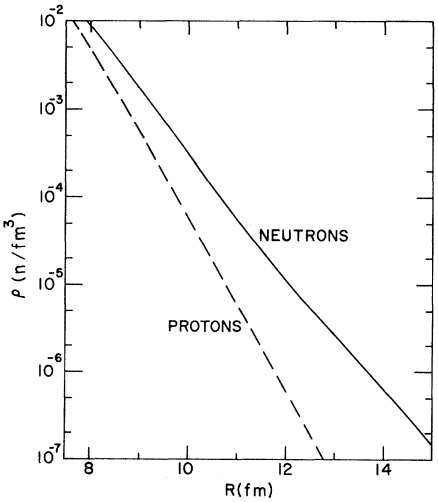

FIG. 1. Neutron and proton densities, derived as described in the text.

are proton densities from the same Woods-Saxon calculation, in which the parameters were adjusted to fit $\langle r^2 \rangle$ measured for the charge distribution.

Such densities should be used in calculations of K^- or antiproton absorption. These rely critically on densities at large radii, and any estimates that do not take the asymptotic separation energy into account are likely to be seriously in error, particularly for heavy nuclei.

A comparison between our results and those of Negele,[15] who has done perhaps the most realistic neclear-matter calculations to date, are not very meaningful because he does not reproduce the absolute binding energies of neutron-hole states *exactly*. Deviations of the order of an MeV become crucial in determining the asymptotic cross sections. The Bethe-Siemens model[16] for K^- capture emphasizes the importance of the neutron tail but tends to overestimate its absolute magnitude by about a factor of 2 at 10 fm.

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New High-Spin States in ^{19}F and a Possible $K = \frac{3}{2}^+$ Band

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New levels in ^{19}F with spins of $\frac{9}{2}^+$ and $\frac{11}{2}$ have been discovered at energies of 6592 ± 2 and 7937 ± 3 keV, respectively. It is suggested that these levels, together with the known 3907-keV ($\frac{3}{2}^+$), 4549-keV ($\frac{5}{2}^+$), and 5464-keV ($\frac{7}{2}^+$) levels, form a $K = \frac{3}{2}^+$ rotational band.

Members of the ground-state rotational band in ^{19}F up to the $(2s, 1d)^3$ limit have now been identified with the discovery of levels of spin $\frac{11}{2}^+$ at 6.50 MeV[1] and spin $\frac{13}{2}^+$ at 4.65 MeV.[2] This band is reasonably well explained by a shell-model calculation with three nucleons outside an inert ^{16}O core. Similarly, the second $\frac{3}{2}^+$ level at 3.91 MeV, the second $\frac{5}{2}^+$ level at 4.55 MeV,[3] and the second $\frac{7}{2}^+$ level at 5.46 MeV[4] seem to comprise a $K = \frac{3}{2}^+$ band, but these levels are not so easily explained in terms of three-particle configurations. In particular, three-particle shell-model