Resolution of the Discrepancy in the Determination of the Cabibbo Angle θ_V from K_{l3} and Nuclear β Decays*

Ephraim Fischbach
Physics Department, Purdue University, Lafayette, Indiana 47907

and

Michael Martin Nieto†
Physics Department, University of California, Santa Barbara, California 93106

and

Henry Primakoff
Physics Department, University of Pennsylvania, Philadelphia, Pennsylvania 19104

and

C. K. Scott
Physics Department, McMaster University, Hamilton, Ontario, Canada

and

Jack Smith
Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11790
(Received 29 April 1971)

The magnitude of the polar-vector Cabibbo angle θ_V obtained from K_{e3} decay has been reevaluated using the Kemmer description of pseudoscalar mesons. We find $\theta_V^{(K)}=0.192_+ \pm 0.016$. This compares favorably with the value $\theta_V^{\beta}=0.188_- \pm 0.007$ obtained for $0^+ \to 0^+$ β decays, and thus removes the discrepancy between $\theta_V^{(\beta)}$ and the value obtained from the conventional Klein-Gordon K_{e3} description, $\theta_V^{(K-G)}=0.235_- \pm 0.019$. Combining the Kemmer and the β-decay results leads to a "best" value of $\theta_V=0.188_+ \pm 0.006$. A new determination of the axial-vector Cabibbo angles $\theta_A^{(K)}$ and $\theta_A^{(K-G)}$ is also obtained.

In a previous paper[1] (hereafter called I) a treatment of K_{l3} form factors was given based on the Duffin-Kemmer-Petiau equation describing the pion and the kaon. It was shown that the Kemmer formulation provides a natural explanation for the observed (large and negative) value of the ratio of the "effective" form factors $\xi \equiv \hat{f}_-(0)/\hat{f}_+(0)$, and in addition predicts a kinematic zero in the effective scalar form factor $\hat{f}_0(t)$. The presence of this zero, which is an unambiguous prediction of our use of the Kemmer equation, will permit a direct test of our theory to be made as soon as better data become available. In the absence of such data we are confronted with two sets of K_{l3} form factors, the conventional Klein-Gordon form factors $f_\pm(t)$ and the Kemmer form factors $g_{V,S}(t)$.

The purpose of the present Letter is to point out that these two bases of form factors lead to different determinations of the magnitude of the polar-vector Cabibbo angle θ_V. Specifically we find that $\theta_V^{(K-G)}=0.235_- \pm 0.019$, and $\theta_V^{(K)}=0.192_+ \pm 0.016$ from the Klein-Gordon (K-G) and the Kemmer (K) form factors, respectively. These values compare to $\theta_V^{(\beta)}=0.188_- \pm 0.007$ obtained[2] from the $\mathfrak{F}t$ values for $0^+ \to 0^+$ β decay. The close agreement between the Kemmer-K_{e3} and the $0^+ \to 0^+$ β-decay values of θ_V further strengthens the case for using the Kemmer equation to describe processes where there is symmetry breaking among the pseudoscalar mesons, and leads to a combined "best" value $\theta_V=0.188_+ \pm 0.006$.

Before going further, we give here a short description of how our new result comes about physically. We first note that the rate ($\Gamma_{e3}^\pm$) for $K^\pm \to \pi^0 e^\pm \nu$ is given in terms of $f_+^2(0)$ by[3]

$$\Gamma_{e3}^\pm = 4\Gamma_0 \sin^2\theta_V^{(K-G)} f_+^2(0) \times (1.1826 + 4.3725\lambda_+) \times 10^{-2}. \quad (1)$$

We remind the reader[1] that the K_{l3} decay rates must be recalculated for the Kemmer formulation, which we have done (see Table I). Then in terms of $g_V^2(0)$, $\Gamma_{e3}^\pm$ is given by

$$\Gamma_{e3}^\pm = 4\Gamma_0 \sin^2\theta_V^{(K)} g_V^2(0) \times (1.7536 + 6.4835\gamma_V) \times 10^{-2}, \quad (2)$$

using the results of Table I. The parameters

TABLE I. The Kemmer expressions for the $K_{l3}{}^{i}$ decay rates ($\Gamma_l{}^i$) and the rate ratios (R^i). $\Gamma_l{}^i = 0.04\Gamma_0 \sin^2\theta_V \times g_V{}^2(0) T_l{}^i$, with $\Gamma_0 = G^2(m_{K^0}+m_{K^-})^5/2^{11}\pi^3 = 3.118\times10^9$ sec^{-1} and $G = 1.435\times10^{-49}$ erg cm^3. The $T_l{}^i$ are obtained by multiplying all the numbers in the $T_l{}^i$ row by their column headings and then summing. The same for the R^i.

	1	γ_V	ρ	$\rho\gamma_V$	$\rho\gamma_S$	ρ^2	$\rho^2\gamma_S$	$\rho^2\gamma_V$
$T_\mu{}^\pm$	1.0167	5.8062	−0.1313	−0.7038	−0.7038	0.0216	0.1789	0.0
$T_e{}^\pm$	1.7536	6.4835	0.0	0.0	0.0	0.0	0.0	0.0
$T_\mu{}^0$	1.0180	5.4340	−0.1318	−0.6518	−0.6518	0.0212	0.1644	0.0
$T_e{}^0$	1.7505	6.0503	0.0	0.0	0.0	0.0	0.0	0.0
$R^\pm = T_\mu{}^\pm/T_e{}^\pm$	0.5798	1.1675	−0.0749	−0.1246	−0.4013	0.0123	0.1020	−0.0455
$R^0 = T_\mu{}^0/T_e{}^0$	0.5815	1.0943	−0.0753	−0.1122	−0.3724	0.0121	0.0939	−0.0419

$f_+(0)$, λ_+, $g_V(0)$, and γ_V are as defined in I. In the limit of exact SU(3) symmetry *either* $f_+(0) = 1/\sqrt{2}$ *or* $g_V(0) = 1/\sqrt{2}$, depending on which basis of form factors is taken as being the more fundamental. If we work in the Kemmer basis, it follows from Eq. (7a) of I that it is $g_V(0)$ which assumes the value $1/\sqrt{2}$, so that the *effective* value $\hat{f}_+(0)$ of $f_+(0)$ is given by

$$\hat{f}_+(0) = \frac{m+\mu}{2(m\mu)^{1/2}} g_V(0) = 1.2177 \times \frac{1}{\sqrt{2}}, \tag{3}$$

where m and μ are the masses of the kaon and pion, respectively. In other words, the assumption that $g_V(0) = 1/\sqrt{2}$ in the Kemmer formulation is equivalent to the assumption that $f_+(0) \to \hat{f}_+(0) = (1/\sqrt{2})[(m+\mu)/2(m\mu)^{1/2}]$ in the conventional (K-G) formulation. Since $f_+{}^2(0)$ multiplies $\sin^2\theta_V{}^{(\text{K-G})}$ in the expression for $\Gamma_{e3}{}^\pm$ it follows immediately from Eq. (3) that the value of $\sin\theta_V$ determined using the Kemmer basis will be ~20% *smaller* than the value obtained from the usual K-G basis. In what follows we show that this smaller value of θ_V is in excellent agreement with that recently obtained[2] from $0^+ \to 0^+$ β decay.

To continue, using the experimental values[4] $\Gamma_{e3}{}^\pm = (3.93 \pm 0.06)\times10^6$ sec^{-1} and[5] $\lambda_+ = \gamma_V = 0.045 \pm 0.012$, we find from Eqs. (1) and (2)

$$\sqrt{2} f_+(0) \sin\theta_V{}^{(\text{K-G})} = 0.213 \pm 0.006,$$
$$\sqrt{2} g_V(0) \sin\theta_V{}^{(\text{K})} = 0.176 \pm 0.006. \tag{4}$$

In order to extract $\sin\theta_V{}^{(\text{K-G})}$ and $\sin\theta_V{}^{(\text{K})}$ from Eqs. (4) we must apply radiative and SU(3)-symmetry-breaking corrections to $f_+(0)$ and $g_V(0)$, respectively. These corrections have the effect of replacing $f_+(0)$ and $g_V(0)$ by $f_+'(0)$ and $g_V'(0)$ where

$$f_+'(0) = f_+(0)(1 + \tfrac{1}{2}\eta_R + \eta_S),$$
$$g_V'(0) = g_V(0)(1 + \tfrac{1}{2}\zeta_R + \zeta_S). \tag{5}$$

In Eq. (5) η_R and ζ_R denote the radiative corrections to $\Gamma_{e3}{}^\pm$, and η_S and ζ_S the effects of SU(3) symmetry breaking. Although η_R and η_S have been calculated by several authors,[6,7] ζ_R and ζ_S have not. However, our experience with the Kemmer formalism shows that $\eta_R = \zeta_R$, and we may in addition expect $\eta_S \cong \zeta_S$ to within 10–20%. Since these corrections are small to begin with, and in any case are uncertain by more than this amount, we will hereafter assume that $\eta_S = \zeta_S$. An estimate of η_R has been given by Ginsberg[6] who finds $\eta_R = \Delta\Gamma_{e3}{}^\pm/\Gamma_{e3}{}^\pm \cong -0.45\%$. Estimates of η_S have been given by several authors[7] who find $\eta_S = -(0.9\text{–}15)\%$. For later purposes we note that all of the values for both η_R and η_S have the effect of *increasing* $\sin\theta_V{}^{(\text{K-G})}$ or $\sin\theta_V{}^{(\text{K})}$ in Eqs. (4). Combining Eqs. (4) and (5) we then find

$$\theta_V{}^{(\text{K-G})} = 0.235_- \pm 0.019,$$
$$\theta_V{}^{(\text{K})} = 0.192_+ \pm 0.016, \tag{6}$$

where the uncertainties represent the combined experimental errors in Eq. (4) and the theoretical uncertainties associated with $\eta_{R,S}$ and $\zeta_{R,S}$.

We turn next to a comparison of the results in Eq. (6) with those obtained from superallowed $0^+ \to 0^+$ β decays. The determination of θ_V from superallowed $0^+ \to 0^+$ β decays has been discussed in detail by Brene, Roos, and Sirlin[8] and by Blin-Stoyle and Freeman[2] whose results we summarize below. Using the recently determined[9] $\mathfrak{F}t$ value for $^{26}\text{Al}^m \to {}^{26}\text{Mg} + e^+ + \nu_e$ (which, because it has the lowest $\mathfrak{F}t$, is considered to be the "best" $0^+ \to 0^+$ $\mathfrak{F}t$ value), Blin-Stoyle and Freeman find[2]

$$G_V(1 + \tfrac{1}{2}\Delta_R{}^{(V)}) = (1.4150 \pm 0.0011)\times10^{-49} \text{ erg cm}^3, \tag{7}$$

where G_V is the polar-vector decay constant for $\Delta S = 0$ β decay and $\Delta_R{}^{(V)}$ is a model-dependent radiative correction. The most recent estimate of

$\Delta_R^{(V)}$, due to Källén,[2,10] is

$$\Delta_R^{(V)} \cong (+0.68 \pm 0.20)\%. \quad (8)$$

Since G_V is related to the muon decay constant G_μ by[11]

$$G_V = G_\mu \cos\theta_V^{(\beta)} = \{(1.4354 \pm 0.004)\times 10^{-49} \text{ erg cm}^3\} \cos\theta_V^{(\beta)}, \quad (9)$$

we find, on combining Eqs. (7)–(9),

$$\theta_V^{(\beta)} = 0.188_- \pm 0.007. \quad (10)$$

Comparing the results of Eqs. (6) and (10) we see that $\theta_V^{(\beta)}$ and $\theta_V^{(\mathrm{K})}$ are in excellent agreement with one another, while $\theta_V^{(\beta)}$ and $\theta_V^{(\mathrm{K\text{-}G})}$ are in relatively poor agreement. This again strongly suggests that the Kemmer description of K_{l3} decays may be superior phenomenologically to the usual Klein-Gordon description. The agreement between $\theta_V^{(\beta)}$ and $\theta_V^{(\mathrm{K})}$ suggests combining these two values to give a "best" value θ_V. We find

$$\theta_V = 0.188_+ \pm 0.006. \quad (11)$$

We mention that this result roughly agrees with the Cabibbo angle $\theta_V^{(B)}$ obtained from an analysis of all of the less precise data on the semileptonic decays of baryons.[12] There, subject to uncertainties in the D/F ratio, in the axial-vector vertex renormalization constant, and in the type of corrections discussed here, the result was

$$\theta_V^{(B)} = 0.206 \pm 0.009. \quad (12)$$

Further, our "best" value θ_V given in Eq. (11) predicts a branching ratio R of pions into the π_{e3} mode of

$$R_{\mathrm{th}} = \Gamma(\pi_{e3})/\Gamma(\pi \text{ total}) = (1.011 \pm 0.003)\times 10^{-8}. \quad (13)$$

In calculating Eq. (13) we have included the radiative correction[6,13] $\Delta\Gamma(\pi_{e3})/\Gamma(\pi_{e3}) = +0.012$. However, the error in the calculation of $\Delta\Gamma(\pi_{e3})$ is uncertain, and hence has not been included in Eq. (13). R_{th} is to be compared with the present experimental value[4] which is

$$R_{\mathrm{exp}} = (1.02 \pm 0.07)\times 10^{-8}. \quad (14)$$

We turn next to a brief discussion of the axial-vector Cabibbo angles $\theta_A^{(\mathrm{K})}$ and $\theta_A^{(\mathrm{K\text{-}G})}$. From the rates $\Gamma(K \to \mu\nu)$ and $\Gamma(\pi \to \mu\nu)$ we find[4,14]

$$\tan\theta_A^{(\mathrm{K\text{-}G})} f_K/f_\pi = 0.2755 \pm 0.0008, \quad (15a)$$

$$\tan\theta_A^{(\mathrm{K})} g_K/g_\pi = 0.1465 \pm 0.0004, \quad (15b)$$

where[1] f_π (f_K) and g_π (g_K) are the pion (kaon) decay constants in the K-G and K formulations, respectively. SU(3) symmetry requires that $f_K/f_\pi = g_K/g_\pi = 1$ and hence, in this limit, $\tan\theta_A^{(\mathrm{K\text{-}G})}$ and $\tan\theta_A^{(\mathrm{K})}$ are given by Eqs. (15a) and (15b), respectively. If, contrariwise, we assume that $\theta_A^{(\mathrm{K\text{-}G})} = \theta_A^{(\mathrm{K})} = \theta_V^{(\beta)}$ then, from Eqs. (10) and (15),

$$f_K/f_\pi \cong 1.45 \pm 0.05, \quad (16a)$$

$$g_K/g_\pi \cong 0.77 \pm 0.02. \quad (16b)$$

Thus the deviation from exact SU(3) symmetry is smaller in the Kemmer formulation of K_{l2} and π_{l2} decays than in the K-G formulation, which again supports the view that the Kemmer equation may provide a more natural description of pseudoscalar mesons than the K-G equation.

We conclude by contrasting our resolution of the "θ_V problem" with that of Blin-Stoyle and Freeman.[2] These authors observe that if $\Delta_R^{(V)}$ were ~(100–200)% larger than the value quoted by Källén,[10] such as could come about in some models of $\Delta_R^{(V)}$ which assume a nonlocal weak interaction Hamiltonian H_w, then $\theta_V^{(\beta)}$ would also be larger and could thus be brought into agreement with $\theta_V^{(\mathrm{K\text{-}G})}$. The nonlocal models of H_w in question are, however, more speculative than the conventional local current-current model to the extent that they require the existence of a hypothetical W boson with a mass in the range of ~20–100 GeV. What we have shown, by contrast, is that the "θ_V problem" can be solved in the framework of the conventional model of H_w by simply using the Kemmer basis of K_{l3} form factors in place of the customary K-G basis. In any event, the two solutions to the "θ_V problem" can be distinguished experimentally as soon as better data become available on π_{e3} and semileptonic baryon decays and overall consistency criteria among them are applied.

---

*Work supported in part by the U. S. Atomic Energy Commission (E.F. and J.S.), the National Science Foundation (M.M.N. and H.P.), and the National Research Council of Canada (C.K.S.).

†Address from September 1971: Physics Department, Purdue University, Lafayette, Ind. 47907.

[1]E. Fischbach, F. Iachello, A. Lande, M. M. Nieto, and C. K. Scott, Phys. Rev. Lett. 26, 1200 (1971).

[2]R. J. Blin-Stoyle and J. M. Freeman, Nucl. Phys. A150, 369 (1970); R. J. Blin-Stoyle, in *Topical Conference on Weak Interactions, CERN, Geneva, Switzerland, January 1969* (CERN Scientific Information Service, Geneva, Switzerland, 1969), p. 495.

[3]H. W. Fearing, E. Fischbach, and J. Smith, Phys. Rev. Lett. 24, 189 (1970).

[4]M. Roos *et al.*, Phys. Lett. 33B, 1 (1970).
[5]L. M. Chounet and M. K. Gaillard, Phys. Lett. 32B, 505 (1970).
[6]E. S. Ginsberg, Phys. Rev. 142, 1035 (1966), and 162, 1570 (1967), and 171, 1675 (1968).
[7]P. B. Jones, Phys. Rev. Lett. 21, 1553 (1968); S. L. Glashow and S. Weinberg, Phys. Rev. Lett. 20, 224 (1968); P. P. Strivastava, Nucl. Phys. B7, 220 (1968); G. Furlan, F. G. Lannoy, C. Rossetti, and G. Segrè, Nuovo Cimento 38, 1747 (1965).
[8]N. Brene, M. Roos, and A. Sirlin, Nucl. Phys. B6, 255 (1968).
[9]J. M. Freeman, J. G. Jenkin, G. Murray, and W. E. Burcham, Nucl. Phys. A132, 593 (1969).
[10]G. Källén, Nucl. Phys. B1, 225 (1967).
[11]See M. Roos and A. Sirlin, CERN Report No. Th. 1294, 1971 (to be published), for this numerical value of G_μ.
[12]H. T. Nieh and M. M. Nieto, Phys. Rev. 172, 1694 (1968).
[13]N. P. Chang, Phys. Rev. 131, 1272 (1963); M. V. Terent'ev, Zh. Eksp. Teor. Fiz. 44, 1320 (1963) [Sov. Phys. JETP 17, 890 (1963)].
[14]R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley, New York, 1969), p. 407.

ERRATA

0^+ STATES IN Mo ISOTOPES FROM (p,t) REACTIONS AND ANOMALIES IN THE NEW TRANSITIONAL REGION. H. Taketani, M. Adachi, M. Ogawa, K. Ashibe, and T. Hattori [Phys. Rev. Lett. 27, 520 (1971)].

The tenth line of the second column, page 522, should read, "The (d,p) spectroscopic factors ⋯" instead of "The spectroscopic factors ⋯."

EVIDENCE FOR ONE-DIMENSIONAL METALLIC BEHAVIOR IN $K_2Pt(CN)_4Br_{0.3}\cdot(H_2O)_n$. D. Kuse and H. R. Zeller [Phys. Rev. Lett. 27, 1060 (1971)].

Two significant misprints occur in this Letter: On page 1062, lines 21 and 22 should read, "This visual appearance persists upon cooling down the crystal to 4.2°K, indicating ⋯" instead of "4.2%."

The second sentence of the caption of Fig. 2 should read, "The solid square on the vertical axis represents the conductivity deduced from ⋯."