Lett. 26B, 161 (1968). This analysis includes theoretical smoothing, but the results are typical (see Ref. 7).

[9] M. Olsson and G. B. Yodh, Phys. Rev. Lett. 10, 353 (1963).

[10] P. Carruthers, in *Lectures in Theoretical Physics*, edited by W. E. Brittin *et al.* (Univ. of Colorado Press, Boulder, Colo., 1965), Vol. VII, pp. 82–135.

[11] There have been a number of other calculations of the D_{13} resonance, most including only the ρ-production mechanism. In general, these have been over a very narrow energy range, with many more parameters than we invoke here, and have not taken three-body effects explicitly into account [Ref. 2; Ref. 10; A. M. Gleeson and K. T. Mahanthappa, Nuovo Cimento 51A, 40 (1967); P. duT. van der Merwe, Phys. Rev. 145, 1257 (1966); and many others]. Lomon and Miller have studied the effects of $N\rho$, $\pi\Delta$, and $N\sigma$ intermediate states in the neighborhood of the D_{13} resonance [E. L. Lomon and A. T. Miller, Phys. Rev. Lett. 21, 1733 (1968); A. T. Miller and E. L. Lomon, Phys. Rev. D 2, 1245 (1970)].

[12] The connection of the ρ-production mechanism between the D_{13} and D_{33} channels was successfully used previously by H. Goldberg and E. L. Lomon [Phys. Rev. 134, B659 (1964)].

[13] R. Aaron, R. D. Amado and R. R. Silbar, Phys. Rev. Lett. 26, 407 (1970).

[14] Cf. J. J. Brehm and G. L. Kane, Phys. Rev. Lett. 17, 764 (1966); J. J. Brehm and L. F. Cook, Phys. Rev. 170, 1387 (1968).

New Form of Duality

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A new kind of finite-energy sum rule is proposed that new dual amplitudes must satisfy in addition to the usual finite-energy sum rules. This new form of duality is suggested by general considerations on the common properties of the Regge-behaved dual models. Some phenomenological implications of this new concept are briefly discussed.

In this Letter we propose a new kind of finite-energy sum rule (FESR) that new dual models must satisfy in addition to the usual FESR's. This new form of duality is suggested by general considerations of the Regge-behaved dual models and is seen to be related to the unitarity problem.

As a basic illustration, we confine ourselves to the simple pion-pion Veneziano amplitude[1,2]

$$A^{I_t=1}(s,t,u) = -\lambda\left\{\frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(1-\alpha(s)-\alpha(t))} - \frac{\Gamma(1-\alpha(u))\Gamma(1-\alpha(t))}{\Gamma(1-\alpha(u)-\alpha(t))}\right\} \tag{1a}$$

$$\sim \frac{\lambda}{\Gamma(\alpha(t))}\frac{\pi}{\sin\pi\alpha(t)}\left[(1-e^{-i\pi\alpha(t)})\left(\frac{\nu}{\nu_0}\right)^{\alpha(t)} + (1-e^{-i\pi[\alpha(t)-1]})\tfrac{1}{2}\alpha(t)(\alpha_0-1)\left(\frac{\nu}{\nu_0}\right)^{\alpha(t)-1} + \cdots\right], \tag{1b}$$

with $\nu=(4\mu)^{-1}(s-u)$, $\nu_0\equiv(2\mu\alpha')^{-1}$, and where we expanded the ratios of two Γ functions into asymptotic series for large values of s. We further assume that the above Regge asymptotic series is valid also on the positive real s axis.[3]

The essential point of our approach is to make use of the "dual" predictions derived from the Regge and resonance description of the dual model. We consider that the Regge form of the dual model, Eq. (1b), would give us information concerning unitarity which is lost in the resonance form, Eq. (1a).[4] We first investigate the physical implications contained in the Regge formula by the partial-wave analysis of the invidivual terms of Eq. (1b).[5] Since the validity of the Regge asymptotic expansion on the positive real s axis has been assumed, the resonance poles on the positive real s axis in Eq. (1a) must be shifted into the second sheet of the s plane in such a way as to produce Regge behavior on the positive real s axis. Therefore we consider that the above displaced resonance poles would appear as Argand-diagram circles in the partial-wave analysis of the Regge formula.

Thus, interpreting the Argand-diagram circles as those resonances displaced into the second sheet of the s plane, we find from the computer calculations the following results on the number and locations of the s-channel resonances generated by the Regge formula, as is indicated in Figs. 1 and 2:

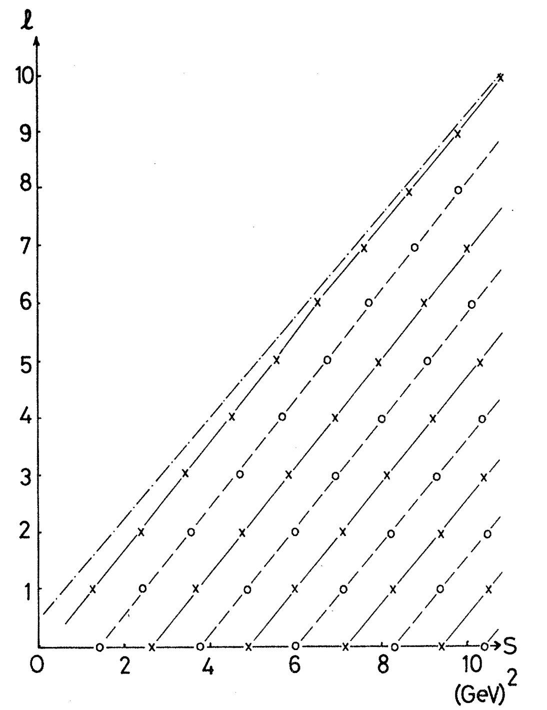

FIG. 1. Top positions of the Argand diagram circles: dot-dashed line, the input t-channel trajectory $\alpha(t) = 0.483 + 0.885t$; crosses, the output s-channel resonances on the leading and even daughter trajectories; circles, the output s-channel resonances on the odd daughter trajectories. By making use of the Regge asymptotic expansion Eq. (1b) (see Ref. 6) the actual numerical computer calculations were performed without approximations. Apart from the systematic distortion of the resonance pattern in the low-energy region, we can see the s-channel resonances on almost linearly rising trajectories with parallel daughter trajectories.

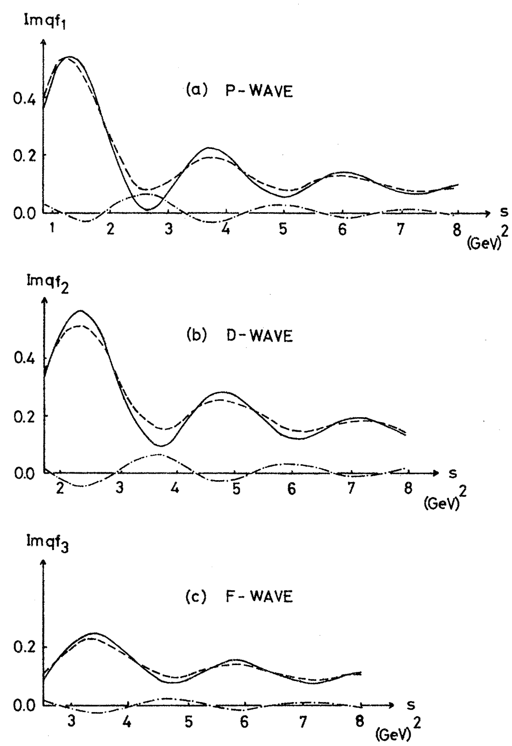

FIG. 2. Imaginary part of the partial wave amplitudes: solid line, imaginary part of the partial-wave decomposition of the leading Regge term proportional to $(\nu/\nu_0)^{\alpha(t)}$; dot-dashed line, imaginary part of the partial-wave decomposition of the first nonleading Regge term proportional to $(\nu/\nu_0)^{\alpha(t)-1}$; dashed line, sum of the contributions from the leading and the first nonleading Regge terms. It is to be noted that the sum of the contributions from the leading and the first nonleading Regge terms generates only half of the resonance spectrum (here q is the center-of-mass momentum).

(i) Each Regge term proportional to $(\nu/\nu_0)^{\alpha(t)-m}$ generates the s-channel resonances either on the even or on the odd daughter trajectories.

(ii) The sum of the terms proportional to $(\nu/\nu_0)^{\alpha(t)}$, $(\nu/\nu_0)^{\alpha(t)-1}$, $(\nu/\nu_0)^{\alpha(t)-2}$, $\cdots$ generates just half of the resonance spectrum, i.e., the s-channel resonances on the leading and the even daughter trajectories.

(iii) The sum of the terms proportional to $(\nu/\nu_0)^{\alpha(t)}$, $(\nu/\nu_0)^{\alpha(t)-2}$, $\cdots$ (which we call the even t-channel exchanges) generates the s-channel resonances on the leading and the even daughter trajectories.

(iv) The sum of the terms proportional to $(\nu/\nu_0)^{\alpha(t)-1}$, $(\nu/\nu_0)^{\alpha(t)-3}$, $\cdots$ (the odd t-channel exchanges) generates the s-channel resonances on the odd daughter trajectories.[6]

In order to confirm the above results of numerical calculations, we recall the arguments of Chiu and Kotanski,[7] who showed (i) that the usual Regge formula has an infinite number of s-channel resonances on linearly rising trajectories with parallel daughters spaced by *two* units of angular momenta,[8] and (ii) that for fixed l, the diameters of the Argand-diagram circles, i.e., the elasticities of resonances, decrease like s^{-s} with increasing s. The absence of the even (or odd) system of trajectories is the most crucial point.[9]

Thus we could conclude that the even (odd) t-channel exchanges generate the s-channel resonances on the even (odd) daughter trajectories through partial-wave analysis. Conversely, we require[10] that the s-channel resonances on the even (odd) daughter trajectories build up the even (odd) t-channel exchanges via FESR's. Then we are led to the following FESR's of a new kind:

$$\frac{1}{\nu_N}\int_0^{\nu_N} d\nu \,\mathrm{Im}A_{\mathrm{even}}{}^{I_t=1}(\nu,t) = \beta_{\mathrm{even}}(t)\frac{(\nu_N/\nu_0)^{\alpha(t)}}{\alpha(t)+1}, \qquad (2a)$$

$$\frac{1}{\nu_N}\int_0^{\nu_N} d\nu \,\mathrm{Im}A_{\mathrm{odd}}{}^{I_t=1}(\nu,t) = \beta_{\mathrm{odd}}(t)\frac{(\nu_N/\nu_0)^{\alpha(t)-1}}{\alpha(t)}. \qquad (2b)$$

Here $\mathrm{Im}A_{\mathrm{even}}{}^{I_t=1}(\nu,t)$ $[\mathrm{Im}A_{\mathrm{odd}}{}^{I_t=1}(\nu,t)]$ involves all resonances on the even (odd) alternate trajectories. It will be important to notice here that the even and odd systems of Regge trajectories belong to different Toller families, so the separation into two families of trajectories is quite consistent with (and even suggested by) the $t=0$ analyticity and $O(4)$ symmetry arguments.[11]

So far we have confined ourselves only to the pion-pion scattering case; however, we can easily find that the above arguments are not at all restricted to the pion-pion case but have general validity to the Regge-behaved dual models and, therefore, we consider the solutions that satisfy the new FESR's, Eqs. (2a) and (2b), as the expression of a new form of duality. It will also be important to notice here that a single Veneziano term like $\Gamma(n-\alpha(s))\Gamma(m-\alpha(t))/\Gamma(l-\alpha(s)-\alpha(t))$ is, of course, a solution of the usual FESR obtained by adding both sides of Eqs. (2a) and (2b), but it does not automatically satisfy the new FESR's.[12]

The solutions of the new FESR bootstrap can be constructed, in the narrow-resonance approximation, by superposing the Veneziano terms and eliminating the alternative trajectories in an iterative way,[13] more details of which are deferred to forthcoming papers.

As we have seen, the concept of the new form of duality has been introduced in our attempt at matching the "dual" predictions from the two complementary aspects of dual models, i.e., the resonance form, Eq. (1a), and the Regge form, Eq. (1b). Since the real physical world could be considered to lie between these two extremes of dual models, we hope that the concept of the new form of duality may be of use in the construction of more realistic dual models and in the deeper understanding of the duality structure in nature.

It will be worthwhile to give a few concluding remarks.

(i) Our dual amplitude for the even family of trajectories can be expanded as a sum of the infinite number of s-channel poles as well as t-channel poles on the even (odd) family of trajectories. In the narrow resonance limit, the amplitude for the even family of trajectories will be of the form

$$\sum_{N=1}\sum_{m=0}\frac{\gamma_{N,N-2m}(t)}{N-\alpha(s)}=\sum_{N=1}\sum_{m=0}\frac{\gamma_{N,N-2m}(s)}{N-\alpha(t)},$$

while the amplitude for the odd family of trajectories will be

$$\sum_{N=1}\sum_{m=0}\frac{\gamma_{N,N-2m-1}(t)}{N-\alpha(s)}=\sum_{N=1}\sum_{m=0}\frac{\gamma_{N,N-2m-1}(s)}{N-\alpha(t)}.$$

(ii) s-channel resonances on the odd daughter trajectories produce only nonleading Regge terms via the new FESR and, therefore, they couple only weakly[14] to the process as compared with the resonances on the even daughter trajectories.

(iii) As we noted before, the elasticities of the resonances diminish qualitatively like s^{-s} as we increase s for fixed l, resulting in very broad resonances even at relatively low values of s, too broad even to be called actual resonances. On the other hand, any Veneziano formula has exponentially decreasing partial widths[15] with fixed s and increasing l due to the high centrifugal barriers. Therefore the total widths of a few highest-lying trajectories are far smaller than those of the bulk of the lower-lying daughter trajectories and remain relatively small even for fairly high excited levels. These results would throw some criticism on the narrow-resonance-approximation philosophy which has been governing us these last few years.

[1]G. Veneziano, Nuovo Cimento 57A, 190 (1968).

[2]C. Lovelace, Phys. Lett. 28B, 264 (1968); J. Shapiro, Phys. Rev. 179, 1345 (1969).

[3]To be more precise, we first approximate the asymptotic series by taking its first several leading terms in the region $|\arg s| > \epsilon$. Then, making use of them, we extropolate to the positive real s axis.

[4]We note that if the Regge amplitude approximates the real physical amplitude at high energy, which we are assuming to be the case, it has to satisfy unitarity approximately there.

[5]C. Schmid, Phys. Rev. Lett. 20, 689 (1968).

[6]In practice we explicitly calculated up to the term proportional to $(\nu/\nu_0)^{\alpha(t)-3}$. Calculation of further nonasymptotic terms would require only more patience, but it is irrelevant since we are dealing with an asymptotic expansion.

[7]C. B. Chiu and A. Kotanski, Nucl. Phys. B8, 553

(1968). See also V. A. Alessandrini, P. G. O. Freund, R. Oehme, and E. J. Squires, Phys. Lett. 27B, 456 (1968), especially Eq. (5).

[8]The double spacing of daughter trajectories has also been noted in H. R. Rubinstein, A. Schwimmer, G. Veneziano, and M. A. Virasoro, Phys. Rev. Lett. 21, 491 (1968); M. Ademollo, H. R. Rubinstein, G. Veneziano, and M. A. Virasoro, Phys. Rev. 176, 1904 (1968).

[9]This fact is simply due to the peculiar property of the Regge residue function $1/\Gamma(\alpha(t))$ having alternate signs on the negative-$\alpha(t)$ axis.

[10]This requirement is strongly suggested by the concept of duality. For example, if the amplitude is written as a sum of the Veneziano terms, it is easy to see that the elimination of the terms proportional to $(\nu/\nu_0)^{\alpha(t)-1}$, $(\nu/\nu_0)^{\alpha(t)-3}$, $\cdots$ $[(\nu/\nu_0)^{\alpha(t)}$, $(\nu/\nu_0)^{\alpha(t)-2}$, $\cdots]$ in the asymptotic expansion automatically implies the elimination of the s-channel resonances on the odd (even) daughter trajectories.

[11]D. Z. Freedman and J. M. Wang, Phys. Rev. 153, 1597 (1967); M. Toller, Nuovo Cimento 53A, 671 (1967).

[12]This point is easy to understand if we consider the amplitude $\Gamma(m-\alpha(s))\Gamma(m-\alpha(t))/\Gamma(2m-1-\alpha(s)-\alpha(t))$ whose resonances are all ghosts asymptotically on alternate trajectories [see Y. Nambu and P. Frampton, in *Essays in Theoretical Physics Dedicated to Gregor Wentzel* (Univ. of Chicago Press, Chicago, Ill., 1969), p. 403] except for the $m=1$ case. This amplitude satisfies the usual FESR with the negative contribution of ghosts included to balance the equation. Therefore the left-hand side of Eq. (2a) cannot balance the right-hand side since the negative contribution of ghosts has been removed from it.

[13]Difficult convergence problems of the infinite Veneziano series would occur if we consider the limit $s\to\infty$. We shall restrict ourselves to $|s|<N$ according to the spirit of FESR bootstrap throughout our approach.

[14]As one may note from Eqs. (2a) and (2b), asymptotically $\Gamma_{N,N-2m-1}{}^{el}/\Gamma_{N,N-2m}{}^{el}\sim O(N)^{-1}$, where $\Gamma_{N,l}{}^{el}$ is the elastic width of the resonance with mass $m_N=[(N-\alpha_0)/\alpha']^{1/2}$ and spin l (the positivity of each resonance residue is assumed).

[15]Nambu and Frampton, Ref. 12.

Examination of Parity Nonconservation by Means of Radiative Neutron-Deuteron Capture*

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We report results for the circular polarization of photons emitted in the reaction $n+d\to H^3+\gamma$ arising from parity nonconservation in the weak nucleon-nucleon interaction. Polarizations are found to be of the order 10^{-6}.

We have attempted to discriminate among models currently in vogue, for the weak, parity nonconserving nucleon-nucleon interaction[1-5] $V_{12}{}^w$ by calculating the circular polarization P_γ in the reaction $n+d\to H^3+\gamma$. P_γ is a manifestation of the presence of a weak component in the nucleon-nucleon force[6,7] which, as is the case with the strong component, arises from the exchange of the mesons $\pi^\pm$, $\rho^\pm$, ρ^0, ω, and φ between nucleons. The models of $V_{12}{}^w$ under investigation are described in Ref. 3.

In the absence of the weak, nucleon-nucleon interaction, the capture of thermal neutrons by deuterons goes mainly through the $M1$ transition. The weak interaction admixes odd-parity states in the initial and final nuclear systems and, therefore, allows for neutron capture through an $E1$ transition as well. P_γ is defined by

$$P_\gamma=\frac{d\sigma^+-d\sigma^-}{d\sigma^++d\sigma^-}=\frac{E^DM^M+E^QM^Q}{(M^D)^2+(M^Q)^2}, \qquad (1)$$

where $d\sigma^+$ and $d\sigma^-$ are the cross sections for right- and left-circularly polarized photons, respectively, and E, M are the electric and magnetic dipole transition matrix elements from the doublet (D) and quartet (Q) initial spin states.

The investigation of parity nonconservation in $n+d\to H^3+\gamma$ presents some unique advantages over similar investigations in heavy and intermediate nuclei[8-12] and in the reaction $n+p\to d+\gamma$.[13-17] In the former case there are difficulties due to uncertainties in the nuclear wave functions.[8,9] In the latter case the parity nonconserving effects are extremely small[15,16] and, hence, cannot be measured experimentally. For example, the circular polarization of photons following n-p capture, calculated with wave functions which are correlated at short range, was found to be of order 10^{-8} to 10^{-9}.[15,16] On the other hand, it is expected that P_γ for n-d radiative capture will be considerably larger.[13] The reason is that in this reaction the regular $M1$ transition proceeds via the smallest component of the triton, namely, the S^1 state only,[18] while the irreg-