the 8p-4h 2^+ state at 7.83 MeV in ^{20}Ne is indicated. A state at 2.79 MeV in ^{21}Ne, which is thought[8] to have $J^\pi = \frac{1}{2}^-$, is tentatively identified as the $\frac{1}{2}^-$ member of this 8p-3h configuration. A few states between 6- and 8-MeV excitation in ^{21}Ne are also observed to be selectively populated by the ^{12}C(^{13}C, α) reaction and might be the $\frac{7}{2}^-$ and $\frac{9}{2}^-$ 8p-3h states.

In conclusion, the asymmetries observed[1] in the ^{12}C(^{13}C, α) angular distributions for the 3.66- and 3.89-MeV levels of ^{21}Ne persist over a wide range of incident energies. Furthermore, the angular distributions and excitation functions of the ^{12}C(^{13}C, α) reaction to the 3.66- and 3.89-MeV levels in ^{21}Ne are observed to be similar to those for the ^{12}C(^{12}C, α) reaction to the 2^+ member[2] of the 8p-4h ([220] quartet) configuration. This striking similarity suggests that the 3.66- and the 3.89-MeV states in ^{21}Ne may be the $\frac{3}{2}^-$ and $\frac{5}{2}^-$ states of an 8p-3h configuration obtained from coupling a $1p_{1/2}$ neutron to the 2^+ [220] quartet state at 7.83 MeV in ^{20}Ne. At present, the question remains open as to the nature of the mechanism responsible for the fact that, in the ^{12}C(^{13}C, α) reaction, the angular distribution of the 3.89-MeV level is *forward* peaked, whereas that of the 3.66-MeV level is backward peaked.

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Bound-Pion Absorption by ^{12}C and Nuclear Short-Range Correlations*

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The absorption of stopped negative pions by ^{12}C inducing nucleon pair emission is studied. The three-body final-state scattering includes a two-body residual interaction and an optical potential. We find that agreement of this model with experiment cannot be obtained without including Jastrow-type short-range correlations with dominant momentum components between 0.3 and 0.4 GeV/c.

It is well known[1] that slow-pion absorption by nuclei favors the subsequent emission of a pair of fast back-to-back nucleons. The low momentum transfer ($\lesssim 25$ MeV/c) combined with the high-energy release ($\sim M_\pi$) is characteristic of a reaction designed to measure nuclear short-range pair correlations (SRC): The continuum nucleon pair wave with relative momentum $\frac{1}{2}|\vec{k}_1 - \vec{k}_2| \sim 350$ MeV/c serves as a probe of the matching momentum components in the bound-pair wave function.

Little if any conclusive direct evidence exists to date,[2,3] though, for the presence in nuclear wave functions of such high-momentum components in addition to those small ones already generated by the shell model (SM). As usual, the two basic ingredients of the SM are taken here to be a central Woods-Saxon potential of the independent particle model (IPM) and a residual two-body interaction[3] without repulsive core.

As a consequence of the impulse approximation and time-dependent perturbation theory, the pion absorption rate is given in terms of the matrix element of the standard nonrelativistic pion-nucleon pseudoscalar interaction taken between the bound and continuum nucleon pair states.[4] Pion rescattering is not considered here.[5]

By orthogonality, nuclear wave functions of the IPM yield vanishing absorption rates by the dominant (π^-, NN) mode. Within the shell model, then, the central question arises as to whether or not the residual interaction (RI) in the final state can restore agreement with experiment or whether additional short-range correlations are needed whose dominant momentum components are expected at significantly larger momenta then the Fermi momentum $q_F \lesssim 250$ MeV/c.[6]

Over the past several years it has become clear that the answer requires a solution of the three-body final-state scattering problem which involves two nucleons in the continuum of the residual nucleus. The latter is taken to be in a singly excited state for each partial absorption rate and, as a consequence, appears only in terms of the optical potential. This assumption seems justified[7] for nucleon pair emission from the $(p_{3/2})^2$ and $(s_{1/2})^2$ states of ^{12}C. Avoiding the three-body problem, a number of modifications of the relative-momentum component of plane NN waves alone have been used with varying results.[8] However, since the well depth of the central potential is of the same order as the energy of each emitted nucleon (≈ 50 MeV), it is expected to be important, and orthogonality of bound and continuum states requires its adequate treatment over the whole nuclear region.[9]

To this end, it is very useful to introduce coordinates which are appropriate for the boundary conditions of the NN emission channels: Let r_1 and r_2 be the distances from the heavy target of two nucleon detectors; then only nucleon pairs are picked up in coincidence whose energy ratio satisfies $E_1/E_2 = (r_1/r_2)^2$ because, for equal time of flight, t, each nucleon travels the distance $r_i = (2E_i/M)^{1/2}t$. Introduction of the natural coordinates (r, α) defined by $\cot\alpha = r_1/r_2$ and $r^2 = r_1^2 + r_2^2$ yields a discrete set (labeled by n) of NN partial waves $\tilde{\psi}_c$ instead of a continuous set labeled by E_1/E_2 as is the case for plane pair waves. As a result, the three-body problem reduces to a coupled-channel partial-wave analysis[10] in the single variable r.

Central and two-body transition potentials, $v_n(r)$ and $v_{nl}(r)$, are defined by multipole expansions of $V(r_1) + V(r_2)$ and $V(|\vec{r}_1 - \vec{r}_2|)$, respectively. A well-known technical difficulty of the three-body system is reflected here in the long range of the transition potentials which supports the (in our model) closed single-nucleon and single-deuteron emission channels and must be cut off. We proceed as follows: In the kinematic region of interest to us, viz. NN emission of almost equal energy ($\alpha \approx \pi/4$) and NN opening angle $\theta \approx 180°$, the crudest approximation for the central potential is to neglect all v_n with $n \geq 1$. This yields the short-ranged potential $v_0(r) = 2V(r/\sqrt{2})\pi^{-1/2}$. By iteration, the other v_n follow from expanding the truncated multipole expansion of $V(r_1) + V(r_2)$ near $\alpha = \pi/4$. The v_{nl} can be handled similarly.[11] Inside the nucleus all v_n and v_{nl} except v_0, v_1, v_{00}, and v_{01} turn out to be negligible.

Next let us consider the central potential alone. The overlap of the continuum- and bound-pair wave functions can be used as a check on the quality of the approximation for v_0 and, in first iteration, for v_0 and v_1, etc. To this end, inclusion of v_1 requires a numerical solution of the coupled equations with n coupling of twelve channels for two p-shell nucleons and for ten s-shell nucleons.

From Table I we conclude that first, the (spurious) plane-wave overlap is reduced by two to three orders of magnitude upon including v_0 alone, and second, the coupled equations including v_1 in addition do not further improve the overlap. As a consequence, for an adequate treatment of the central potential it is sufficient to keep the short-ranged v_0 only. This also demonstrates that the pion absorption rates depend effectively on a two-body operator since upon switching off both the two-body final-state interaction and the SRC, they are essentially given by the foregoing overlaps. The bound-state wave functions are well approximated by harmonic-oscillator wave functions.

TABLE I. Overlaps of the bound-pair wave function with various continuum-pair waves.

NN shells	$V=0$ (Plane wave)	v_0	v_0 and v_1
$(s_{1/2})^2$	5.46×10^{-2}	$(0.73 - i1.98)10^{-4}$	$(6.65 + i3.49)10^{-4}$
$(s_{1/2}p_{3/2})$	3.39×10^{-2}	$(0.74 + i5.94)10^{-4}$	$(6.54 - i0.22)10^{-4}$
$(p_{3/2})^2$	2.26×10^{-2}	$(4.47 + i7.59)10^{-4}$	$(2.60 - i2.71)10^{-4}$

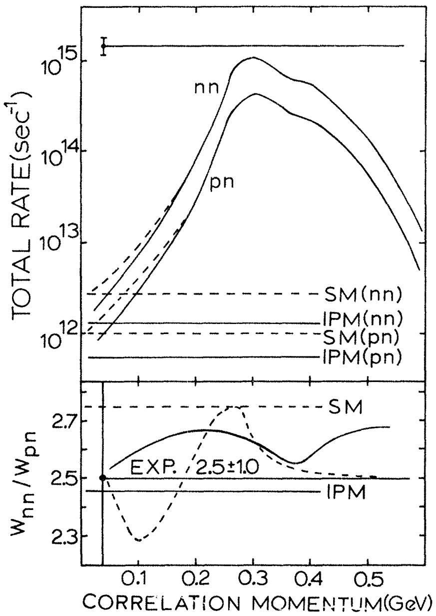

FIG. 1. Upper curves: total neutron-neutron (nn) and proton-neutron (pn) $2P$-pion absorption rates by ^{12}C. The experimental point is from Huguenin (Ref. 7). Lower curves: total rate ratios. The experimental point is from Nordberg (Ref. 12). Solid (dashed) curves are calculated without (with) final-state RI and are plotted versus correlation momentum q. The straight lines do not include SRC.

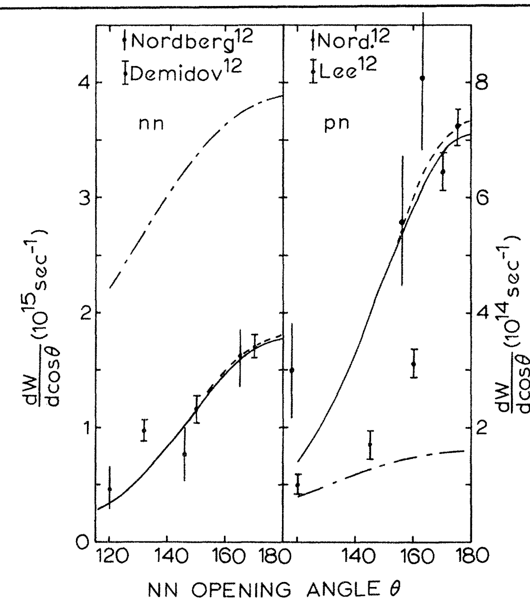

FIG. 2. Total angular distributions for neutron-neutron (nn) and proton-neutron (pn) emission from ^{12}C. Solid and dashed lines have the same meaning as in Fig. 1 and are calculated for SRC, $q = 370$ MeV/c. The dot-dashed curves have been scaled up by a factor of 10^3 for nn and 10^2 for pn emission and include the final-state two-body potential but no SRC. Nordberg's (Ref. 12) arbitrarily normalized data are fitted once at the largest opening angle in the nn case.

The only non-negligible two-body coupling potential v_{01} is less than 10% of v_{00} for $r \lesssim 2$ fm. Therefore, it has been treated in first-order perturbation theory using the partial-wave Green's functions of the v_0 plus v_{00} wells. The three-body problem is now substantially simplified and reduced to a single-particle potential scattering problem involving partial waves up to $l_i = 3$ and $n = 10$.

All the pion absorption rates of Figs. 1–3 include a central Woods-Saxon potential of well depth -61 MeV, range 2.87 fm and diffuseness 0.65 fm which reproduces the experimental s- and p-shell separation energies. The total rates of Fig. 1 are quite sensitive to SRC. By contrast, the dashed straight lines show the rates (SM) without SRC but including an attractive two-body Gillet potential in the final state. Its depth is taken to be -44 MeV, its range 1.7 fm, and spin-isospin mixture $a_0 = 0$, $a_\tau = -0.3$, $a_\sigma = -0.2$, $a_{\sigma\tau} = -0.1$. When both SRC and this final-state RI are turned off, the rates decrease drastically as required by the IPM. This agrees with results of Chung, Danos, and Huber in Ref. 9. It is because of the repulsive components of the final-state RI for the partial rates of the $(s_{1/2}p_{3/2})$ shells that the SM rates (without SRC) are higher than our approximate IPM rates; yet they are

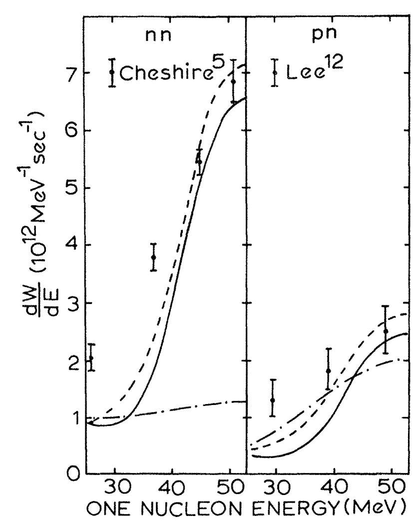

FIG. 3. Single-nucleon energy distributions for the $(p_{3/2})^2$ shell. See caption of Fig. 2. The dot-dashed curves are scaled up by 10^3. The experimental data are normalized at the value closest to equal energy sharing $E_{max}/2$.

far below experiment. Presumably we underestimate the total rates by our restriction to the dominant (π^-, NN) channel. Although the ratio of total rates in Fig. 1 is a rapidly varying function both of SRC and the final-state RI, the uncertainty of the experimental value[12] 2.5 ± 1.0 is too large for distinguishing among the different models. The slope and width of the backward peak of the angular distributions in Fig. 2 are not very sensitive to SRC; the height is, but the experimental normalization is not well known. Consistently, the final-state RI acts as a small perturbation compared to SRC for $0.2\ \mathrm{GeV}/c \lesssim q \lesssim 0.4\ \mathrm{GeV}/c$. The structure of the single-nucleon energy distributions of Fig. 3, however, is very sensitive to SRC and differs noticeably from experiment upon switching off SRC.[13] The best fit to the energy distributions from all three nucleon pair shells is found at a correlation momentum of about $q = 370\ \mathrm{MeV}/c$.

Concluding, we feel that the general agreement represents a strong test in favor of SRC. The results of this model indicate that short-range modifications of two-particle shell-model wave functions are not only sufficient but also necessary for fitting recent data on nucleon pair emission following bound-pion absorption by ^{12}C; they are required in *bound* shell-model wave functions if the two-body final-state interaction does not contain a strongly repulsive short-range core.[11]

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