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Fluctuation Theory of the Superconducting Transition in Restricted Dimensionality*

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The results of a calculation of the properties of a dirty superconductor near the transition temperature are described. The density of states and conductivity are obtained within an approximation that corresponds to BCS theory above T_c. In two dimensions or less no singularity is encountered at T_c; instead, the conventional BCS results are obtained for $T \ll T_c$, even though the order parameter is strictly zero. The anomalous contribution to the conductivity reported by Maki is found to be well behaved and finite in the absence of external pair breaking.

In this Letter we give a simplified derivation of several results of a self-consistent microscopic calculation[1] of the effects of fluctuations on the properties of a superconducting alloy near the transition. The electrons are taken to interact via the usual pairing term $V(k-k')=-|V|$ and with a random distribution of impurities. The proper inclusion of the finite lifetime of the interacting electrons leads to a finite dc conductivity, in contrast to a previous calculation.[2]

Just above the transition, resonance scattering occurs between electrons of opposite spin and momentum, foreshadowing the formation of bound pair states. The anomalously large contribution of such repeated scattering must be included from the outset by replacing the bare interaction V with the t matrix, which within mean-field theory has the form

$$t^{-1}(p,\nu)=N_0[\epsilon+(\pi|\nu|/8T+Dp^2)], \tag{1}$$

where $\epsilon=\ln(T/T_c)$, $D=v_\mathrm{F}l/3$ is the diffusion constant, and N_0 is the density of states at the Fermi surface. As an effective electron-electron interaction, the t matrix leads to the following decay rate for the electron states ($\hbar=k_\mathrm{B}=1$, $d=$dimensionality):

$$\frac{1}{\tau_n}=2\pi\sum_{p,\nu}\left|\frac{t(p,\nu)}{2}\right|^2 N_0 \propto \epsilon^{-2+d/2}. \tag{2}$$

The decay rate (2) may be regarded as the rate at which electrons leave a "normal fluid" to combine in a "superfluid" of virtual pair states at an energy above the Fermi surface. The inverse rate at which electrons decay from the superfluid back to the Fermi surface is given by the time-dependent Ginzburg-Landau equation:[3] $1/\tau_s=8T\epsilon/\pi$. A simple detailed-balance argument gives the (small) change in the density of states $N(0)$ at the Fermi surface:[4]

$$\frac{N(0)-N_0}{N_0}=\frac{\tau_s}{\tau_n}\simeq\left(\frac{\epsilon}{\epsilon_d}\right)^{-3+d/2}. \tag{3}$$

We use Eq. (3) to define a characteristic reduced temperature ϵ_d at which the relative change in $N(0)$ becomes of order unity. The temperature

ϵ_d is much larger than that of the Ginzburg criterion,[5] ϵ_c, for the validity of mean-field fluctuation theory: $\epsilon_d \simeq \epsilon_c{}^\alpha$, where $\alpha = \frac{1}{3}, \frac{1}{2}, \frac{3}{5}$, and $\frac{2}{3}$ in three, two, one, and zero dimensions, respectively. The result is the existence of two distinct regions within mean-field theory above the transition:

$$\epsilon_c \ll \epsilon_d \ll 1. \tag{4}$$

For $\epsilon > \epsilon_d$, the density of states $N(\omega)$ deviates only slightly from that in the normal state, but for $\epsilon < \epsilon_d$, strong electron correlations exist and $N(\omega)$ is similar to that of a gapless superconductor. The microscopic result[1] for $N(\omega)$ in zero dimensions (e.g., a tungsten film 1 μm square) is shown in Fig. 1, and has the simple form[6]

$$\frac{N(\omega)}{N_0} = \mathrm{Re}\left\{\left[1 - \frac{\Delta^2}{(\omega + i/2\tau_s)^2}\right]^{-1/2}\right\}. \tag{5}$$

The parameter Δ, which represents an effective gap when $\Delta\tau_s \gg 1$, satisfies the equations

$$\Delta^2 = T/N_0\Omega\eta, \tag{6a}$$

$$\eta = \epsilon + \tfrac{7}{8}\zeta(3)\Delta^2/(\pi T)^2, \tag{6b}$$

where Ω is the volume of the sample.

From Eqs. (6) it is clear that the renormalized temperature difference η never vanishes.[7] However, as $\eta \to 0$, Eq. (6b) becomes the BCS gap equation, and (5) the BCS density of states. A typical transition (Fig. 1) appears sharp as a function of ϵ, even though the width in the peak of the density of states remains finite ($1/\tau_s \propto \eta \neq 0$).

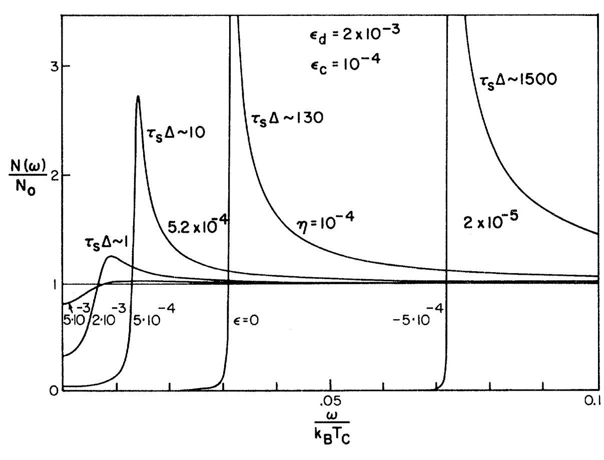

FIG. 1. The density of states $N(\omega)$ for a zero-dimensional superconductor $[\xi(T) \gg L_1, L_2, L_3]$ as a function of temperature. Both the renormalized temperature given by Eqs. (6) and $\epsilon = (T - T_c)/T_c$ are shown. For $\epsilon > \epsilon_c$, $\eta \simeq \epsilon$.

The decay (2) of the normal states is particularly important in the conductivity process investigated by Maki,[2] which is shown in Fig. 2 in the time ordering giving the anomalous contribution. The wavy line denotes a t matrix, the directed lines electrons and holes. The evaluation of the conductivity proceeds from the linear response expression (valid for $\omega \ll T$)

$$j_\alpha(\omega) = \int_0^\infty dt\, \langle j_\alpha(0) j_\beta(t)\rangle e^{i\omega t} E_\beta(\omega), \tag{7}$$

where j_α is the current operator. In the presence of impurities, an isolated particle propagates as $\exp[(iE - 1/2\tau_i - 1/2\tau_n)t]$ and a hole as $\exp[(-iE - 1/2\tau_i - 1/2\tau_n)t]$, where $1/\tau_i$ is the electron-impurity scattering rate. However, because the virtual pair state has a small total momentum p, the process of Fig. 2 has the property that, in a system with many impurities, the pair decays into a long-wavelength diffusion mode rather than single particles. A diffusion mode in the normal state has the decay rate Dp^2, which would lead to infinite conductivity in two or fewer dimensions because of the large phase space at small p.[8] However, the diffusion mode can last no longer than the individual states whose decay rate is given by (2), and thus has the effective lifetime $(Dp^2 + 1/\tau_n)^{-1}$.

Figure 2 then results in the following expres-

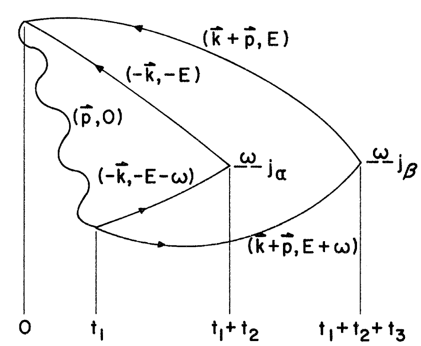

FIG. 2. The physical process giving rise to the anomalous contribution to the conductivity. Wavy line denotes t-matrix interaction, directed lines electrons. The three time intervals are summed over to give Eq. (8).

sion:

$$\sigma(\omega)=\frac{2N_0e^2v_F^2}{3}\int_{-\infty}^{\infty}\frac{dE}{\cosh^2(E/2T)}\sum_p t(p)\int_0^\infty dt_1\exp\left[\left(-2iE-Dp^2-\frac{1}{\tau_n}\right)t_1\right]\int_0^\infty dt_2\exp\left[\left(-2iE-Dp^2-\frac{1}{\tau_n}\right)t_2\right]$$
$$\times\exp\left[\left(2iE+2i\omega-Dp^2-\frac{1}{\tau_n}\right)t_2\right]\int_0^\infty dt_3\exp\left[\left(iE+i\omega-\frac{1}{2\tau_i}-\frac{1}{2\tau_n}\right)t_3\right]\exp\left[\left(-iE-\frac{1}{2\tau_i}-\frac{1}{2\tau_n}\right)t_3\right]$$
$$=\frac{\sigma_0}{1-i\omega\tau_i}\frac{\pi}{4\Omega}\sum_p t(p)\frac{1}{-i\omega+1/\tau_n+Dp^2}. \qquad (8)$$

In the limit $\tau_n\to\infty$, Eq. (8) gives the expression obtained by Maki.[2] It is clear that the appearance of the decay rate (2) in the denominator in Eq. (8) can not be obtained by a perturbation expansion in the interaction t. The nature of the result (8) has been confirmed by a complete microscopic calculation[1] in which fluctuations and impurities are treated self-consistently.[9]

The general microscopic result for the conductivity has the form

$$\sigma(\omega)=\sigma_{AL}(\omega)+\sigma_{BCS}(\omega)+\sigma'(\omega), \qquad (9)$$

where σ_{AL} is the Aslamazov-Larkin contribution,[10] and $\sigma_{BCS}+\sigma'$ includes the normal conductivity plus Maki's terms; σ_{BCS} is the contribution one would calculate within the BCS theory using the density of states (5), and σ' is an anomalous term whose nature is similar to that found by Gor'kov and Éliashberg with the time-dependent Ginzburg-Landau equation.[11] For $\epsilon>\epsilon_d$, σ' reduces to Eq. (8) and provides the largest correction to the normal conductivity σ_0, while for $\epsilon<\epsilon_d$, σ' is smaller than both σ_{AL} and σ_{BCS}. In the latter region σ_{BCS} is given by

$$\frac{\sigma_{BCS}(0)}{\sigma_0}=1+\frac{\Delta}{2T}\left[\ln\left(\frac{\pi\Delta}{\eta}\right)-1\right]. \qquad (10)$$

In one and two dimensions, respectively, Eq. (6a) is replaced by

$$\Delta^2=\frac{T}{2N_0S\xi(0)\eta^{1/2}}, \qquad (11a)$$

$$\Delta^2\cong\frac{T}{4\pi N_0d\xi(0)^2}\ln\left(\frac{1}{\eta}\right). \qquad (11b)$$

In one dimension the following formula is obtained for the region $\epsilon>\epsilon_d$:

$$\frac{\sigma-\sigma_0}{\sigma_0}=\left(\frac{\eta_0}{\eta}\right)^{3/2}\left[4\sqrt{2}-3+\frac{2\sqrt{2}}{\pi}\frac{\eta^{5/4}}{\eta_0^{3/4}}\ln\left(\frac{\pi}{4\eta}\right)\right];$$
$$\eta_0^{3/2}=\frac{\pi e^2\xi(0)}{16\sigma_0S}\simeq\epsilon_c^{3/2}, \qquad (12)$$

and S is the cross-sectional area.

Satisfactory agreement with experiment has been found.[12] In two dimensions the following formula interpolates between the two regions (4) for which Eq. (9) is valid:

$$\frac{\sigma-\sigma_0}{\sigma_0}=\frac{\eta_0}{\eta}\left[1+\frac{2}{1-\delta/\eta}\ln\left(\frac{\eta}{\delta}\right)\frac{1.7+3\eta^2/\eta_0}{1+3\eta^2/\eta_0}\right],$$
$$\frac{1}{\delta}=\frac{1}{2.9\sqrt{\eta_0}}+\frac{\eta}{1.1\eta_0}, \qquad (13)$$
$$\eta_0=1.52\times10^{-5}R_{sq}\simeq\epsilon_c;$$

R_{sq} is the resistance of a square film sample.

Comparison with experiment[13] reveals essential agreement over four decades of R_{sq} for $\epsilon<0.1$. Apart from T_c, there are no adjustable parameters. For $\epsilon>0.1$, the experimental conductivities uniformly drop below the predicted values, as might be expected because of the instability of the fluctuation mode far from T_c.

The prediction of Eq. (10) is also made for the spin-lattice relaxation rate R_s/R_n, in which the decay rate $1/\tau_s$ leads to a *finite* logrithmic singularity below the transition in restricted dimensions.

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Fluctuation-Induced Conductivity of a One-Dimensional, Nearly Ideal BCS Superconductor*

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We report detailed measurements of the fluctuation conductivity above T_c of one-dimensional aluminum microstrips. These results support a new theory by Patton which apparently resolves the recent puzzle concerning the anomalous contribution to the fluctuation conductivity reported by Maki and Thompson. Comparison of experiment with theory indicates that the samples studied are the first characterized by lifetime effects closely approaching the intrinsic limit for a BCS superconductor.

The results of a new calculation by Patton[1] of the fluctuation conductivity above T_c in a BCS superconductor are presented in the preceding Letter.[2] This approach, which utilizes nonequilibrium electron propagators in the calculation of the electromagnetic response, indicates that the paraconductivity in one- and two-dimensional superconductors is finite even in the absence of pair breaking, contrary to previous results.[3] We have studied the temperature and magnetic-field dependence of the paraconductivity in one-dimensional Al microstrips [with one dimension larger than the temperature-dependent coherence length $\xi(T)$], and we find agreement with Patton's predictions.

Patton's calculation implies that the excess conductivity $\sigma' = \sigma - \sigma_N$, where σ_N is the normal conductivity, can be written as

$$\sigma' = \sigma_{AL}' + \sigma_{BCS}' + \sigma_A'. \tag{1}$$

We have probed the temperature region where σ_A' dominates the other terms in Eq. (1), i.e., in the region where the single-fluctuation (Maki) diagrams[4] dominate the two-fluctuation [Aslamazov-Larkin (AL)] diagrams[5] and the Cooper pairs are sufficiently sparse and short-lived to make the fluctuation-induced change in the density of states and hence σ_{BCS}' relatively small. For a one-dimensional sample this temperature region is defined by

$$\epsilon > \epsilon_c^{3/5} \equiv \epsilon_d, \tag{2}$$

where ϵ_c is the reduced temperature associated with the Ginzburg criterion for the validity of the mean-field theory:

$$\epsilon_c = \left[\pi\left(\frac{e^2}{16\hbar}\right)\frac{R_N}{L}\xi(0)\right]^{2/3}. \tag{3}$$

R_N is the normal-state resistance and L the length of the one-dimensional sample. Patton's result for this regime may be written

$$\sigma' = \sigma_{AL}' f_1(\epsilon, \epsilon_c, \tau_c), \tag{4a}$$

where σ_{AL}' is the Aslamazov-Larkin[5] contribution,

$$\sigma_{AL}'/\sigma_N = (\epsilon_c/\epsilon)^{3/2}, \tag{4b}$$

with $\epsilon = (T - T_c)/T_{c0}$, T_{c0} and T_c being the transition temperatures in the absence and presence of pair breaking, respectively. The increase over