ciated with nongravitational fields of any integer spin.

We are pleased to thank Kip S. Thorne and Richard H. Price for suggesting this problem and for helpful discussions.

*Work supported in part by the National Science Foundation under Grants No. GP-27304, No. GP-19887, No. GP-28027, and No. GP-15267 at the University of Washington.

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Matter in Superstrong Magnetic Fields: The Surface of a Neutron Star*

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(Received 30 August 1971)

In huge magnetic fields ($B \gtrsim 10^{12}$ G) matter forms a tightly bound, dense ($\gtrsim 10^4$ g cm^{-3}) solid with properties of a one-dimensional metal and a work function of the order of a keV. Electron field emission from the sharp surface of a pulsar is much easier than ion emission; it is estimated to be cut off when the stellar rotation period exceeds several seconds.

The enormous magnetic fields ($B \sim 10^{12} - 10^{13}$ G) which are assumed to thread the surface of canonical neutron stars (pulsars) dominate the motion of electrons and the structure of matter in the stellar surface. The form of such matter and its properties are discussed below, together with a possible consequence for pulsar observations.

In minimum-energy degenerate eigenstates for electrons in a uniform magnetic field $\vec{B}$ the particles are effectively confined to move in tubes of radius

$$\hat{\rho} \equiv \frac{\hbar}{mc}\left(\frac{m_e{}^2c^3}{\hbar eB}\right)^{1/2} \sim \frac{2.6\times 10^{-4}}{B^{1/2}} \text{ cm}. \quad (1)$$

around a flux line. The infinite degeneracy associated with the arbitrariness in position of the flux line is most conveniently exploited in problems with cylindrical symmetry by using cylindrical eigenstates of approximate radius

$$\rho_n = (2n+1)^{1/2}\hat{\rho} \quad (n = 0, 1, 2, \cdots) \quad (2)$$

on which the electron motion is centered ("Landau orbitals").[1] All lowest-state electrons have the same spin direction (antiparallel to $\vec{B}$). Excited states which describe spin flip or motion greater than zero point perpendicular to $\vec{B}$ are excited by integer multiples of $e\hbar B(mc)^{-1} \sim 12B_{12}$ keV (where B_{12} is the magnetic field in units of 10^{12} G). The canonical magnetic fields of pulsars are large enough that the excited states do not enter significantly into the description of free atoms or condensed stellar surface matter.

The strength of a magnetic field on an atom of atomic number Z is characterized by the dimensionless parameter

$$\eta \equiv \frac{a_0}{Z\rho_z} = \left(\frac{B}{4.6\times 10^9 Z^3}\right)^{1/2}, \quad Z \gg 1, \quad (3)$$

which is the ratio of the Bohr radius of the most tightly bound electron when $\vec{B} = 0$ to the cylinder radius of the atom formed by putting exactly one electron into each Landau orbital. There are three qualitative regimes[2,3]: (i) $\eta \gg 1$ (ultrastrong B), (ii) $1 \gg \eta \gg Z^{-3/2}$ ("strong" B), and (iii) $Z^{-3/2} \gg \eta$ (perturbative B). In regime (iii) $\vec{B}$ is sufficiently weak that conventional perturbation treatments are adequate. In the ultrastrong field regime (i) the lowest-energy state of a single atom is achieved by successively putting single electrons into Landau orbitals which keep them (in directions perpendicular to $\vec{B}$) much closer to their nuclei than can the nuclear Coulomb field alone. The resulting atoms[2-4] are small and elongated along $\vec{B}$ with energies (relative

to all electrons free but still in $\vec{B}$) approaching

$$E_a^{(1)}r \sim -\tfrac{9}{8}(Z^3e^2/a_0)\ln^2\eta \quad (Z \gg 1). \tag{4}$$

Atoms (and molecules[5]) will coalesce to form more tightly bound solids for two reasons. They have relatively large quadrupole moments with symmetry axes along $\vec{B}$; this will give a classical electrostatic potential between two such quadrupoles that varies like $(9-90\cos^2\theta+105\cos^4\theta)r^{-5}$ with θ the angle between $\vec{B}$ and the line joining them. This interaction alone would give a bound body-centered orthorhombic lattice in which nearest neighbors attract, next nearest repel. More important, however, is a quantum-mechanical binding which causes the nuclei to arrange themselves in a one-dimensional lattice parallel to B, surrounded by a cylindrical electron sheath.

A linear chain of charge-Z nuclei (lattice constant l) surrounded by a uniform-density cylinder of electrons (radius $r \gg \hat{\rho}$) has an energy per "atom"

$$E_a = -\frac{(Ze)^2}{l}\left[\ln\left(\frac{2l}{r}\right) - (\gamma - \tfrac{3}{4})\right] + \frac{2Z^3\pi^2\hbar^2}{3ml^2}\left(\frac{\hat{\rho}}{r}\right)^4. \tag{5}$$

The first two terms on the right-hand side are the classical Coulomb energy (γ is the Euler constant). The last is the quantum-mechanical degeneracy energy of the uniform density of electrons relative to their energy at zero density but in the same $\vec{B}$. A minimization with respect to l and r gives[6]

$$r = 1.3a_0Z^{-1}\eta^{-4/5}, \tag{6}$$

$$l = 2.4a_0Z^{-1}\eta^{-4/5}, \tag{7}$$

$$E_a = -0.5Z^3e^2\eta^{4/5}/a_0. \tag{8}$$

In regime (i) E_a greatly exceeds the binding energy of an isolated atom or molecule,[7] and Eq. (8) would then also give the binding energy of the atom in the lattice.[8] A single magnetic chain is extraordinarily strong (elastic modulus ~ 10 dyn for $B \sim 10^{13}$ G and $Z \sim 10$) and inflexible. Adjacent chains will have strong Coulomb attraction when one is displaced along $\vec{B}$ half a lattice length relative to the other. An array of chains will then cohere so that the nuclei form a body-centered orthorhombic lattice which is almost bcc.[9] Its mass density (σ) is $2m_NZ(\pi r^2 l)^{-1}$:

$$\sigma \sim Z^4 m_N \eta^{12/5}/6a_0^3 \sim 10^6(Z/26)^4\eta^{12/5}\ \text{g cm}^{-3} \tag{9}$$

with m_N the nucleon mass. Such matter behaves like a one-dimensional metal parallel to $\vec{B}$ and an insulator perpendicular to $\vec{B}$.

In the "strong"-field regime the lowest-energy atom is achieved by filling the closer Landau orbitals with many electrons. A Thomas-Fermi model[2,3] gives a spherical atom of radius

$$R \sim a_0Z^{-1}\eta^{-4/5}. \tag{10}$$

As in the case of field-free atoms this model is not valid for the very outer electrons. The inner ones form a core for which we will approximate Pauli principle effects by assuming the core to be impenetrable by the few outer electrons.[9] Initially we choose $\hat{Z}$ extracore electrons around the core of radius R with $\hat{Z} \ll a_0R^{-1}$. Extracore wave functions with a node have binding energies $< e^2\hat{Z}^2(2a_0)^{-1}$ and will remain unoccupied; rather, the $\hat{Z}$ electrons will successively fill Landau orbitals with radii in the interval $R < \rho_n < R + \hat{Z}\hat{\rho}^2R^{-1}$. This thin hollow sheath of nodeless electron wave functions has a structure like that of the single atom in the regime $\eta \gg 1$ except that η is replaced by $\hat{\eta} = a_0(ZR)^{-1} \gg 1$. As in the ultrastrong field case the energy of these extracore electrons is lowered by arranging the cores in a one-dimensional lattice surrounded by a thin hollow cylindrical sheath of inner radius R. A calculation exactly analogous to that of Eq. (1) gives a lattice spacing

$$l \sim 2R, \tag{11}$$

a lattice binding energy per atom

$$E_a^{(\text{ii})} \sim 0.5(\hat{Z}e)^2/R, \tag{12}$$

and sheath thickness

$$\delta \sim \eta^{4/5}R. \tag{13}$$

A more detailed numerical exploration of this atom model indicates that the above description of extracore electrons should be valid for $\hat{Z}$ up to about $a_0R^{-1} \sim Z\eta^{4/5}$. Therefore in regime (ii) *the lattice binding energy per atom* is of order

$$E_a^{(\text{ii})} \sim -Z^3e^2\eta^{12/5}/a_0. \tag{14}$$

Two adjacent chains of atoms, displaced longitudinally by half a lattice length, will also attract each other just as in regime (i). In the transition region as η increases from (ii) to (i), the atomic core radii shrink ($\sim \eta^{-4/5}$) until the inner sheath radius becomes negligible relative to the outer one. The density σ of Eq. (9) holds for both regimes.[10] From Eqs. (11) and (13) it also follows that this same σ is also appropriate for the sheath region of regime (ii).

For a neutron star with a surface field[11] $B \sim 5\times 10^{12}$ G composed of He, $\eta \sim 12$, the mass density $\sigma \sim 10^4$ g cm^{-3}, and $E_a \sim -800$ eV from Eq. (8). For a probably more realistic assumption of an iron nuclei surface, $\eta \sim 0.3$, $\sigma \sim 4\times 10^4$ g cm^{-3}, and $E_a \sim -30$ keV from Eq. (14). Unfortunately neither η is sufficiently extreme to justify the approximations of either regime (i) or (ii). However, the transition region gives the same σ and similar E_a for both sets of approximations. The estimated surface temperatures after 10^5 yr are of order 10^5 °K ~ 10 eV because of the reduction in surface opacity from $B \gtrsim 10^{12}$ G.[12] Therefore the surface magnetic metal would be very cold relative to its evaporation temperature.

The work function φ in a metal generally equals the electron Fermi energy. In a magnetic metal with electron density n only the component parallel to $\vec{B}$, $E_F(\|)$, is relevant and

$$\varphi \sim E_F(\|) = 2\pi^4\hbar^2\hat{\rho}^4 n^2 m^{-1}. \quad (15)$$

From Eq. (9) it follows in the uniform-density approximation[8] that

$$\varphi \sim 0.3 Z^2 e^2 \eta^{4/5}/a_0 \quad (16)$$

in regime (i) and also in the conduction band sheath of regime (ii). For $Z = 5$ and $B = 2\times 10^{12}$ G, $\eta = 2$ and Eq. (16) predicts a work function of 360 eV, about the same as a numerical estimate of the single-electron ionization energy of the isolated atom in the same field. For $B \sim 5\times 10^{12}$ G and iron nuclei, $\eta \sim 0.3$ and $\varphi \sim 1.8$ keV. Therefore a work function $\varphi = \nu$ keV with $\nu \sim 1$ is plausible. Because of stellar rotation a neutron star surface has electric fields (in V/cm) $\mathcal{F} \sim 10^{-8} \times (2\pi R_s P^{-1} B)$, where R_s is the stellar radius ($\sim 10^6$ m), and P is its period (in sec). Ions remain in the lattice despite such fields. The field-emission electron current generated by components of $\mathcal{F}$ parallel to B at the smooth stellar surface is given by application of the Fowler-Nordheim equation:

$$j(\mathrm{A/cm^2}) \sim 10^3 \mathcal{F}^2 \exp(-30\nu^{3/2} P B_{12}^{-1}). \quad (17)$$

The field-emission current may effectively be cut off for rotation periods exceeding a few seconds (unless the surface is bombarded by energetic x rays or particles): This may be why long period pulsars are not observed. Quite generally it is much easier to remove electrons than ions from the condensed stellar surface, and pulsar magnetospheric currents may therefore consist entirely of electron flow.

I thank Professor Larry Spruch, Professor Thomas Gold, and Professor David Pines for very helpful discussions and the Aspen Center for Physics for its kind hospitality.

*Research supported in part by the National Science Foundation.

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[6]If instead of a uniform-density cylinder the electrons have a Gaussian density distribution in ρ, the resulting E_a is increased about 40% in magnitude.

[7]In each lattice spacing there is only $\eta^{-1/5} < 1$ electron per Landau orbital so that an electron is shared among nuclei. This considerably reduces its kinetic energy relative to confinement in a single atom. More Landau orbitals must be used to do this but this gives only a small logarithmic reduction in Coulomb binding.

[8]An alternative approach which gives the E_a of Eq. (8) consists in minimizing the energy of nuclei distributed in a uniform density (n) of electrons. The electron kinetic energy density is $2\pi^5 n^3 \hat{\rho}^4 \hbar^2 (3m)^{-1}$. It is well known that the unscreened Coulomb repulsion among nuclei is minimized by a bcc configuration.

[9]This works reasonably well for closed-shell cores in normal atoms, viz. Li or Na; in one-dimensional, many-electron problems, which contain main features of atoms in "strong" fields, impenetrability is exactly equivalent to the Pauli principle.

[10]R. Mueller, A. Rau, and L. Spruch, to be published, consider effects of smaller atoms on the equation of state at positive pressures below the surface. Because in their calculations atoms do not cohere into a solid at zero pressure, their surface-atmosphere transition is continuous rather than sharp.

[11]The rms surface field is probably considerably greater than that of the conventionally estimated dipole field alone, $\beta \sim 3\times 10^{12}$ G.

[12]S. Tsuruta, V. Canuto, J. Lodenquai, and M. Ruderman, to be published.