P_γ are sufficiently different for the different models so that experimental results, when available, will be able to discriminate among them.

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Slope Diffraction Scattering and a Remarkable Kinematic Variable

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Group-theoretical analysis suggests the introduction of a new kinematical variable. Data on diffraction scattering exhibit remarkable universality in this variable. An s-dependent slope formula is obtained which fits both K^+p and pp elastic scattering at *all* measured energies.

It is by now well known that the t-channel partial-wave helicity amplitudes for an arbitrary s-channel process

$$A+B\rightarrow C+D$$

have to satisfy equations of constraint at certain special kinematical points, e.g., at $t=0$.[1] The structure of these constraint equations is very sensitive to the masses of the external particles involved. In a group-theoretical approach[2] the constraint equations are a consequence of the change in the symmetry properties of the scattering amplitude under transformations which leave the vector $\Delta = p_A - p_C$ invariant as $t=\Delta^2$ varies. For example in elastic scattering in the s channel as $t\to 0$, Δ changes from a spacelike four-vector to a null vector, and the symmetry of the scattering amplitude "contracts" from O(2, 1) to O(3, 1). However, at $t=0$, for an inelastic process Δ depends upon the external mass situation. Strict little-group analysis at $t=0$ would imply Toller families for elastic scattering but representations of the two-dimensional Euclidean group[3]—a completely different object—for inelastic processes. However, even with spinless particles it is in just this *unequal-mass* situation that *analyticity* requires Toller-type families.[4]

To avoid this confusion and ambiguity we have sought and found a new momentum vector whose little-group structure is mass independent. It is the four-dimensional normal to the reaction plane,

$$n_\mu = 2\epsilon_{\mu\nu\rho\sigma} p_A{}^\nu p_B{}^\rho p_C{}^\sigma/(p_A+p_B)^2. \tag{1}$$

The normalization is *in principle* arbitrary (up to a scalar factor) but the above choice will be seen later to be extremely convenient. We note that

$$n^2 = -\varphi(s,t)/s^2, \tag{2}$$

where φ is the Kibble[5] function defining the physical region boundaries.

We come now to the crucial feature: n_μ is a *null* vector for forward scattering in *any* reaction, independently of the external masses. Thus the group analysis will be uniform for all physical processes.

We can now treat the scattering amplitude as a function of, say, n, $p=p_B+p_D$, and $q=(p_A+p_C)$ and perform a group-theoretic expansion with respect to the little groups of n. In the forward direction this is an O(3, 1) expansion. Of course $t=0$ is the forward direction for elastic scattering, while forward scattering for an inelastic process only has $t\to 0$ as $s\to\infty$. Thus we see that in the general mass cass the scattering amplitude does have Lorentz symmetry at $t=0$ *if the energy is asymptotic*, giving some additional justification to past treatments of this problem.

Let us return to our new little-group analysis. If when n is null we assume that the generalized partial-wave amplitudes have a Lorentz pole in the complex λ plane[6] at $\lambda = a(0)$ with Lorentz quantum number M, then detailed analysis,[7] allowing for arbitrary spins, shows that at high energies the forward scattering amplitude is dominated by this pole and has the following behavior:

$$f_{\lambda_C\lambda_D;\lambda_B\lambda_A}{}^s(s,n=0) \sim \delta_{\lambda_A-\lambda_C,\lambda_B-\lambda_D}(p\cdot q/|p||q|)^{a(0)-1-|M-|\lambda_A-\lambda_C||}. \tag{3}$$

To leading order in s this is in complete agreement with the results obtained in other group-theoretic treatments[1,2,8] and in the analyticity approach if $a(0)=\alpha(0)$. One of our Lorentz poles will of course give the well-known structure of daughter poles[9] in our j plane, but now in all mass configurations.

Within the s-channel physical region we can similarly perform an O(2, 1) expansion of the scattering amplitude since there $n^2<0$. If we assume that the "partial-wave" amplitudes have poles in the complex j plane, these will occur at $j=a(n^2)$ with residues $b(n^2)$. This will give us, in the case of spinless particle scattering in the forward hemisphere,

$$f(s,n^2) \sim b(n^2)(p\cdot q/|p||q|)^{a(n^2)} \tag{4}$$

for s tending to infinity along a constant-n^2 line.

We now require that our result reduce to the usual Regge asymptotic form for s large and t small, i.e.,

$$f(s,n^2) \sim b(n^2)s^{a(n^2)} \underset{t\ \text{small}}{\sim} \beta(t)s^{\alpha(t)}. \tag{5}$$

In fact our normalization (1) has just been designed to ensure that

$$n^2 \to t \text{ for } s\to\infty,\ t \text{ small}, \tag{6}$$

so that (5) is automatically consistent. It should be noted that although (6) holds asymptotically, n^2 is quite different from t at large angles and noninfinite s values.

We plotted differential cross-section data for $pp\to pp$ at all angles and laboratory momenta greater than 5 GeV/c as a function of s and n^2 and were amazed to discover that all the data fell essentially onto one universal n^2 curve. We then realized that for *equal-mass elastic scattering*, n^2 reduces exactly to the variable $\beta^2 p_\perp{}^2$ which was introduced by Krisch[10] on totally different physical grounds! It is thus possible to think of n^2 as a natural generalization of the Krisch variable to arbitrary-mass reactions. We have also plotted data for np, K^+p, and $\pi^\pm p$ elastic scattering, and φ photoproduction, and in each case the data are seen to cluster around a universal *function of* n^2 only.

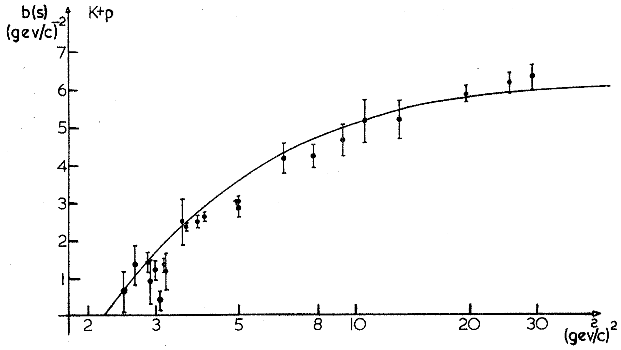

FIG. 1. A plot of the slope of the forward peak for K^+p elastic scattering [taken at $|t| \simeq 0.1\,(\mathrm{GeV}/c)^2$] as a function of s. The data points are from Ref. 11. The curve is the prediction of Eq. (9) with $\beta = 6.5$.

Since a single dominant pole will give

$$d\sigma/dt = A(n^2)s^{2[\alpha(n^2)-1]}, \tag{7}$$

we conclude that the leading pole has

$$\alpha(n^2) = 1.$$

The effect of nonleading poles appears of course in nondiffractive processes and also, for example, in $\pi^\pm p$ elastic scattering below 6 GeV/c. In the latter the large dips and bumps evident in $d\sigma/dt$ plotted as a function of t appear, when plotted against n^2, as small oscillations about the overwhelmingly dominant asymptotic $[\alpha(n^2) = 1]$ term. We shall not discuss the parametrization of these processes in this Letter, but turn rather to a most striking consequence of the n^2 universality in diffractive processes—namely, the formula for the slope of the forward peak.

If we put

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt}\right)_{n^2=0} \exp(\beta_0 n^2) = \left(\frac{d\sigma}{dt}\right)_{t=0} \exp[b(s)t], \tag{8}$$

then our data plots indicate that β_0 is a constant. Equation (8) then yields the slope formula (for $m + M \to m + M$)

$$b(s) = \beta_0[1 - 2(M^2 + m^2)/s + (M^2 - m^2)^2/s^2] \tag{9}$$

in contrast, e.g., to the Regge prediction

$$b(s) = b_0 + 2\alpha' \ln s. \tag{10}$$

A fit by eye to the pp and K^+p differential-cross-section data at laboratory momenta of 7 and 7.3 GeV/c, respectively, yields

$$\beta_0^{K^+p} = 6.5\ (\mathrm{GeV}/c)^{-2} \text{ and } \beta_0^{pp} = 11.5\ (\mathrm{GeV}/c)^{-2}.$$

We have plotted $b(s)$ as given by (9) for all values of s from threshold to 2000 $(\mathrm{GeV}/c)^2$ (see Figs. 1 and 2). For the pp case both the Serpukhov[13] and the new intersecting storage-ring[14] (ISR) results are included. Also shown is the Regge result[14] with $\alpha_p' = \frac{1}{2}$.

It is remarkable that $b(s)$ provides a reasonable interpolation over such a vast energy range. It is also worth noting that the term involving the mass differences $M^2 - m^2$ in (9) is essential in accounting for the difference between the pp and K^+p slopes at lower energies.

We have also studied the slope parameter in $\pi^\pm p$ and $\bar{p}p$ scattering. In these reactions the presence of low-energy resonances causes the slope to oscillate markedly and we find that our theoretical prediction gives a not unreasonable mean interpolation. However a detailed discussion is

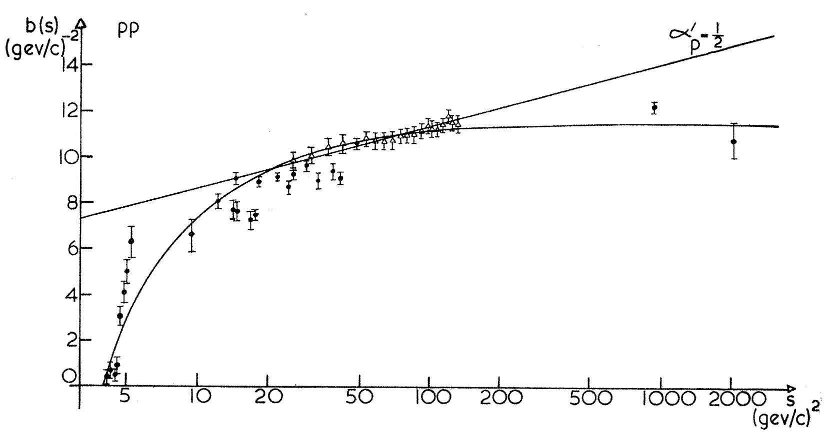

FIG. 2. A plot of the slope of the forward peak for pp elastic scattering [taken at $|t| \simeq 0.1\,(\mathrm{GeV}/c)^2$] as a function of s. The data points (closed circles) are from the Particle Data Group compilation (Ref. 12), triangles are the Serpukhov data (Ref. 13), and crosses the ISR results (Ref. 14). The curve is our prediction, Eq. (9) with $\beta = 11.5$, while the straight line is the Regge-pole prediction with $\alpha_p' = \frac{1}{2}$ given in Ref. 14.

deferred to a future publication.

It is clear that n^2 has remarkable properties as a kinematical variable and that it is meaningful to consider amplitudes as a function of s and n^2. It remains a mystery, however, as to why the dynamics seem to yield an s-independent result. Nevertheless we would conjecture that the dominant asymptotic form for all diffractive processes will turn out to be a function of n^2 only.

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