plasmas remains open. But in those circumstances, if any exist, in which a strongly magnetized plasma *can* be considered to move in two dimensions, Eq. (7) provides a possible route for estimating a substantially higher resistivity than the conventional expression based on binary encounters in three dimensions.

We give for reference purposes a formal expression for the ac conductivity as well:

$$\sigma_{ac}(\omega) = 2\frac{N}{L^2}\frac{e^2}{lkT}\frac{c^2}{B^2}\int_0^\infty e^{+i\omega\tau}Q(\tau)d\tau, \qquad (8)$$

though the actual value of $\sigma_{ac}(\omega)$ depends upon numerical evaluation of the definite integral in (8), which we have done and shall report in detail elsewhere. Equation (7) is a satisfactory approximate expression for frequencies less than $4\pi ec/\lambda_D^2 Bl$.

Equations (7) and (8) are examples of what Kubo terms a "generalized Einstein relation," though it was not *a priori* obvious that such relations would exist for the guiding-center plasma.

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Metastable Thermodynamic States Near the Critical Point of He^3†

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We report the first measurements of the specific heat of a metastable, superheated, pure liquid (He^3). The specific heat shows no evidence of any singularity near the onset of metastability. We have also observed the lifetimes of metastable states as the He^3 is cooled at constant density. For densities near the critical density, lifetimes decrease abruptly in an extremely narrow, reproducible temperature interval. This may indicate the onset of an "intrinsic" instability.

We report the first measurements of the specific heat (C_v) of a liquid in states which are metastable with respect to formation of bubbles of vapor. Since these measurements were made near the critical point of He^3, they are of interest from several points of view. First, the "equilibrium" properties of the metastable states can be compared with those model equations of state which may be continued into a metastable region within the liquid-vapor coexistence curve. Such comparison will be particularly valuable near the singularity at the critical point. Secondly, the observation of lifetimes of metastable states will enable the testing of theories which predict the rate of homogeneous nucleation of instability by local thermodynamic fluctuations. Finally, it is conceivable that observations on metastable states may demonstrate the existence of a frequently conjectured spinodal line, where "intrinsic" or large-scale instability occurs.

Since measurements of the "equilibrium" properties of metastable states of fluids are quite rare,[1,2] we now describe our apparatus and qualitative observations. We also mention the difficulty encountered in attempting to extrapolate the "linear model" equation of state[3] to the observed limit of stability. Detailed comparison of the specific-heat data with model equations of state will be described elsewhere. Detailed comparison of lifetimes with nucleation calculations will require additional experiments, some of which are in progress.

An idealized sketch of our calorimeter is shown in Fig. 1. Most of the 1.06-cm^3 sample of He^3 was contained in an accurately horizontal cylindrical volume 3.97 cm in diameter and 0.087 cm high. A vertical fill hole 0.72 cm long and 0.035 cm in diameter leads up from the main volume

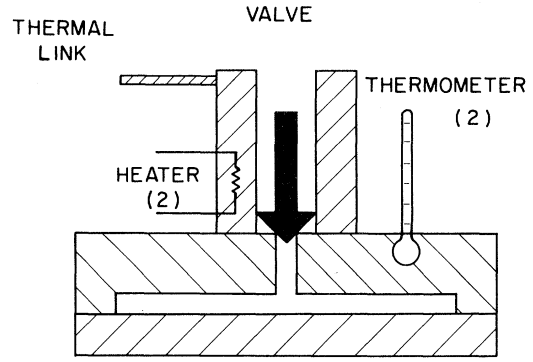


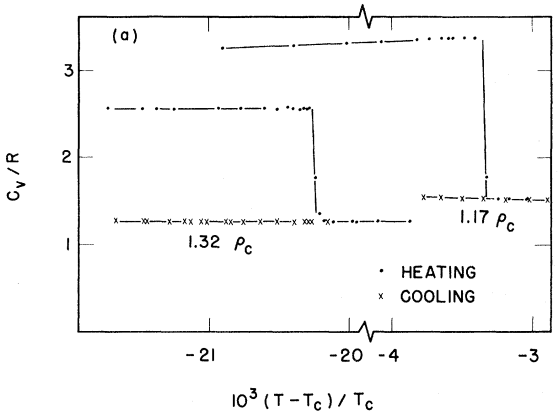
FIG. 1. Schematic diagram of calorimeter.

to a valve. The fill hole contains 0.06% of the sample. We did not expect the He^3 in this small volume to contribute noticeably to the specific heat, but this volume is important for interpreting the observed limit of stability (see below). The valve, fill tube, and calorimeter are well anchored together thermally. They are connected to the cooler, pumped He^4 bath at 1.3 K by a weak adjustable thermal link which allows specific-heat measurements to be made while either warming or cooling.

To determine the critical temperature, the C_v data for $t<0$ are fitted with the function $C_v = A(-t)^{-0.1} + B$. [Here $t=(T-T_c)/T_c$ is the reduced temperature measured from the critical temperature. A, B, and T_c are adjusted to fit the data.] This function has been shown to describe He^3 and He^4 specific-heat data accurately.[4,5] The critical density (ρ_c) is determined relative to the densities where we have data by requiring the observed coexistence curve to be symmetrical about ρ_c. This determines ρ_c to within $\pm 0.1\%$.

After filling the calorimeter with a known quantity of He^3 (with an impurity of 40 ppm He^4), metastable states were prepared by warming the calorimeter well into the one-phase region and then cooling. When the filling density exceeded approximately $1.08\rho_c$, metastable states were observed *every* time the calorimeter was cooled through the coexistence curve. Although we prepared these states by cooling and they become less stable as cooling at constant volume progresses, these states are usually referred to as those of a "superheated liquid." We made no unusual effort to clean our calorimeter, and so, in fact, we have measured the heat capacity of a "dirty" bubble chamber.

Figure 2 shows the normalized specific heat at constant volume, C_v, of He^3 as a function of t.



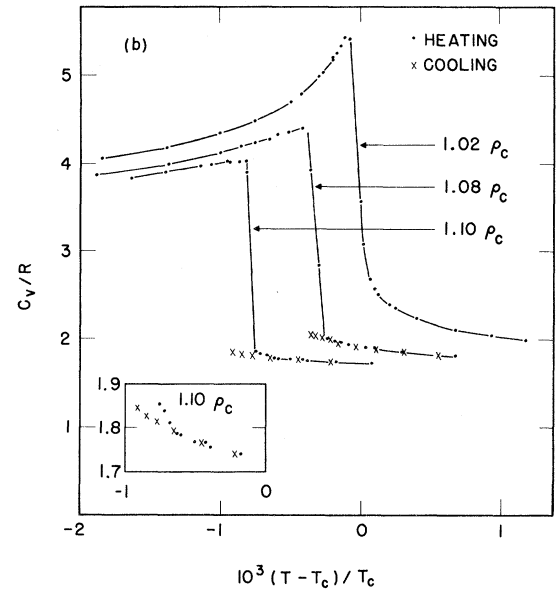


FIG. 2. Normalized (by the gas constant) specific heat at constant volume of He^3 as a function of t at five values of ρ/ρ_c. At most densities data from several runs are included. Inset to (b) shows detail for $\rho = 1.10\rho_c$. The difference between heating and cooling data in the inset is a consequence of the presence of a phase boundary in the fill tube upon heating.

The dots represent data taken upon warming from low temperatures where the calorimeter contains both liquid and vapor. As the temperature is raised the liquid expands until it fills the calorimeter at t_{coex}. This point is marked by an abrupt decrease in C_v. The crosses on Fig. 2 represent data taken by cooling. C_v appears to be a completely smooth function of temperature as the temperature is reduced through t_{coex} into metastable one-phase states. This observation is in agreement with the common textbook assumption that the properties of metastable thermodynamic states can be described by an analytic extrapola-

tion from nearby stable states. Of course, this observation is also consistent with those theoretical arguments[6,7] which suggest that thermodynamic quantities will have a weak essential singularity at t_{coex} (such that all temperature derivatives remain finite). Our measurement of C_v, a second temperature derivative of the free energy, is a sensitive confirmation of the smoothness of the free energy at t_{coex}.

It took approximately 50 sec to secure the data for each cooling point on Fig. 2. The apparent thermal relaxation time of the filled calorimeter was less than 3 sec throughout the metastable region. As we attempted to take cooling data at still lower temperatures, the calorimeter began to warm spontaneously. It took as long as 15 min for the calorimeter to settle to a new higher temperature. The new temperature and subsequent C_v data (not shown) were consistent with the He^3 in the calorimeter having returned to the stable two-phase state with the same internal energy.

We attempted to nucleate deliberately the transition from metastable to stable states in the region where lifetimes exceed 1 h. A sharp blow to the cryostat merely warmed the calorimeter. A fast neutron source providing a flux of about 10^3 neutron/sec through the calorimeter was successful in reducing the lifetimes to some tens of seconds. At lower temperatures the neutron source caused prompt nucleation. This suggests that some cosmic rays (which pass through our calorimeter at a rate[8] of 0.2/sec [8]) may be responsible for the spontaneous nucleation we observe.

In Table I we have summarized some of the information we have obtained about lifetimes. As the density approaches ρ_c the transition from long to short lifetimes occurs in a range of temperatures which narrows and moves comparatively further from the coexistence curve. To interpret these facts we were led to consider the gravitationally produced density gradient in our calorimeter. Near T_c and ρ_c this gradient is substantial; therefore, the He^3 near the top of the fill hole is less dense than the bulk of the sample and more likely to be the site of either homogeneous nucleation or intrinsic instability. Because this site is in effect a sample of very small volume ($< 10^{-3}$ cm^3), it has a negligible cross section to cosmic rays. To determine the density in this small volume we used the "linear-model" equation of state.[3] For this application the linear model has several advantages compared with other proposed equations of state: (1) It has the fewest adjustable parameters. (2) It is in agreement with data near the critical points of many systems including the comparable fluid He^4. (3) The linear-model μ-ρ isotherms extrapolate naturally into the metastable region until a cusp is reached (μ is the chemical potential). This cusp occurs before the isothermal compressibility $(\partial\rho/\partial\mu)_t/\rho^2$ becomes infinite; therefore, it does not represent a point of intrinsic instability, but it is a limit of applicability of the linear model.

TABLE I. The reduced temperatures at which the lifetimes are approximately 100 sec (t_{100}) and 3 sec (t_3) are compared with the coexistence temperature. Also, t_3 is compared with the temperature at which the cusp in the linear-model equation of state is calculated to appear at the top of the calorimeter (t_{cusp}).

ρ_{mean}/ρ_c	t_{100}/t_{coex}	t_3/t_{coex}	t_3/t_{cusp}
1.078	1.23	1.23–1.29	1.27–1.33
1.105	1.22	1.22–1.26	0.71–0.73
1.171	1.14	1.16–1.24	0.57–0.61
1.32	1.05	1.09–1.22	0.53–0.59

We determined the two coefficients and the two exponents in the linear model from the data of Wallace and Meyer[9] on the coexistence curve and critical isotherm of He^3. We used the fact that the mean density of the He^3 in the calorimeter (ρ_{mean}) equals the local density at some height in the main volume and integrated the equation $d\mu = -gdz$ upward to calculate the density as a function of height in the fill hole at various temperatures. For each value of ρ_{mean} there is a lowest temperature (t_{cusp}) at which this integration may be carried through to the top of the fill hole. At lower temperatures the cusp in the model isotherm occurs beneath the top; therefore, the linear model cannot be applied to calculate the density at the top. At $\rho_{mean} = 1.078\rho_c$ we observed metastability at temperatures significantly below the limit of applicability of the linear model (see Table I). Nevertheless, it is reasonable to assume that the trend indicated by the linear model is correct. If so, the ratio of t_3 to the t_{coex} appropriate to the estimated density at the *top* of the calorimeter increases rapidly as $\rho_{mean} - \rho_c \to 0^+$. This increase is consistent with a "critical slowing down" of the rate of spontaneous nucleation. Eggington *et al.*[10] have found such a slowing down in a calculation of the rate of nucleation of droplets in a supercooled vapor. Their calculation also predicts a narrowing of the temperature interval in which nucleation rates increase. Be-

cause our data are at large "superheating," a quantitative comparison must await either equation-of-state data in metastable states or further lifetime measurements without the complication of a fill hole. We cannot now state whether the abruptness of the decrease in lifetimes is in agreement with nucleation theory or results from another source such as the onset of large scale instability.

We *never* saw evidence of metastability when ρ_{mean} had the values $0.68\rho_c$, $0.90\rho_c$, $0.94\rho_c$, $1.02\rho_c$, and $1.05\rho_c$. The observations at $0.68\rho_c$, $0.90\rho_c$, and $0.94\rho_c$ imply that there is no barrier to nucleation of a droplet in the calorimeter, a situation which is expected if He^3 wets the walls of the calorimeter. The observations at $1.02\rho_c$ and $1.05\rho_c$ lead us to infer that very near T_c the density at the top of the fill tube was less than ρ_c. (The linear model predicts that metastability will not occur in our calorimeter if $\rho_{mean} \leq 1.064\rho_c$ and wetting occurs.)

To summarize, we have reported the comparative ease of obtaining metastable thermodynamic states of He^3 and measuring their properties. We find that specific heat is continuous at the onset of metastability. Near the critical point the range of metastable states changes in qualitative agreement with a calculation showing a decrease in the rate of homogeneous nucleation. A detailed test of nucleation rates should be possible with further measurements of this type.

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Nonlinear Electron-Cyclotron Drift Instability and Turbulence

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The cross-field anomalous resistivity due to the electron-cyclotron drift instability is shown to persist in the nonlinear regime as a result of mechanisms which depend critically on the magnitude of the magnetic field.

The nonlinear development of current-driven electron-cyclotron turbulence perpendicular to a background magnetic field[1,2] is important for understanding the anomalous resistance observed in collisionless-shock experiments.[3] Based on the results of numerical simulations, we outline a model of the strong turbulence which is quite different from the picture of Lampe *et al.*[4] and is in disagreement with many of their conclusions, particularly in the parameter range of high-β shock experiments. We find that there are three physical regimes exhibiting different nonlinear electron behavior depending on the relative size of the wavelength λ of the turbulence compared with the electron gyroradius r_e. For high-β shock experiments where $\lambda = 2\pi/k \ll r_e$, the electrons can be trapped in potential wells for longer than an electron gyroperiod, thus giving rise to a nonlinear electron-cyclotron instability which does not have the properties of the nonmagnetic ion-acoustic instability. This instability causes substantial electron heating and cross-field dif-