and

$$\mathrm{Ar}^{++}(^1D)+e\to\mathrm{Ar}^{+}(^2S) \tag{8}$$

are seen to have RE's sufficiently close to the ionization potential of D_2 to produce the wide Ar^+ kinetic energy distribution with $\Delta E\simeq 0$.

The authors would like to thank Dr. P. B. James and Dr. G. R. North for many helpful discussions.

*Work supported in part by a Fredrick Gardner Cottrell grant from Research Corp.

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Plasma Heating by High-Current Relativistic Electron Beams *

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(Received 14 June 1971)

A mechanism is proposed for the heating of a plasma with a high-current relativistic electron beam which makes essential use of the plasma return current induced by the beam. From overall energy conservation it is concluded that a large fraction of the beam energy is converted into plasma thermal energy. For reasonable parameters the heating occurs through ion sound turbulence generated by the plasma return current.

Recent developments in technology have led to the generation of beams of electrons with energies in the range 500 keV to 10 MeV and currents in the range of 50 kA to 1 MA, of pulse durations of the order of 50 nsec. The energy content in these beams is as large as 10^5 J. The possibility of using these beams in controlled fusion experiments for purposes of heating a plasma to thermonuclear temperatures is of considerable interest. In this Letter we point out one important mechanism by which a high-current beam[1] ($\nu/\gamma \gtrsim 1$) can heat a plasma, and we estimate the rate at which this heating occurs. The mechanism does not involve the collective interaction of the beam electrons with the plasma, which is expected to be weak for high-energy beams and small beam-plasma density ratios.[2]

The injection of an electron beam into a cold dense plasma ($n_p \gg n_B$, where n_p and n_B are the plasma and beam electron densities, respectively) is accompanied by a return current which acts to neutralize the magnetic field of the beam if $\lambda_E/a \ll 1$ [where a is the beam radius, $\lambda_E = c/\omega_p$ is the electromagnetic skin depth, and $\omega_p=(4\pi e^2 n_p/m_e)^{1/2}$ is the plasma frequency]. This result may be understood as follows (in the rest frame of the plasma): Assume that on a macroscopic scale the plasma may be described by the generalized Ohm's law

$$\left(\frac{\partial}{\partial t}+\frac{1}{\tau_*}\right)\vec{J}^P=\frac{\omega_p^2}{4\pi}\vec{E}+\frac{e}{m_e c}\vec{J}^P\times\vec{B}, \tag{1}$$

where τ_*^{-1} is the effective collision frequency and $\vec{J}^P(\vec{x},t)$ is the plasma current density. External magnetic fields are not included; however, in the main the results below appear to hold also for beams propagating parallel to an external magnetic field, and indeed such fields may be essential for stability of the beams. In addition, we neglect for the moment the self-magnetic field due to the plasma and beam currents so that the Hall contribution in Eq. (1) is absent. Then by operating on Eq. (1) with $\nabla\times\nabla\times$, and using Faraday's and Ampere's laws (assuming overall charge neutrality[3]), we obtain

$$\lambda_E^2\left(\frac{\partial}{\partial t}+\frac{1}{\tau_*}\right)\nabla^2\vec{J}^P=-\frac{\partial}{\partial t}(\vec{J}^P+\vec{J}^B), \tag{2}$$

where the total current density $\vec{J}(\vec{x},t)=\vec{J}^P(\vec{x},t)+\vec{J}^B(\vec{x},t)$ is written as the sum of the plasma and beam contributions. Estimating the scale of the gradient operator in Eq. (2) to be of the order of the beam radius a, it is clear that for $\lambda_E/a \ll 1$ and short times we have $\vec{J}^P \approx -\vec{J}^B$, so that $\vec{J}^P$ flows within the beam channel. The theory of the induced "return current" $\vec{J}^P$ has been discussed in detail by a number of authors.[4,5] The current $\vec{J}^P$ is produced by the plasma electrons streaming in the direction opposite that of the beam with the small velocity $v_p = -(n_B/n_p)v_B \ll v_B$, where $v_B \approx c$ is the beam propagation velocity. Observations supporting the theory have been described previously.[6] Note that the near cancelation of the beam and plasma currents lends support to the neglect of the self-magnetic field in Eq. (1).

For slow time variations of the return current, $\partial\vec{J}^P/\partial t \ll \vec{J}^P/\tau_*$, Eq. (2) may be rewritten as

$$\left(\frac{\partial}{\partial t} - \frac{\lambda_E^2}{\tau_*}\nabla^2\right)\vec{J}^P = -\frac{\partial}{\partial t}\vec{J}^B. \qquad (3)$$

For the case of a beam with a sharp front propagating in the $+z$ direction, for example of the form $\vec{J}^B(\vec{x}, t) = \vec{J}^B(x,y)U(v_Bt - z)$ [where $U(x)$ is the unit step function, $U(x>0)=1$, $U(x<0)=0$], the right-hand side of Eq. (3) is nonzero only at the beam front. Behind the front the plasma current density $\vec{J}^P(\vec{x}, t)$ obeys a diffusion equation. Thus the characteristic decay time is $t_D = \tau_*(a/\lambda_E)^2$, a result previously obtained by a different method by Lee and Sudan.[5] The corresponding decay length is $L = t_D v_B$, and we assume $L/a \gg 1$.

An important consequence of the plasma response to a relativistic beam not previously appreciated is that the electric field, which sustains the return current, at the same time exerts a substantial drag on the beam. This drag may completely stop an intense beam ($\nu/\gamma \gtrsim 1$) in a distance short compared with the decay length L. A large fraction of the energy lost by the beam goes into heating the plasma.

For a discussion of the disposition of the beam, plasma, and field energies consider an infinitesimal volume contained between z and $z+dz$ (with the z axis in the beam propagation direction) and extending in the perpendicular x-y plane to infinity. The total electric field and current density are written as $\vec{E} = \vec{E}_0 + \delta\vec{E}$ and $\vec{J} = \vec{J}_0 + \delta\vec{J}$, where $\vec{E}_0$ and $\vec{J}_0$ are slowly varying macroscopic quantities appearing in Eq. (1), and $\delta\vec{E}$ and $\delta\vec{J}$ are rapidly fluctuating irrotational quantities discussed below. The beam current density $\vec{J}^B$ is assumed to be only slowly varying.[2] The appropriate Poynting's theorem for this volume is[7]

$$\frac{\partial}{\partial t}\int d^2x\,\frac{\vec{B}^2}{8\pi} = -\int d^2x\,\vec{J}_0\cdot\vec{E}_0 - \int d^2x\,\langle\delta\vec{J}\cdot\delta\vec{E}\rangle - \frac{\partial}{\partial t}\int d^2x\,\frac{\langle\delta\vec{E}^2\rangle}{8\pi}, \qquad (4)$$

where the angular brackets denote time averages over the rapid fluctuations and where it has been assumed that the fields decay for radial distances large with respect to the beam radius. The field energy $\propto \vec{E}_0^2$ and the Poynting flux $\propto \vec{E}_0$ are neglected as is valid if the plasma conductivity σ is sufficiently high, $\sigma = \omega_p^2\tau_*/4\pi \gg c/\lambda_E$ or $\omega_p\tau_* \gg 4\pi$. It is helpful to rewrite Eq. (4) in terms of the current densities and magnetic fields associated with the plasma and with the beam, $\vec{J} = \vec{J}^P + \vec{J}^B$ and $\vec{B} = \vec{B}^P + \vec{B}^B$. One then finds

$$\frac{\partial}{\partial t}\int d^2x\,\frac{(\vec{B}^P)^2}{8\pi} + \int d^2x\,\frac{\vec{B}^P}{4\pi}\cdot\frac{\partial}{\partial t}\vec{B}^B = -\frac{\partial}{\partial t}(K+W), \qquad (5a)$$

$$\int d^2x\,\frac{\vec{B}^B}{4\pi}\cdot\frac{\partial}{\partial t}\vec{B} = -\int d^2x\,\vec{J}^B\cdot\vec{E}_0, \qquad (5b)$$

where $\partial(K+W)/\partial t$ is the rate of change of the plasma internal energy,

$$\partial K/\partial t = \int d^2x\,\vec{J}_0^P\cdot\vec{E}_0 + \int d^2x\,\langle\delta\vec{J}^P\cdot\delta\vec{E}\rangle$$

is the rate of change of plasma kinetic energy, and $W = \int d^2x\,\langle\delta\vec{E}^2\rangle/8\pi$ is the field fluctuation energy. The rate of decrease of the beam energy is given by $-\int d^2x\,\vec{J}^B\cdot\vec{E}_0$.

For simplicity consider an electron beam with a well-defined front. Suppose that the initial beam electron energy γ_0 (in units of m_ec^2) is sufficiently larger than unity that any changes in this energy $\delta\gamma$, even $\delta\gamma/\gamma_0 \approx 1$, represent only a small change in the beam velocity, $\delta v_B/v_B \approx \delta\gamma/\gamma_0^3 \ll 1$, and thus only a negligible change in the beam current density $\vec{J}^B$. Hence in Eq. (5) we may take $\partial\vec{B}^B/\partial t \approx 0$ behind the beam front. In a time interval $\Delta t = t_f < t_D$, the plasma field $\vec{B}^P$ decays from an initial value of $\vec{B}^P \approx -\vec{B}^B$ to a value which may be written approximately as $\vec{B}^P \approx -(1-f)\vec{B}^B$. For $\Delta t = 0$, $f = 0$, and for $\Delta t \approx t_D$ we have $f \approx 1$. It follows from Eqs. (5) that in the interval Δt, the energy lost by the beam is $2fU_B$, the energy put into the plasma is $(2f - f^2)U_B$, and the energy put into the total magnetic field is f^2U_B, where $U_B = \int d^2x\,(\vec{B}^B)^2/8\pi$ is the magnetic field energy of the unneutralized beam. Evidently a fraction $1-f/2$ of the energy lost by the beam goes into the plasma, and the fraction $f/2$ goes into the magnetic field.

Only a small value of f is required for electrons of an intense beam to lose most of their initial energy γ_0 to the plasma. The fractional beam-energy loss may be written as

$$\frac{\delta\gamma}{\gamma_0}=\frac{2fU_B}{\pi a^2 n_B \gamma_0 m_e c^2}=f\frac{\nu}{\gamma_0}d \tag{6}$$

for a beam of uniform density n_B, where d is a numerical factor close to unity.[8] For say $\nu/\gamma_0 \gtrsim 2$, we have $\delta\gamma/\gamma_0=1$ when $f=\gamma_0/\nu d \lesssim \frac{1}{2}$, and the efficiency of transfer of beam energy to the plasma, $1-f/2 \gtrsim \frac{3}{4}$, is close to unity. As the return current decays, the ratio of the energy of the total magnetic field to the energy of the beam increases and reaches a value corresponding to the Alfvén-Lawson limit[9] for $f=(d/2)\{(1+4\gamma_0/\nu d^2)^{1/2}-1\}$. However, with a suitable magnetic guide field, larger values of f ($\approx\gamma_0/\nu d$) may be achieved.

The energy $(2f-f^2)U_B$ put into the plasma is partitioned between the kinetic energy K (of electrons and ions) and the field fluctuation energy W. The drift of the plasma electrons at velocity v_p is held relatively constant ($\delta v_p/v_p \approx f$) by the high "inductance" of the beam-plasma system and therefore the electron kinetic energy is mainly thermal. If the fraction of the energy $(2f-f^2)U_B$ which goes into kinetic energy is denoted by α (≈ 1), then the final temperature $(T_e+T_i)_f$ may be written thus:

$$\frac{(T_e+T_i)_f}{m_e c^2}=\tfrac{2}{3}d\nu\alpha f\frac{n_B}{n_p}\left(1-\frac{f}{2}\right). \tag{7}$$

Most of the beam energy is extracted ($\delta\gamma/\gamma_0=1$) with $f=\gamma_0/\nu d$, and for this f the plasma temperature increases to $(T_e+T_i)_f=\gamma_0 m_e c^2(n_B/n_p)$. It is clear that the time t_f required to achieve this temperature must be less than t_D by a factor of order γ_0/ν. If the return current were to flow for a time of order t_D, then the energy dissipated by the return current would exceed the beam energy by a factor ν/γ_0. Thus for a beam with high ν/γ we have $t_f \ll t_D$. The precise value of t_f and the factor α must be derived from a theory of the microturbulence which we now discuss.

The microscopic processes involved in heating by return currents are equivalent to heating with currents induced by external fields. However, the use of return currents avoids the skin-effect limitations of currents induced by external fields. Initially, the electron drift velocity $v_{de}=-v_p=(n_B/n_p)v_B$ (in the $+z$ direction) is assumed to be larger than the electron thermal spread $v_e(t=0)$ and that of the ions $v_i(0)$, and we may reasonably set the ion drift velocity[7] $v_{di}(0)=0$. Consequently, in the very early stages the electron-ion interaction develops as a conventional two-stream instability. The unstable wave grows until it traps the electrons in a few growth times and spreads them symmetrically about the wave phase velocity over its trapping width. However, the induction electric field $\vec{E}_0$ acts to hold the current density constant so that at the end of this stage the electrons have a "thermal" spread $v_e \gtrsim v_{de}$, the ions have not been significantly affected, and the instability passes into the ion-acoustic stage.

Heating rates for both electrons and ions and their final temperatures may be estimated by assuming that Fokker-Planck–type equations hold for the distribution functions $F(\vec{v},t)$. The first moments of these equations provide

$$\frac{\partial}{\partial t}\vec{J}^P-\frac{\omega_p^2}{4\pi}\vec{E}_0=-\sum_{j=i,e}e_j\int d^3v\,\overleftrightarrow{D}_j\cdot\frac{\partial}{\partial\vec{v}}F_j, \tag{8}$$

where

$$\overleftrightarrow{D}_j=(e_j/m_j)^2\langle\delta\vec{E}(\vec{x},t)\int_{-\infty}^{t}dt'\,\delta\vec{E}(\vec{x}(t'),t')\rangle,$$

$\delta\vec{E}$ is the stochastic electrostatic field, and $\vec{E}_0$ is the systematic electric field of Eq. (1). The return current is maintained by the relativistic beam through $\vec{E}_0$ and therefore $\partial\vec{J}^P/\partial t \approx 0$. Thus with $\vec{E}_0$ determined by Eq. (8), the second moments of the Fokker-Planck equations give the time evolution of the mean thermal energies. Hence the heating rates are determined in principle from the statistics of the electric field fluctuations. Here, however, more qualitative estimates will suffice.

The particle heating is now dominated by the ion acoustic instability for which $v_i<\omega_k/|\vec{k}|<v_{de}<v_e$. Without going into the intricacies of this instability we note that Sagdeev[10] suggests an effective collision frequency $\tau_*^{-1}\approx 10^{-2}(v_{de}/C_s)(T_e/T_i)\omega_{pi}$, where ω_{pi} is the ion plasma frequency. It is reasonable to assume from plasma turbulence experiments[11] ion temperatures $T_i \lesssim T_e$, and we have $(m_i/m_e)^{1/2}>v_{de}/C_s>1$, so that $10^2(m_e/m_i)^{1/2}(T_i/T_e)<\tau_*\omega_{pi}<10^2$. Thus we have crude bounds on $t_D=\tau_*(a/\lambda_E)^2$. Neglecting other processes this instability continues until either (a) the ions are trapped, or (b) there is linear stability, $C_s \gtrsim v_{de}$, or (c) an equilibrium is reached because of waves convecting energy out of the beam channel.

As an illustration consider a beam with ν/γ_0

$=3.5$, $\gamma_0=2.5$, and $n_B=5\times10^{12}$ cm^{-3} (i.e., a 1.3-MeV, 150-kA beam of 1.4 cm radius) and a hydrogen plasma of density $n_p=10^{14}$ cm^{-3} so that $n_B/n_p\approx0.05$. The final temperature is $(T_e+T_i)_f\approx30$ keV by Eq. (7) with $f\approx\gamma_0/\nu$. The time required to achieve this temperature falls in the range 30 nsec $<t_f<1$ μsec.

We thank Dr. R. E. Lee for valuable discussions on the theory of relativistic beams and return currents.

[1]The notation is $\nu=Nr_e$; N is the number of beam electrons per unit beam length, r_e is the classical electron radius, and $\gamma\approx\gamma_{\parallel}=(1-v_B^2/c^2)^{-1/2}$.

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Fermi-Liquid Behavior of ^{3}He Adsorbed on Liquid Helium*

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Some properties of ^{3}He adsorbed on the surface of liquid helium are determined from measurements of the surface and interfacial tension of ^{3}He-^{4}He mixtures. As a mixture approaches phase separation, the ^{3}He adsorbed on the surface grows continuously into the upper, ^{3}He-rich phase. At all temperatures the limit of low surface density is well described by the Andreev model, although there is some evidence for a weak, attractive quasiparticle interaction.

Andreev[1] has shown that the behavior of the surface tension of dilute solutions of ^{3}He in ^{4}He at high temperatures ($T>0.5$ K) demonstrates that some ^{3}He is adsorbed on the surface of the liquid. In Andreev's model the adsorbed ^{3}He is assumed to be in a set of independent quasiparticle states with energy spectrum[2]

$$\epsilon=-\epsilon_0+p^2/2M. \quad (1)$$

The surface tension of the solution, σ, is then the surface tension of liquid ^{4}He, σ_4, reduced by the two-dimensional "pressure" of the ^{3}He quasiparticle gas on the surface. From measurements of $\Delta\sigma\equiv\sigma_4-\sigma$ at $T>0.5$ K and assuming that the surface quasiparticle gas could be treated by Boltzmann statistics, Zinov'eva and Boldarev[3] determined approximate values for the surface binding energy ϵ_0 and the effective mass M.

There is considerable interest in studying the many-body properties of adsorbed helium particularly in this case since the substrate (liquid helium) is uniquely perfect and homogeneous. We therefore have made further measurements of σ down to low temperatures (0.04 K) and over a wide range of X, the concentration in the bulk phase (30 ppm to saturation). Under these conditions the adsorbed ^{3}He can be changed from a fraction of an atomic layer, when it behaves like