ous theories.[2,3,7] One can conclude that it may be possible for pulsar radiation to penetrate to large distances in the medium of the Crab nebula, thus allowing the possibility of an energy source in these large volumes. On the other hand, penetration of the filaments with their higher densities and shorter scale lengths appears doubtful. Thus, the results presented here would support the hypothesis that the "amorphous-mass" region of the Crab nebula is filled with strong electromagnetic radiation, since the electron density in this region is less than 1 cm^{-3}.[8]

We have benefitted from discussions with J. Arons, P. Kaw, R. Kulsrud, and J. Ostriker.

*Work supported by the U. S. Air Force Office of Scientific Research under Contract No. F44620-70-C-0033.

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Mobility of Electrons on the Surface of Liquid ^{4}He

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(Received 4 January 1971)

The mobility of electrons on the surface of liquid ^{4}He has been measured from 0.9 to 3.2°K. Although scattering from atoms in the vapor appears to be the dominant scattering mechanism, the mobility of the surface electrons is significantly lower than that of free electrons in the vapor. No evidence for surface-wave scattering is discernible.

The existence of extrinsic image-potential–induced surface electron states in certain insulators, including liquid helium, has been discussed by Cole and Cohen.[1] Subsequently, several mechanisms which might influence the properties of electrons in surface states on liquid helium have been proposed. Cole[2] has predicted that scattering by quantized surface waves would be the dominant scattering mechanism at low temperatures and would lead to an essentially temperature-independent mobility below about 1°K. More recently Crandall and Williams[3] have discussed the possibility of crystallization of the surface electrons.

To investigate the interactions of electrons in surface states on liquid ^{4}He, we have measured the mobility of surface charges between 0.9 and 3.2°K using the apparatus shown schematically in Fig. 1(a). A movable support positions the chamber containing the electrodes so that the liquid surface is at a height h of 1 to 2 mm above the set of three identical submerged electrodes. Electrons are provided by a gas discharge located in the vapor about 8 cm above electrode 2. The potential difference V_{dc} between electrode 2 and the submerged electrodes, along with the distance of these electrodes below the surface, determines the surface charge density. A signal from the internal oscillator of a lock-in amplifier is applied to an outer electrode of the submerged set. The signal coupled by the surface charge to the other outer electrode is amplified by the lock-in amplifier and compared in phase with the input signal, yielding data from which the surface electron mobility may be determined. Direct capacitive coupling between the two outer electrodes is reduced to a negligible value by the grounded central electrode.

In normal operation the electron source was operated continuously. When accumulated charge reduced the field between electrode 2 and the surface to zero, the surface would accept no more charge and the entire potential difference V_{dc} would appear between the surface and the submerged electrodes. If C represents the capacitance per unit length between the surface and the submerged electrodes then it will be seen that in

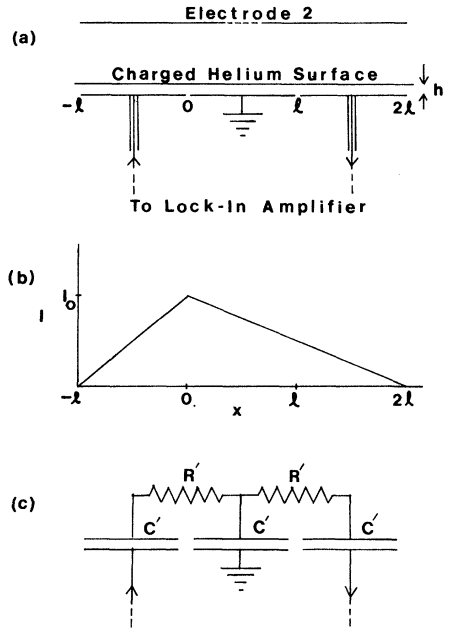

FIG. 1. (a) Schematic of apparatus used to measure electron surface mobility. Width of electrode array was about 2.5 cm and length ($3l$) was about 3 cm. Spacing between electrode 2 and set of submerged electrodes was 1 cm. (b) Ramp-function approximation for current variation along electrode array. This approximation, with $I_0=\frac{2}{3}V_0\omega Cl$ (where V_0 is the amplitude of the applied signal and C is the capacitance per unit length), is appropriate for low phase shifts and low liquid levels. It was used for calculating the energy dissipation in the electron layer in the derivation of Eq. (2). (c) Simple discrete-element circuit which may be used to check Eq. (2) and to estimate effects of second-order terms in the phase shift.

normal operation the charge per unit length is CV_{dc} and the resistance per unit length R is $(\mu CV_{dc})^{-1}$ where μ is the mobility of the surface charge. By using these relationships (and by letting $C'=Cl$ and $R'=Rl$, where l is the length of an individual electrode) a crude, but instructive, approximation for the relation linking the surface mobility to the observed phase shift may be made by applying ordinary circuit analysis to the discrete circuit shown in Fig. 1(c). This approximation is of the same functional form and differs by only about 14% in magnitude from the more elaborate calculation described below, which is based on analysis of the energy losses and spatial phase variations in the surface current.

The surface current is governed by the equation

$$\partial^2 I/\partial x^2 = RC\partial I/\partial t. \quad (1)$$

If the height h of the liquid is small compared to the dimensions of an individual electrode, so that edge effects may be neglected, then R and C will be constant over the electrode region, and the solutions to Eq. (1) are hyperbolic functions of complex argument. If, in addition, the phase shifts are limited to small values, then the current amplitude as a function of position may be approximated by the ramp functions shown in Fig. 1(b). (This result depends on the input impedance of the lock-in amplifier and lead cables connected to the receiving electrode being sufficiently small that the receiving electrode remains close to ground potential.) The phase of the peak current I_0, passing above the junction between the transmitting electrode and grounded central electrode, may be found by equating the energy dissipated in the charge layer (calculated using the ramp-function approximations for the current) to the energy given up by the applied signal. The total phase shift is then obtained by adding a contribution due to the spatial phase variation across the central electrode, calculated from the complex hyperbolic expression for the current. The final expression relating the mobility to the phase shift is found[4] to be

$$\mu = 7\pi f l^2/3V_{dc}(\varphi_0 - \varphi), \quad (2)$$

where f is the frequency of the applied signal and $\varphi_0 - \varphi$ is the phase shift introduced by the finite conductivity of the surface charge.

The agreement of Eq. (2) with the approximation obtained from the simple discrete circuit of Fig. 1(c) was tested further by a numerical calculation of the phase shift for discrete element circuits in which each capacitor of Fig. 1(c) was divided into a number of equal segments linked by resistors equivalently reduced in value. With each capacitor in Fig. 1(c) divided into thirty parts, the computed phase shifts are within 2% of the values predicted by Eq. (2). The results of analysis of the discrete circuits indicate that the deviation from the small-angle approximation at phase shifts of 30° (the largest values used in the experimental measurements) is roughly 6%.

The experimental technique employed here had been applied previously by one of us,[4] in a more primitive form and over a limited temperature range, to observe the behavior of electrons on the surface of ^{4}He. In these earlier measurements the applied signal, ranging from 200 mV to 1 V, had produced some electron heating, as evidenced by a dependence of measured mobility

values on signal amplitude. The signal amplitude of 20 mV rms used in the present measurements was sufficiently small to avoid electron heating, as shown by the absence of mobility variation with applied signal amplitude in test runs ranging from 10 to 50 mV. It should be pointed out that the maximum field experienced by each surface electron due to the applied signal is greatly reduced by shielding arising from the collective motion of the surface charge. This maximum field is just RI_0 and at the lowest temperatures reached in these measurements, where electron heating would normally be most likely to occur, was never greater than a few hundred microvolts per centimeter.

In order to calculate the mobility by using Eq. (2) the phase shift corresponding to infinite surface conductivity, φ_0, had to be experimentally determined, since the observed phase of the output signal included instrumental contributions. At the lowest temperatures reached, 0.9°K, φ_0 could be found by measuring φ as a function of V_{dc} at constant f and solving for φ_0 using Eq. (2). Then φ was measured as a function of temperature. This technique produced values for $\varphi_0 - \varphi$ at low temperatures which were, within experimental error, proportional to the density of atoms in the vapor (i.e., to P/T—the ratio between vapor pressure and temperature). For this reason φ_0 was determined for most of the data presented by a least-squares fit between φ and P/T, and the original technique for determining φ_0 was dispensed with.

Our mobility data are shown in Fig. 2. The data displayed are from individual runs having the least internal scatter out of some thirty runs total. Overall scatter from all sets of data indicates a possible random error of about 25%. In the region below 2°K the surface electron mobility is seen to have a temperature dependence very similar to that calculated for a free electron in the vapor using semiclassical theory[5] and the accepted scattering length[6] of 0.62 Å and corresponding cross section of 4.9×10^{-16} cm^2. The absolute values of our surface mobility data, however, are 4 to 5 times smaller than the values calculated for electrons in the vapor in this temperature region. Above about 2°K the surface mobility starts to undergo a transition to a lower mobility regime. This transition has previously been investigated for electrons in helium vapor by Levine and Sanders[7] and discussed theoretically for three-dimensional systems by Neustadter and Coopersmith[8] and by Eggarter and Cohen.[9]

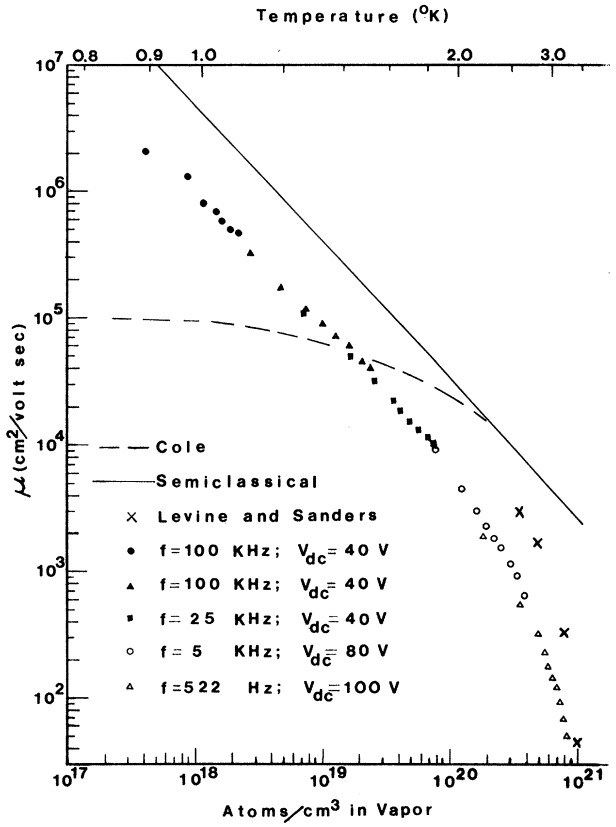

FIG. 2. Mobility of surface electrons on liquid ^{4}He. Data are obtained by observing change in phase of the output signal from apparatus shown schematically in Fig. 1(a) as the temperature is swept. Individual sets of data are distinguished and operating parameters are specified for each set. Comparison is made with the semiclassical calculation of mobility for electrons in ^{4}He vapor and with experimental values of electron mobility in dense ^{4}He vapor, as well as with a mobility calculation intended for the surface electrons.

Our data indicate that in this region the surface electron mobility is also significantly lower than the values measured by Levine and Sanders for electrons in helium vapor.

Although the calculated binding energy[1,2] for surface electrons is significantly greater than thermal energies in the 1 to 2°K range, the ratio of the two is sufficiently small that the lifetime for an electron to remain in an image-potential–induced bound state would be expected to be rather short, as recent experiments have attempted to show.[10] To test whether the field configuration used in our experiment resulted in electrons being predominantly on the surface rather than in the vapor, some runs were made in which, after the phase shift had been measured in the normal

way, the electron source was turned off, after which V_{dc} was increased by a factor of about 2.5. The value of V_{dc} to be used in Eq. (2) remains that value which determined the surface charge density. Therefore if the electrons were initially on the liquid surface, the only effects to be expected when V_{dc} is increased while the electron source is off would be small effects due to redistribution of charges around the periphery of the electrode array and possibly to small changes in mobility produced by perturbation of the electron surface-state wave functions. On the other hand if a significant fraction of the electrons originally had been in free states in the vapor and had a mobility significantly different from the surface-state mobility, then the change in V_{dc} would be expected to produce a sizable change in phase shift. Application of the additional field would reduce the density of states for unbound thermal electrons by reducing the region of space available to them, so that the probability of electrons remaining unbound would greatly decrease. When V_{dc} was increased in this fashion, the change in phase shift was never larger than that due to the random difference between runs and in most cases was much less, indicating that the electrons were initially in surface states.

Our data show no evidence for surface-wave scattering of the type predicted by Cole. The fact that the mobility in the region below 2°K has a temperature dependence very close to that expected for free electrons in the vapor, and yet is significantly lower in absolute value, suggests that careful consideration should be given to calculations of effective mass and cross section for this type of surface state. As no detailed description has been given for the transport properties to be expected for electrons in the crystallized state hypothesized by Crandall and Williams, it is not possible to tell whether this hypothesis is consistent with our data. In the transition region above 2°K the low mobility of the surface electrons compared to the values for electrons in the vapor may depend on a different mechanism from that which is operative in the low-temperature region. In their analysis of this type of transition in a three-dimensional system of hard-core scattering, Eggarter and Cohen[9] consider a simple cubic system of cells, some of which are allowed to the electrons and some of which are prohibited, by virtue of density fluctuations. Their quantitative calculation employs the use of a quantity called percolation probability which was computed by Frisch, Hammersley, and Welsh[11] using Monte Carlo methods. Reference to these Monte Carlo results reveals that a significantly higher fraction of cells would have to be allowed cells for an electron to percolate through a two-dimensional square lattice than would be the case for a three-dimensional cubic lattice. On the basis of this model the difference in dimensionality alone would be expected to result in lower values of mobility in the transition region for an electron on the surface than for a free electron in the vapor.

While consistent values for surface electron mobility were obtained over a fairly wide range of surface charge densities from our phase-shift measurements and Eq. (2), we wish to mention one effect observed in the course of our measurements which is not readily explained in terms of the simple model of signal coupling used to derive Eq. (2). According to this model the amplitude of the output signal should be independent of surface charge density provided that the charge density is high enough that the phase shift is not excessive. However, for values of V_{dc} below 25 V the amplitude of the coupled output signal was often observed to depend strongly on the value of V_{dc}, particularly at low temperatures. The phase shifts were quite small and roughly as predicted by Eq. (2) and the mobility values given here. The variation of the output signal amplitude with changing V_{dc} would terminate dramatically at about $V_{dc} = 25$ V where, within a voltage interval of about 0.5 V, the output would be observed on a monitoring oscilloscope to fluctuate rapidly between two signals with different amplitudes but the same phase. The signal amplitude, which had varied nearly linearly with V_{dc} below this narrow interval, increased by roughly a factor of 2 between the bottom and top of the interval and was quite independent of V_{dc} above the interval. Any attempt to explain these phenomena would seem to require the hypothesis that for voltages below $V_{dc} = 25$ V there are regions of surface charge which for some reason are totally immobile, and hence restricted from contributing to the effective value of C, surrounded by surface charge of well-defined mobility. Experimental examination of these effects with altered electrode configurations will be required to determine if they have any fundamental basis.

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Nonlinear Landau Damping of Alfvén Waves*

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It is shown that large-amplitude linearly or elliptically polarized Alfvén waves propagating parallel to $\vec{B}_0$ can be dissipated by nonlinear Landau damping. The damping is due to the longitudinal electric field associated with the ion sound wave which is driven (in second order) by the Alfvén wave. The damping rate can be large even in a cold plasma ($\beta \ll 1$, but not zero), and the mechanism which we propose may be the dominant one in many plasmas of astrophysical interest.

This communication shows that large-amplitude Alfvén waves propagating parallel to the average magnetic field $\vec{B}_0$ can be dissipated by Landau damping. The physical basis[1] is straightforward. A (first-order) Alfvén wave which is linearly or elliptically polarized produces a (second-order) gradient in the magnetic field pressure. This pressure gradient drives a (second-order) ion sound wave and an associated longitudinal electric field. This electric field leads to Landau damping in the usual fashion.

Our calculation of the damping rate is analogous to Stix's[2] calculation of the Landau damping of plasma waves. Consider a linearly polarized[3] Alfvén wave propagating parallel to $\vec{B}_0$ in the $+z$ direction. The wave magnetic field is $B_x = \alpha \times \cos(kz - \omega t)$; the transverse electric field follows from Faraday's law, $E_y = -(\alpha\omega/ck)\cos(kz - \omega t)$. The longitudinal electric field is $E_z = E_0 \times \sin(2kz - 2\omega t)$, where E_0 will be calculated below. Our approach is to calculate the energy gained by test particles moving in this wave field; equating the energy gained by the particles to the energy lost by the field determines the damping rate. Like Stix's, our analysis is appropriate only in situations where the damping is weak, i.e., $|\gamma/\omega| \ll 1$, where γ is the damping rate.

The motion of a test particle (charge q, mass m) in the electromagnetic field specified above is determined by the equations of motion:

$$(d/dt \pm i\Omega)v_\pm = \pm iqm^{-1}(E_y + v_z c^{-1} B_x), \tag{1}$$

$$dv_z/dt = qm^{-1}(E_z - v_y c^{-1} B_x), \tag{2}$$

where $v_\pm \equiv v_x \pm iv_y$, and $\Omega \equiv qB_0/mc$ is the gyrofrequency. The homogeneous solutions of (1) are $v_{x0} = V_\perp \cos(\Omega t - \varphi_0)$ and $v_{y0} = -V_\perp \sin(\Omega t - \varphi_0)$, where $V_\perp$ is a constant, and φ_0 is the phase of the particle at $t = 0$. Similarly, the homogeneous solution of (2) is $v_{z0} = V_z = \text{const}$.

To solve (1) and (2) we use an expansion procedure. We write

$$z = z_0 + V_z t + \int_0^t v_{z1}\,dt + \int_0^t v_{z2}\,dt + \cdots, \tag{3}$$

where z_0 is the position of the particle at $t = 0$, and v_{z1}, v_{z2}, etc. are, respectively, the first-order, second-order, etc. solutions for v_z. The ordering is done in terms of the parameter α, so that $v_{z1} \sim \alpha$, $v_{z2} \sim \alpha^2$, etc. Equation (3) is inserted into the argument, $kz - \omega t$, of the electromagnetic field components which are then expanded under the simplifying assumption

$$k\int_0^t v_{z1}\,dt + k\int_0^t v_{z2}\,dt + \cdots \ll 1. \tag{4}$$

The field components may also be ordered in terms of powers of α. Then by equating terms of