with electron measurements at lower energies (~ 3 GeV), a differential spectral index at the source of ~ 2.4, and the upper limit on sidereal anisotropy of cosmic rays. Within this model, Vela X produces about 10% of the observed electrons at 750 GeV with the remaining flux being produced by local supernovae with observable radio remnants.

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Single-Particle Distributions of π Mesons Produced in K^-p Interactions at 9 GeV/c*

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We present distributions of transverse momentum squared and of longitudinal momentum in the laboratory, center-of-mass, and projectile frames for pions produced in K^-p interactions at 9 GeV/c. Comparisons made with the corresponding distributions published by other laboratories for pion production in K^+p, π^+p, pp, and π^-p interactions indicate that in the proton fragmentation region the distributions for pions produced from K^-p interactions at 9 GeV/c have not yet reached a limiting behavior.

A number of physicists have suggested that the cross section $(d\sigma/dP_{\parallel}dP_{\perp}^2)$ for producing a given type of particle at a given point in momentum space approaches a limit as the energy of the interaction increases.[1] Here $P_{\parallel}$ indicates the momentum of the particle parallel to the beam and $P_{\perp}$, the momentum perpendicular to the beam. There has been much speculation on the rate at which this limit should be approached for various types of interactions. With apologies to the theorists who would like to see the invariant distributions, we present some single-particle distributions for pions produced in K^-p interactions at 9 GeV/c in a form that facilitates comparison with corresponding distributions published by other laboratories for pion production in K^+p, π^+p, pp, and π^-p interactions.[2-4]

The data for the K^-p interactions at 9 GeV/c are from an exposure of 100 000 pictures taken in the Brookhaven National Laboratory (BNL) 80-in. bubble chamber. The film has been scanned twice and events of all topologies have been measured. The plots we present represent about half of our available data. The statistical errors in the regions of high statistics are typically less than 5%. The uncertainties indicated for our cross sections reflect a 15% systematic uncertainty as well as the statistical uncertainties.

In estimating the systematic uncertainty, we have considered scanning and measuring losses, as well as uncertainties in beam contamination, beam count, hydrogen density, and contamination of the pion spectra with kaons or with protons. We minimize the problem of contamination by considering π^- slow in the laboratory frame and π^+ slow in the projectile frame, since the K^- tends to be produced fast and the proton slow in the laboratory. We estimated the contamination from slow K^- and fast protons by examining the momentum distributions of K^- and of protons identified from four-constraint–fitted events. We also studied the momentum distributions of all neutral kaons, making comparisons of momentum distributions of neutral and negative kaons from fitted events with the same number of particles in the final state, and using statistical weights based on isotopic spin and the number of independent degenerate states.[5] From these studies we found that the contamination of the pion spectra is less than 10% for $|P_{\parallel}|<1$ GeV/c, regions of momentum space which may be considered target fragmentation or beam fragmentation. We have made a small correction to the π^- spectra for slow K^-. No correction to the π^+ spectra has been made for fast protons.

In Fig. 1 is presented the longitudinal momentum distribution for π^- in the laboratory frame. The curves drawn on the graph are polynomial fits for distributions presented in the BNL, Rochester, Wisconsin collaboration of Ref. 2. The curves for $X+P\to\pi+$anything have been normalized by the ratio of the asymptotic values of the total cross sections, $\sigma_{K^-p}/\sigma_{Xp}$.[6] This normalization is used since it has been predicted[7] that the limit approached for single-particle cross sections from various interactions should be proportional to the asymptotic total cross section. In Fig. 2 is presented the distribution in transverse momentum squared for negative pions whose longitudinal momenta in the laboratory are less than 0.5 GeV/c. This cut tends to isolate the pion fragments from the target.

The comparison made in Ref. 2 indicates a possible limiting behavior in the proton fragmentation region already at moderate energies for the channels

$$K^+ + p \to \pi^- + \text{anything}, \quad p+p\to\pi^- + \text{anything},$$
$$\pi^+ + p \to \pi^- + \text{anything}.$$

They found that the normalized cross sections for small longitudinal momentum for these reac-

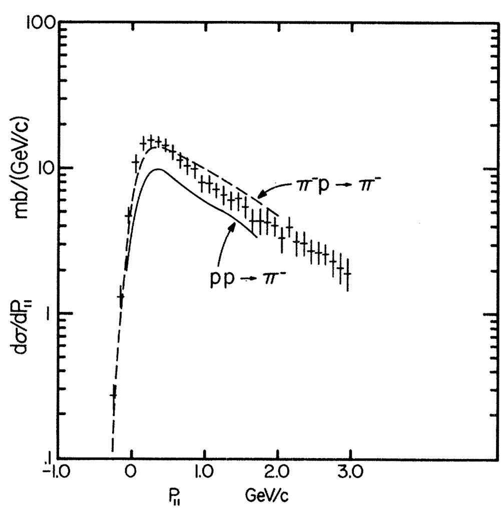

FIG. 1. Longitudinal momentum distribution in the laboratory system for $K^-+p\to\pi^-+$anything at 9 GeV/c. Solid line is the normalized distribution for $p+p\to\pi^-$ +anything, and for small values of longitudinal momentum it also roughly represents the normalized distributions for $K^++p\to\pi^-+$anything and $\pi^++p\to\pi^-+$anything. Dashed line is the normalized distribution for $\pi^-+p\to\pi^-$ +anything (Ref. 2).

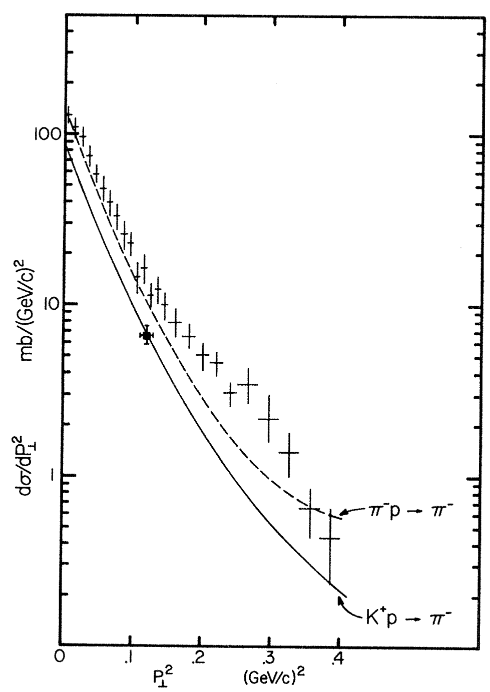

FIG. 2. Distribution in transverse momentum squared for $K^-+p\to\pi^-+$anything at 9 GeV/c for longitudinal momentum in the laboratory less than 0.5 GeV/c. For the same cut in longitudinal momentum, the solid line represents the normalized distribution for $K^++p\to\pi^-+$anything and the dashed line represents the normalized distribution for $\pi^-+p\to\pi^-+$anything (Ref. 2).

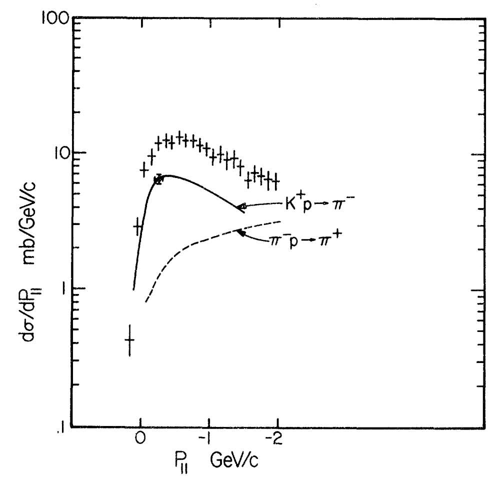

FIG. 3. Longitudinal momentum distribution in the projectile frame for $K^-+p\rightarrow\pi^++$anything at 9 GeV/c. Solid line is the normalized distribution for $K^++p\rightarrow\pi^-$ +anything. Dashed line is the normalized distribution for $\pi^-+p\rightarrow\pi^++$anything.

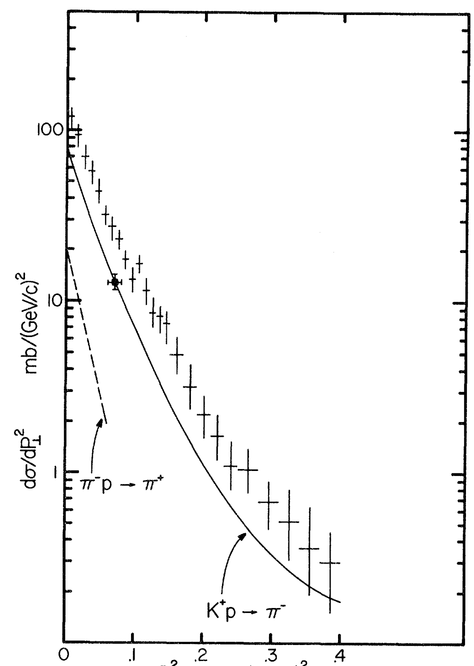

FIG. 4. Distribution in transverse momentum squared for $K^-+p\rightarrow\pi^++$anything at 9 GeV/c for absolute value of the longitudinal momentum in the projectile less than 0.5 GeV/c. For the same cut in longitudinal momentum, the solid line represents the normalized distribution for $K^++p\rightarrow\pi^-+$anything and the dashed line represents the normalized distribution for $\pi^-+p\rightarrow\pi^+$ +anything (Ref. 2).

tions are in agreement with each other, while the normalized cross section for $\pi^-+p\rightarrow\pi^-+$anything lies above the others. We find that for small values of longitudinal momentum of the negative pion in the laboratory, $p_\parallel \leq 0.5$ GeV/c, the pion distributions from K^-p interaction lie about 50% higher than the normalized distributions from K^+p, π^+p, and pp interactions. If π^- distributions from K^+p, π^+p, and pp interactions already exhibit a limiting behavior, then this is an indication that in the proton fragmentation region the distributions for negative pions produced from K^-p interactions at 9 GeV/c have not yet reached a limiting behavior.

Since the quantum numbers of $K^-p\pi^+$ are nonexotic, as are the quantum numbers of $\pi^-p\pi^+$, our data provide some additional support for the hypothesis of Chan *et al.*[7] that nonexotic channels may approach a limiting behavior more slowly than exotic channels. By the definition of these authors, negative pions produced from K^+p, π^+p, and pp interactions constitute exotic channels since the quantum numbers of $K^+p\pi^+$, $\pi^+p\pi^+$, and $pp\pi^+$ are exotic.

In Fig. 3 is presented the longitudinal momentum distribution for π^+ in the projectile frame. In Fig. 4 is presented the distribution in transverse momentum squared for the positive pions whose longitudinal momenta in the projectile frame are less than 0.5 GeV/c in absolute value. The distributions for $K^-+p\rightarrow\pi^++$anything and $K^++p\rightarrow\pi^-+$anything should be the same in the limit of only Pomeranchukon exchange, but the distributions of π^+ fragments from K^- are almost a factor of 2 larger than the normalized distributions of π^- fragments from K^+.

In Fig. 2 it is seen that the slope of the transverse momentum distribution for pion fragments from the target does not seem to depend very strongly on the nature of the projectile. It is also interesting to compare distributions of pions fragmenting from different kinds of projectiles. In Fig. 4 it is seen that for the same cut in longitudinal momentum, the distribution in $P_\perp^2$ of pions from $K^\pm+p\rightarrow\pi^\mp+$anything has a smaller slope than that of pions from $\pi^\pm+p\rightarrow\pi^\mp+$anything. The kinematic limits play a role here, however. In the projectile frame, pions from Kp interactions may have momenta as high as 0.23 GeV/c in the forward direction, whereas pions from πp interactions are constrained by kinematics to go backward. The distributions of pions might better be tested with cuts at the same distance from the kinematic boundary. Figure 5 presents distributions of transverse momentum squared for

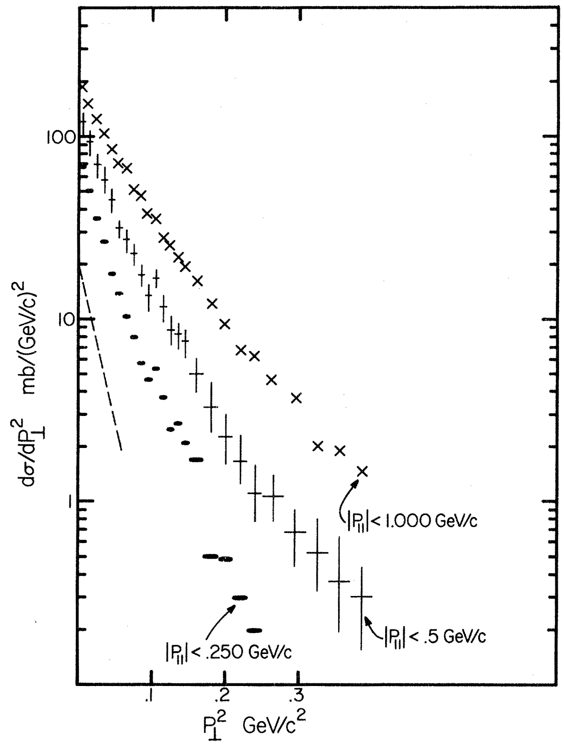

FIG. 5. Distributions in transverse momentum squared for $K^- + p \to \pi^+ +$ anything at 9 GeV/c for various cuts in the longitudinal momentum of the π^+ in the projectile frame. Dashed curve is repeated from Fig. 4.

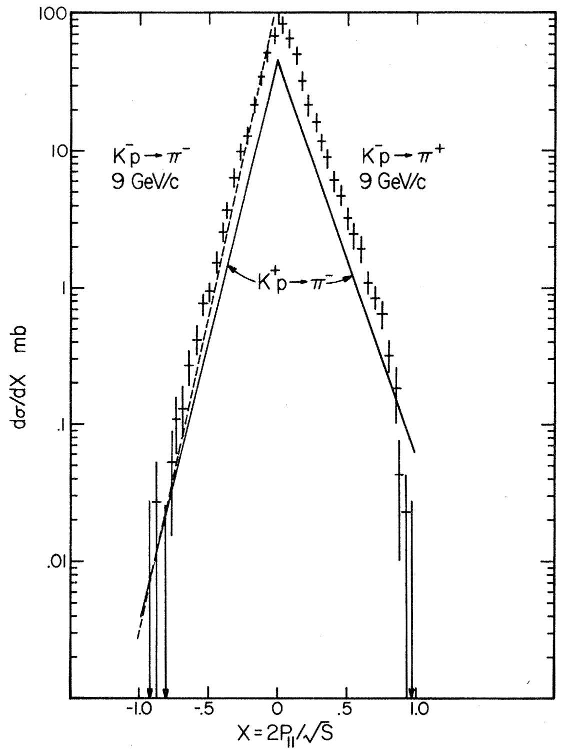

FIG. 6. Distribution of $d\sigma/dx$ for $K^- + p \to \pi^+ +$ anything, $x > 0$; $K^- + p \to \pi^- +$ anything, $x < 0$. The solid line represents the exponential fit for the normalized distribution of $K^+ + p \to \pi^- +$ anything at 12 GeV/c (Ref. 3). The dashed line represents the exponential fit in the backward direction for the normalized distribution of $\pi^- + p \to \pi^-$ + anything at 25 GeV/c (Ref. 4).

several cuts in longitudinal momenta in the projectile frame. The slope does indeed become steeper as the selected region approaches the kinematic boundary. For the cut in $|P_\parallel|$ less than 0.25 GeV/c in the projectile frame, the kinematically allowed region for pion fragments from kaons more closely approximates that for pion fragments from pions with the cut in $|P_\parallel|$ less than 0.5 GeV/c.

We may also make comparisons in the center-of-mass frame. Figure 6 presents the distribution in x, where $x = 2P_\parallel/\sqrt{s}$, $P_\parallel$ is the longitudinal momentum and $\sqrt{s}$ is the total energy, both in the center-of-mass frame. In the forward direction we have plotted $K^- + p \to \pi^+ +$ anything. In the backward direction we have plotted $K^- + p \to \pi^-$ + anything. The distributions for $K^- + p \to \pi +$ anything at 9 GeV/c again lie about a factor of 1.5 to 2 higher than the normalized distribution of $K^+ + p \to \pi^- +$ anything at 12 GeV/c.[3] For detailed comparisons in the center-of-mass frame it would be necessary to use an invariant distribution, but we note that $\sqrt{s}$ in 9- and 12-GeV/c Kp interactions differ by only 10% and our general conclusions would not be altered.

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Inelastic Effects and π-N Resonances

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Inelastic effects (ρ production, Δ production) and particle-exchange effects are shown to determine the major features of the π-N D_{13}, D_{33}, and P_{13} waves. The $D_{33}(1670)$ is associated with a resonance pole but the $P_{13}(1860)$ is not.

A rapidly opening inelastic process can produce a resonance.[1] In the π-N system the connection of the $D_{13}(1520)$ resonance with ρ production has been known for some time.[2] Similarly the importance of particle-exchange or potential terms has been known since Chew and Low first related the $P_{33}(1238)$ and nucleon exchange.[3] However, there have been few attempts to combine such effects in a systematic dynamical calculation in the π-N system, in part because of the absence of a tractable dynamical framework in which to imbed these effects. Recently such a framework has been developed[4-6] and in this Letter we apply it with remarkable success to a calculation of the D_{13}, D_{33}, and P_{13} π-N amplitudes. We find that ρ and Δ production dominate the D_{13} channel, whereas in the D_{33} and P_{13} channels exchange and production effects combine. In all cases we obtain excellent agreement with experiment with reasonable values of our parameters. Phase-shift analyses of the π-N system yield $D_{33}(1670)$ and $P_{13}(1860)$ resonances of doubtful pedigree.[7] We find a clear resonance in the D_{33} state and equally clear evidence of *no* resonance in the P_{13} state near 1900 MeV. In the P_{33} state we find that nucleon exchange is an important ingredient in producing the well-known $\Delta(1238)$, but that higher inelastic states are also quite important. We also discuss the $S_{11}(1700)$ resonance.

Since we are dealing with the π-N system in an energy region dominated by a few strong inelastic processes, it seems clear that we must include these in a unitary way. At the same time we must include particle exchange or potential effects since all π-N waves are not driven purely by inelastic channels. The dynamical scheme into which we put these effects must be one that deals correctly with unitarity and with the dynamical driving terms. Such a scheme is provided by relativistic three-body equations based on the method of Blankenbecler and Sugar.[4] These equations have two- and three-body unitarity exactly and no further scattering singularities. The finite widths of resonances produced are included in a natural manner. We view the BS equations as a new relativistic dynamical scheme designed to deal exactly with a few degrees of freedom of the system while freezing out into "instantaneous potentials" all other degrees of freedom. The BS formalism allows any "potential" to be added as a left-hand cut, either phenomenologically or from some theory.

In the medium-energy π-N system ($T_{\rm lab} \lesssim 2$ BeV), the dominant inelastic processes are ρ production with threshold at 1700 MeV and $\Delta(1238)$ production with threshold at 1380 MeV. The π-N channels corresponding to S-wave production of the above particles are most likely to exhibit dramatic effects of inelasticity. These are S and D waves for ρ production and D waves for Δ production. P-wave production can also significantly affect elastic channels as we shall see in discus-