over the mobility calculations by Levine and Sanders[1] and by Harrison.[6] These authors assume for simplicity the existence of two well-defined electron states: free electron or bubble. Their results show a transition from classical mobility [Eq. (8)] to bubble mobility (≈ 0.1 cm^2/V sec) which is too abrupt. Their curves, if represented in Fig. 3, would look almost like step functions (see, for example, Fig. 11 in Ref. 1). We get better results at intermediate densities because (a) we have a more realistic picture of free electrons, taking into account scattering of low-energy electrons from fluctuations in V_{eff}, and (b) we do not restrict our pseudobubbles to the most probable radius, but carry out the appropriate thermal average which includes all possible pseudobubble sizes, each with its correct weight $n(E)\exp(-\beta E)=\exp[-\beta F(E)]$. This averaging process is important at intermediate densities, where pseudobubbles of different sizes may coexist.

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Feedback Suppression of a Large-Growth-Rate Flute Mode*

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A very large growth rate, large saturation amplitude, hard-onset flute mode in a multipolelike magnetic field is suppressed by a factor of 60 in power by negative current feedback to the plasma. Upon suppression of the instability, the center-line density increased 7%. The required feedback gain and phase are in rough agreement with a linear slab-model theory.

Stabilization of a variety of plasma instabilities has been obtained through use of externally modulated electron sinks immersed in the plasma in previous work. Most modes that have succumbed to external stabilization have been characterized by small growth rates,[1-3] including the flute mode of Ref. 3. This paper reports negative-feedback suppression of a grossly unstable magnetohydrodynamic (MHD) mode for which the growth time to nonlinear saturation is about a single perturbation period. This type of instability is a worst case for feedback stabilization.

The apparatus on which the experiment was carried out is shown in Fig. 1. The plasma is produced in a Lisitano-type electron cyclotron-resonance heating structure at one end of the machine. With a continuous microwave input of 50 W at 2.4 GHz, the density on the axis is 10^{10} cm^{-3}, the electron temperature is 6 eV, the ions are cold, and the background pressure is 4×10^{-5} Torr of hydrogen. The peak magnetic field on the axis is 1.4 kG, and the plasma-source collimator has a continuously adjustable aperture under vacuum.

If, for the magnetic field configuration of Fig. 1, the collimator edge lies outside the separatrix ψ_s, the column is found stable and $\delta n/n_0 < 2\%$. But as the collimator aperture is reduced, the fluctuation level increases abruptly to 30% on ψ_s (hard onset). This behavior is in rather dramatic accordance with MHD stability theory which predicts the flute instability for $(\partial p/\partial\psi)\times(\partial/\partial\psi)\oint ds/B<0$, where ψ is the flux function. Since $\oint ds/B$ diverges on the separatrix, the flute instability should occur if the pressure gradient is negative inside the separatrix. The advantage of this particular configuration is that as the collimator aperture is reduced, a discrete spectrum of large-growth-rate modes appears instead of the turbulence that usually accompanies gross in-

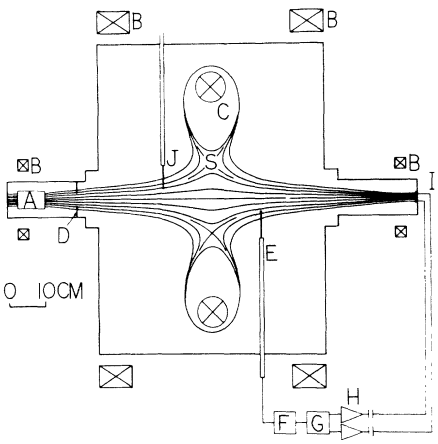

FIG. 1. Schematic diagram of experimental apparatus: A, Lisitano-coil plasma source; B, external magnetic-field coils; C, internal supported current-carrying hoop; D, variable collimator; E, sensor probe; F, bandpass filter; G, phase shifter, attenuator, and phase splitter; H, power amplifiers; I, suppressor electrodes; and J, diagnostic probe. The curved lines represent axially symmetric magnetic-flux surfaces, and S, is the separatrix.

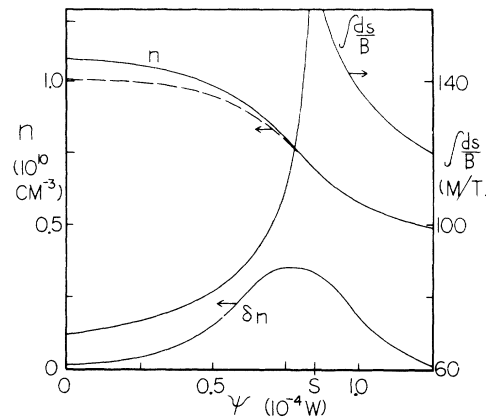

FIG. 2. Density, density perturbation, and $\oint ds/B$ as a function of ψ. The lower density curve is for the absence of negative feedback, and the upper curve is for 20-dB feedback suppression of the $m=1$ mode.

stability. The discrete spectrum can be used to test feedback theory.

Further measurements indicate that the instability is in fact a flute mode traveling azimuthally in the direction of the electron ∇B drift. The instability frequency is 18 kHz, including a Doppler shift of 8 kHz due to steady-state radial electric fields. The $m=1$ mode is usually predominant, but up to six harmonics could be measured on a spectrum analyzer. Figure 2 shows n_0, δn, and $\oint ds/B$ as a function of ψ. Waves of all m values have about the same radial dependence and appear to be standing waves in the radial direction.

MHD stability in the presence of current feedback can be considered by including a source term in the electron continuity equation in a "slab" model in the usual way[1,4]:

$$-i\delta n(\omega - kv_{0e}) - \frac{ik\varphi}{B}\frac{\partial n_0}{\partial x} - i\frac{n_0 k^2\varphi}{B\Omega_i}(\omega - kv_{0e}) = |\sigma| e^{i\theta}\delta n, \tag{1}$$

where $\varphi = \varphi_0 \exp[i(ky - \omega t)]$ is the electrostatic potential, v_{0e} is the ∇B drift, $|\sigma|$ is the feedback loop gain, and θ is the phase angle by which δn leads the feedback particle source. The dispersion relation follows directly from Poisson's equation; and, for $T_e > T_i = 0$, it yields the stability condition on $|\sigma|$ and θ:

$$-|\sigma|^2 e^{2i\theta} - 2i|\sigma| e^{i\theta}\left(kv_{0e} - \frac{2\Omega_i}{k}\frac{1}{n_0}\frac{\partial n_0}{\partial x}\right) + k^2 v_{0e}^2 - \frac{4\Omega_i v_{0e}}{n_0}\frac{\partial n_0}{\partial x} \geq 0. \tag{2}$$

This reduces to the conditions

$$\theta = -\tfrac{1}{2}\pi, \quad |\sigma| \geq 2\gamma_0^2/\omega_0, \tag{3}$$

where γ_0 and ω_0 are, respectively, the MHD growth rate and angular velocity in the absence of feedback. As Taylor and Lashmore-Davies[5] have pointed out, the phase requirement for this reactive type of instability is very stringent; but θ and $|\sigma|$ other than optimum can still reduce the growth rate, and partial suppression may then occur, depending on how the nonlinear amplitude-saturation mechanisms interact with the linear growth rate.

The predicted gain requirement can be compared with the experimental gain according to

$$i_f = e\int S_e dV = \frac{e|\sigma|}{\pi m}\int F(\psi)\delta n(\psi)\,d\psi, \tag{4}$$

where i_f is the measured feedback current and $F(\psi) = \oint ds/B$. Computer-generated values for the ψ-space integrals then provide the proportionality constant by which the gain is defined:

$$|\sigma| = \eta i_f/\delta n(\psi_s), \tag{5}$$

where

$$\eta = \left[\frac{e}{\pi m}\int F(\psi)\frac{\delta n(\psi)}{\delta n(\psi_s)}d\psi\right]^{-1}$$

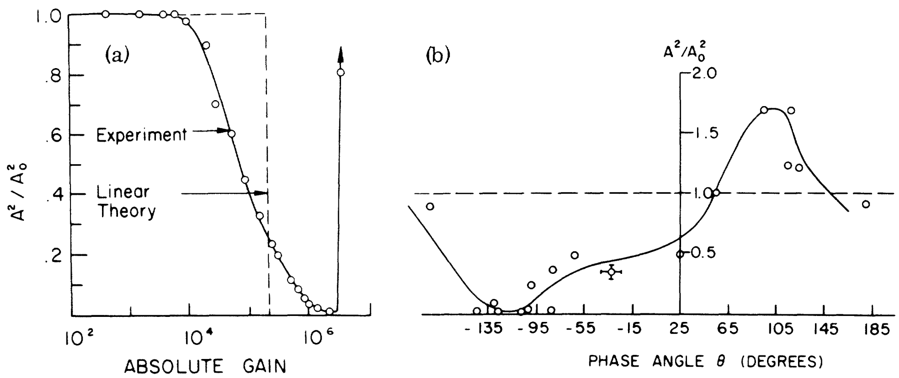

FIG. 3. Normalized wave amplitude squared as a function of (a) gain $|\sigma|$ for $\theta = -100°$ and (b) phase angle θ for $|\sigma| = 1.3\times 10^6$.

The feedback loop consists of a sensor probe biased to collect ions, a bandpass filter and phase shifter, amplifiers, and a pair of suppressor electrodes mounted outside the magnetic mirror on a window opposite the source (Fig. 1). The two suppressors are wedge shaped and cover alternate quadrants of the beam. They are capacitively coupled to separate amplifiers and driven 180° out of phase with each other. The filter is tuned to pass the fundamental frequency and is necessary to prevent excitation of very low-frequency fluctuations at high gain as well as dispersion in the phase shifter. Suppression is measured on other probes at different azimuths.

Figure 3(a) shows the normalized wave amplitude squared as a function of gain for fixed $\theta = -100°$. Substantial suppression occurs at the predicted gain $|\sigma| = 2\gamma_0^2/\omega_0 = 2.3\times 10^5\ \text{sec}^{-1}$, where γ_0 is measured directly by gating off the feedback voltage. However, complete stabilization did not occur even for gains a factor of 10 higher than predicted. This is probably because the theoretical model uses a constant fictitious gravitational acceleration to simulate ∇B drifts, while the wave-particle interactions that come from a distribution of drift velocities can have a destabilizing effect.[6] The amplitude squared as a function of phase angle θ, for fixed gain $|\sigma| = 1.3\times 10^6$, is shown in Fig. 3(b). The most effective phase angles lie between $-105°$ and $-125°$, in reasonable agreement with the predicted value of $-90°$. For optimum $|\sigma|$ and θ, the $m=1$ mode is suppressed 18 dB, or a factor of 60 in power. Higher modes do not grow as the fundamental declines.

The plasma radial loss due to $\vec{E}\times\vec{B}$ transport by the flute, integrated over the flux surface defined by the collimator, is $5\times 10^{16}\ \text{sec}^{-1}$, about the same as losses to the internal hoop supports and current feed. But microwave-cavity perturbation measurements of plasma lifetime indicate total losses of $5\times 10^{17}\ \text{sec}^{-1}$ in the absence of the wave, so the flute is not expected to be a dominant loss mechanism. In addition, the gain in confinement accrued by fluctuation suppression is offset by feedback-current losses. (These losses could be greatly diminished by the use of electron injection instead of the electron collection used in this work.) Nevertheless, the flute transport appears to be responsible for the 7% decrease in center-line density shown in Fig. 2 since the decrease is observed proportional to the square of the wave amplitude for both negative-feedback suppression and positive-feedback enhancement. But because of the poorly understood processes that occur in the source, it is difficult to say anything quantitative about the effect of feedback on confinement time.

This work demonstrates that negative-feedback suppression of a large-growth-rate, hard-onset, reactive instability can be achieved, that it requires a large gain, and that such suppression leads to improved confinement properties. Genuine stabilization, for which the feedback current drops to an infinitesimal level, occurs only for field configurations very near to the onset condition. At larger deviations from onset, even at the maximum suppression obtainable, the feedback current remains finite.

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Particle Containment in Mirror Traps in the Presence of Fluctuating Electric Fields*

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Particle motion in magnetohydrodynamically stable magnetic mirrors and in the high-frequency electric fields characteristic of the collective modes of such a system is shown to be restricted by two simple constraints. These constraints, which reduce to the usual magnetic moment and kinetic energy in the absence of fluctuating fields, dictate that while even small-amplitude waves induce large magnetic-moment excursions, the particle-containment properties of the trap are not greatly reduced by the fluctuations.

It is well known that the non-Maxwellian character of equilibrium plasmas in open-ended mirror machines induces numerous types of instabilities.[1] Suppression of all of these instabilities by machine design is not feasible, but what is more, experimental evidence indicates that particle containment is not always reduced by the presence of large-amplitude fluctuations.[2] In order to understand the data, and at the same time determine what kind of instabilities are most deleterious to confinement, a proper nonlinear analysis of ion motion in mirror geometry and finite-amplitude fluctuating electric fields is formulated. By the use of standard secular perturbation theory with the electric fields assumed to be given and of the resonant loss-cone type (frequencies close to an integral multiple of the ion cyclotron frequency), it is shown that while the magnetic moment undergoes large excursions, the interaction is only local. As a result, the ion, on transiting the mirror, forgets most of these deviations and reflects almost as though the plasma were fluctuation free.

For resonant modes the relevant ion motion is determined by introducing the usual guiding-center coordinates[3] and averaging the equations of motion over a cyclotron period. For a particle with phase-space coordinates $(\vec{r}, \vec{v})$ the substitutions $\vec{r}=\vec{R}+\vec{\rho}$, $\vec{v}=\vec{V}+\vec{\omega}$ are made where $\vec{R}$ and $\vec{V}$ are assumed slowly varying in a cyclotron period and $\vec{\rho}$, $\vec{\omega}$ are coordinates perpendicular to the local $\vec{B}$ field, which account for the modulated Larmor motion. The equations obtained are

$$m\frac{d\vec{V}}{dt}=e\langle\vec{E}\rangle+\left(\frac{e}{c}\right)\vec{V}\times\vec{B}(\vec{R})-\mu\nabla_{\vec{R}}B(\vec{R}), \tag{1a}$$

$$m\frac{d\vec{\omega}}{dt}=e(\vec{E}-\langle\vec{E}\rangle)+\frac{e}{c}\vec{V}\times(\vec{\rho}\cdot\nabla)\vec{B}(\vec{R})+\frac{e}{c}\vec{\omega}\times\vec{B}(\vec{R}). \tag{1b}$$

$\langle\vec{E}\rangle$ is the time-averaged electric field,

$$\langle\vec{E}\rangle=(\Omega/2\pi)\int_t^{t+2\pi/\Omega}dt'\,\vec{E}(\vec{R}(t')+\vec{\rho}(t'),t');$$

Ω is the ion-cyclotron frequency at $\vec{R}$; and μ is the magnetic moment, $\frac{1}{2}m\omega^2/B$. For convenience all terms corresponding to field curvature and twist will be neglected. This is sufficient for times of the order of a few transit periods. Equation (1a) then reduces to

$$m\frac{dv_{\parallel}}{dt}=e\langle E_{\parallel}\rangle-\mu\frac{\partial B}{\partial R_{\parallel}},\qquad \vec{V}_{\perp}=\frac{c\vec{B}}{B^2}\times\left(-\langle\vec{E}\rangle+\frac{\mu}{e}\nabla B\right). \tag{2a}$$

Introducing the Larmor radius ρ and the velocity-space angle θ for $\vec{\omega}$ as $\vec{\omega}=\rho\Omega(\cos\theta,\sin\theta)$, then $\vec{\rho}\simeq\rho(-\sin\theta,\cos\theta)$. With a right-handed set of orthonormal vectors, $\hat{e}_1$, $\hat{e}_2$, and $\hat{e}_{\parallel}=\vec{B}/B$, the relevant equations are obtained by taking the scalar product of $\vec{\omega}$ with (1b) and the $\hat{e}_{\parallel}$ component of the equation for $\vec{\omega}\times d\vec{\omega}/dt$. Averaging the resulting equations over a cyclotron period, with ρ and $d\theta/dt+\Omega$ varying slowly in this period and with the additional assumption of $\vec{E}=-\nabla\varphi$,