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Inelastic Effects and π-N Resonances

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Inelastic effects (ρ production, Δ production) and particle-exchange effects are shown to determine the major features of the π-N D_{13}, D_{33}, and P_{13} waves. The $D_{33}(1670)$ is associated with a resonance pole but the $P_{13}(1860)$ is not.

A rapidly opening inelastic process can produce a resonance.[1] In the π-N system the connection of the $D_{13}(1520)$ resonance with ρ production has been known for some time.[2] Similarly the importance of particle-exchange or potential terms has been known since Chew and Low first related the $P_{33}(1238)$ and nucleon exchange.[3] However, there have been few attempts to combine such effects in a systematic dynamical calculation in the π-N system, in part because of the absence of a tractable dynamical framework in which to imbed these effects. Recently such a framework has been developed[4-6] and in this Letter we apply it with remarkable success to a calculation of the D_{13}, D_{33}, and P_{13} π-N amplitudes. We find that ρ and Δ production dominate the D_{13} channel, whereas in the D_{33} and P_{13} channels exchange and production effects combine. In all cases we obtain excellent agreement with experiment with reasonable values of our parameters. Phase-shift analyses of the π-N system yield $D_{33}(1670)$ and $P_{13}(1860)$ resonances of doubtful pedigree.[7] We find a clear resonance in the D_{33} state and equally clear evidence of *no* resonance in the P_{13} state near 1900 MeV. In the P_{33} state we find that nucleon exchange is an important ingredient in producing the well-known $\Delta(1238)$, but that higher inelastic states are also quite important. We also discuss the $S_{11}(1700)$ resonance.

Since we are dealing with the π-N system in an energy region dominated by a few strong inelastic processes, it seems clear that we must include these in a unitary way. At the same time we must include particle exchange or potential effects since all π-N waves are not driven purely by inelastic channels. The dynamical scheme into which we put these effects must be one that deals correctly with unitarity and with the dynamical driving terms. Such a scheme is provided by relativistic three-body equations based on the method of Blankenbecler and Sugar.[4] These equations have two- and three-body unitarity exactly and no further scattering singularities. The finite widths of resonances produced are included in a natural manner. We view the BS equations as a new relativistic dynamical scheme designed to deal exactly with a few degrees of freedom of the system while freezing out into "instantaneous potentials" all other degrees of freedom. The BS formalism allows any "potential" to be added as a left-hand cut, either phenomenologically or from some theory.

In the medium-energy π-N system ($T_{1\text{ab}} \lesssim 2$ BeV), the dominant inelastic processes are ρ production with threshold at 1700 MeV and $\Delta(1238)$ production with threshold at 1380 MeV. The π-N channels corresponding to S-wave production of the above particles are most likely to exhibit dramatic effects of inelasticity. These are S and D waves for ρ production and D waves for Δ production. P-wave production can also significantly affect elastic channels as we shall see in discus-

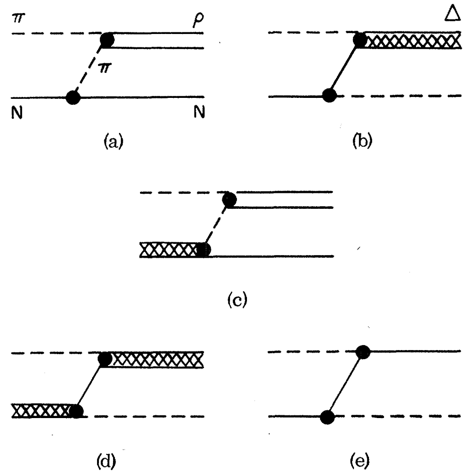

FIG. 1. Diagrammatic representations of the driving terms for our equations.

sion of the π-N P waves. The mechanisms included in our theory are shown diagrammatically in Fig. 1. We incorporate these diagrams into the BS framework and solve numerically the resulting three coupled linear integral equations. The treatment of Figs. 1(a)–1(e) has been given before; the inclusion of $\pi\Delta$ in our formalism is a new technical feature and will be discussed further in a more extensive paper. With the above driving terms our equations become a unitary, off-shell, linear, isobar model. Since all the coupling constants, resonance positions, and widths needed to construct the Born terms are known, there are no unknown constants except for cutoff parameters associated with each vertex. These are needed to make the integral equations be of Fredholm type, and also serve to give the vertices the finite size they presumably have. The detailed results of the calculation are sensitive to the cutoff parameters, although all reasonable values of the cutoffs usually lead to similar physical consequences. One expects that the results should depend in detail on the details of the interaction size, etc. In a true first-principles calculation, there should be no such free parameters, but we do not know how to do one. The best we can hope for here is that the number of parameters should be small, that their physical significance should be clear, and that the values needed to fit the data should be reasonable.

The results of our calculation of the D_{13} wave including ρ and Δ production, but with no phenomenological left-hand cuts, are shown in Fig. 2(a) where they are compared with a recent phase-shift analysis.[8] As can be seen, the fit is excellent over a wide energy region. The cutoff values needed to obtain the fits in Fig. 2(a) are all around $60m_\pi^2$, a reasonable value. We find that ρ production is the primary cause of the resonance, but Δ production is needed as well to get the details, particularly the dip in the inelasticity. The importance of Δ production is also seen empirically in the π-production analysis of Olsson and Yodh.[9] It is not surprising that we get such good results. We have included the important production mechanisms while, as has been shown by Carruthers,[10] the sum of the particle-exchange Born terms that we have neglected (e.g., ρ and Δ

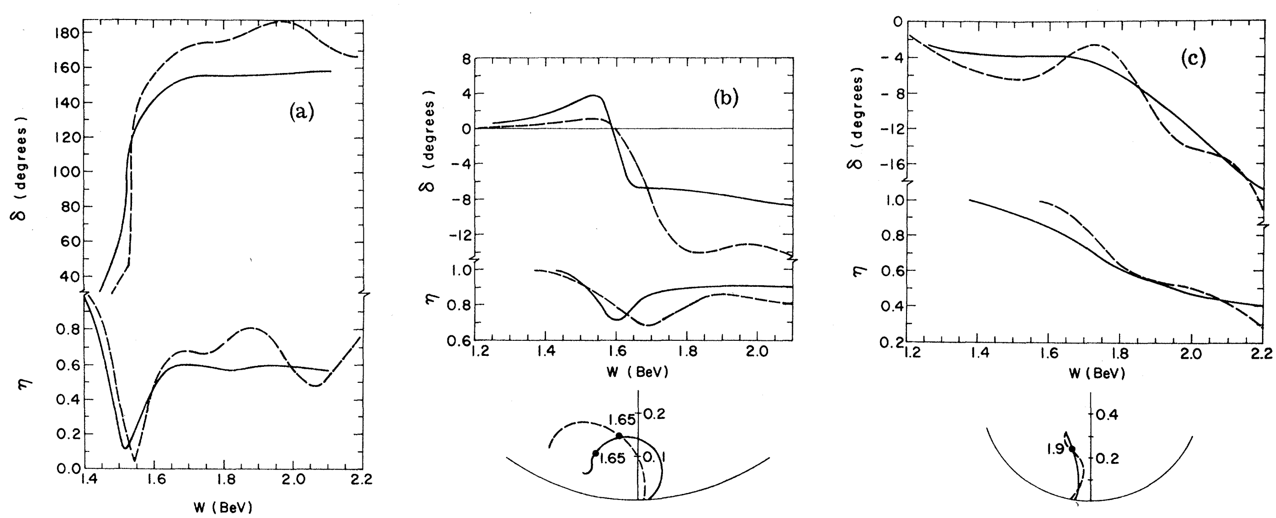

FIG. 2. The phase shift and inelasticity parameter for the (a) D_{13}, (b) D_{33}, and (c) P_{13} π-N states. The dashed curve is the CERN theoretical phase-shift analysis, Ref. 8; the solid curve, our calculation. The calculated Argand plots are also shown for D_{33} and P_{13} compared with the CERN analysis.

exchange) is quite small in this channel.[11]

The D_{33} wave is characterized by nearly no scattering to about 1600 MeV and then a sharp *drop* of the phase shift and accompanying dip in the inelasticity parameter near 1700 MeV, corresponding to a resonance-like loop in an Argand diagram. In this channel Carruthers[10] found that the sum of the potential terms is rather strongly repulsive. Our calculation including a phenomenological repulsion (as a left-hand cut) and Δ and ρ production in a BS framework gives the results shown in Fig. 2(b).[12] The attractive effects of the production mechanism below threshold are canceled by the repulsive left-hand cut. Above threshold this cut cannot cancel production and the inelasticity dips. Empirical phase-shift analyses differ on the question of whether this Argand loop is a resonance or not.[7] Since we have a dynamical model we can investigate this question. We find a clear zero of the real part of the denominator function associated with the Argand loop and conclude that the $D_{33}(1670)$ is a resonance.

In the P_{13} channel we once again obtain results which agree well with the recent phase-shift analysis as can be seen in Fig. 2(c). As in the case of the D_{33}, ρ and Δ production combine with repulsive exchange effects to produce these results. Here also phase-shift analyses differ on the question of whether or not the excursion in the Argand plot corresponds to a resonance around 1860 MeV.[7] We find that the real part of our denominator function is nearly constant near unity in this region and hence, within our dynamical framework, there is definitely no resonance in the P_{13} state near 1900 MeV connected with the phase-shift behavior shown in Fig. 2(c). In the P_{13} state ρ and Δ production occur in P waves rather than S waves as in the D_{33}. This may well account for the difference in the resonance behavior of the two states.

We comment briefly upon other results. We obtain the familiar $P_{33}(1238)$ resonance with nucleon exchange an important ingredient but find that Δ production itself in addition to ρ production is also important and, in particular, affects the inelasticity. It seems to be quite difficult (in our framework or any other) to make the phase shift "hang up" near 180° at higher energies as it does experimentally. Hence, the gross features of the P_{33} wave may be simple, but its details are not, and inelasticity seems again to play an important role. In the S_{11} channel there is experimental evidence of resonances at 1535 MeV and 1700 MeV.[7] The former has been related to η production (threshold 1480 MeV)—We find that the 1700-MeV resonance is probably driven by ρ production. The mechanism of Fig. 1(a) can occur in S waves in this channel, and is large in $I=\frac{1}{2}$. Δ production occurs first in D waves and we neglect it. We have done a calculation including ρ production, but without η production. We do not obtain detailed agreement, but the presence of a resonance is unmistakable and it stays over a wide range of parameters. This resonance and the D_{13} are the strangeness-zero analogs of the exotic resonances (Z^*'s) we predicted earlier.[13]

We are presently investigating a large number of other states, including strange particles, to see if we can extend to them the ideas presented here. It seems surprising that although these inelastic mechanisms have been known for some time, they have not been used even qualitatively to try to correlate the vast amount of resonance data. We believe we have shown here that detailed calculations can be done and sensible answers emerge from trying to correlate the effects of inelastic channels and particle exchange in the π-N system.

Other authors have tried to generalize to SU(3) the mecahnism of Fig. 1(a),[10,14] but its large effect may depend on the small pion mass and hence imposing SU(3) may unnecessarily restrict or prejudge the outcome of a calculation. We wish to stress that the inelastic effects we consider are present—through unitarity—in any scheme. What their connection is with symmetry notions is an interesting question we would like to be able to answer.

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[11]There have been a number of other calculations of the D_{13} resonance, most including only the ρ-production mechanism. In general, these have been over a very narrow energy range, with many more parameters than we invoke here, and have not taken three-body effects explicitly into account [Ref. 2; Ref. 10; A. M. Gleeson and K. T. Mahanthappa, Nuovo Cimento 51A, 40 (1967); P. duT. van der Merwe, Phys. Rev. 145, 1257 (1966); and many others]. Lomon and Miller have studied the effects of $N\rho$, $\pi\Delta$, and $N\sigma$ intermediate states in the neighborhood of the D_{13} resonance [E. L. Lomon and A. T. Miller, Phys. Rev. Lett. 21, 1733 (1968); A. T. Miller and E. L. Lomon, Phys. Rev. D 2, 1245 (1970)].

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New Form of Duality

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A new kind of finite-energy sum rule is proposed that new dual amplitudes must satisfy in addition to the usual finite-energy sum rules. This new form of duality is suggested by general considerations on the common properties of the Regge-behaved dual models. Some phenomenological implications of this new concept are briefly discussed.

In this Letter we propose a new kind of finite-energy sum rule (FESR) that new dual models must satisfy in addition to the usual FESR's. This new form of duality is suggested by general considerations of the Regge-behaved dual models and is seen to be related to the unitarity problem.

As a basic illustration, we confine ourselves to the simple pion-pion Veneziano amplitude[1,2]

$$A^{I_t=1}(s,t,u) = -\lambda\left\{\frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(1-\alpha(s)-\alpha(t))} - \frac{\Gamma(1-\alpha(u))\Gamma(1-\alpha(t))}{\Gamma(1-\alpha(u)-\alpha(t))}\right\} \tag{1a}$$

$$\sim \frac{\lambda}{\Gamma(\alpha(t))}\frac{\pi}{\sin\pi\alpha(t)}\left[(1-e^{-i\pi\alpha(t)})\left(\frac{\nu}{\nu_0}\right)^{\alpha(t)} + (1-e^{-i\pi[\alpha(t)-1]})\tfrac{1}{2}\alpha(t)(\alpha_0-1)\left(\frac{\nu}{\nu_0}\right)^{\alpha(t)-1} + \cdots\right], \tag{1b}$$

with $\nu=(4\mu)^{-1}(s-u)$, $\nu_0\equiv(2\mu\alpha')^{-1}$, and where we expanded the ratios of two Γ functions into asymptotic series for large values of s. We further assume that the above Regge asymptotic series is valid also on the positive real s axis.[3]

The essential point of our approach is to make use of the "dual" predictions derived from the Regge and resonance description of the dual model. We consider that the Regge form of the dual model, Eq. (1b), would give us information concerning unitarity which is lost in the resonance form, Eq. (1a).[4] We first investigate the physical implications contained in the Regge formula by the partial-wave analysis of the invidivual terms of Eq. (1b).[5] Since the validity of the Regge asymptotic expansion on the positive real s axis has been assumed, the resonance poles on the positive real s axis in Eq. (1a) must be shifted into the second sheet of the s plane in such a way as to produce Regge behavior on the positive real s axis. Therefore we consider that the above displaced resonance poles would appear as Argand-diagram circles in the partial-wave analysis of the Regge formula.

Thus, interpreting the Argand-diagram circles as those resonances displaced into the second sheet of the s plane, we find from the computer calculations the following results on the number and locations of the s-channel resonances generated by the Regge formula, as is indicated in Figs. 1 and 2: