Single-π^+ Electroproduction and the Vector-Meson-Dominance Model*

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Positive pion electroproduction data are compared to predictions of the vector-meson-dominance model. Reasonable agreement is found for all parts of the cross section, except for the longitudinal-transverse interference term.

Recently, data on π^+ electroproduction in the forward direction at intermediate energies ($W \approx 2$ GeV) have become available.[1,2] As is known,[3,4] this reaction provides further tests of the vector-meson-dominance (VMD) model,[5] as long as the virtual-photon mass is not too large.[6]

The differential cross section for this reaction can be written as[7]

$$\frac{d\sigma}{dE'd\Omega_e dt d\varphi} = \Gamma\frac{d\sigma^\gamma}{dtd\varphi} = (2\pi)^{-1}\Gamma\{\sigma_U + \epsilon\sigma_L + \epsilon\cos(2\varphi)\sigma_T + [2\epsilon(\epsilon+1)]^{1/2}\cos(\varphi)\sigma_I\}, \tag{1}$$

where $d\sigma^\gamma/dtd\varphi$ is the virtual photoproduction cross section which, for $K^2=0$, reduces to the real photoproduction cross section.

The data of Ref. 1 were taken for $\varphi = 0°, 180°$ for very small t values. In Ref. 2, the t range is larger, but the bin size is also larger. In the latter experiment, $\sigma_U + \epsilon\sigma_L$, σ_T, and σ_I were separated in the range $0.20\ (\mathrm{GeV}/c)^2 \le K^2 \le 0.75\ (\mathrm{GeV}/c)^2$.

VMD relates the four quantities σ in Eq. (1) to the density-matrix elements for vector-meson production by pions[8]:

$$2\pi\frac{d\sigma^\gamma}{dtd\varphi} = \left(\frac{e}{f_\rho}\right)^2\left(\frac{m_\rho^2}{m_\rho^2+K^2}\right)^2\{\rho_{11} + \epsilon\rho_{00}c^2 - \epsilon\cos(2\varphi)\rho_{1-1} + 2[\epsilon(\epsilon+1)]^{1/2}\cos\varphi\,\mathrm{Re}\rho_{10}c\}$$
$$\times\frac{q^2}{k}\frac{2W}{W^2-m^2}\frac{d\sigma}{dt}(\pi^- p \to \rho^0 n) + \text{additional terms}, \tag{2}$$

where W, q, and k are, respectively, the energy, the pion momentum, and the virtual-photon momentum in the c.m. system. The additional terms are the ω and φ contribution and the interference terms. The factor c describes the variation of the longitudinal amplitudes with K^2. Since the latter must vanish for $K^2=0$, c has a strong variation.

In order to compare Eq. (2) with the electroproduction data, we proceed as follows:

(1) Since σ_U is adequately predicted by VMD for π^+ photoproduction, we only need to use the experimentally determined unpolarized photoproduction cross section and the ratios ρ_{00}/ρ_{11} and $\mathrm{Re}\rho_{10}/\rho_{11}$ from ρ production by pions. For ρ_{1-1}/ρ_{11}, we take the ratio as determined in photoproduction, and not from ρ production, since the VMD prediction is known to fail for this quantity.[5] The photoproduction data used are taken from Burfeindt *et al.*[9] These data are interpolated and transformed from $E = 3.4$ GeV to the required energies, using the empirical fact that $(W^2-m^2)^2 d\sigma/dt$ is approximately constant with energy.

(2) All density matrices are taken in the helicity frame since model studies[10] and experiment[5] favor the VMD extrapolation procedure from $K^2 = -m_\rho^2$ to $K^2=0$ in this frame. The density matrices which we used are taken from the compilation in Scharenguivel *et al.*[11] at $q_{\mathrm{lab}} = 2.7$ GeV/c, i.e., the energy nearest to the electroproduction experiments. Whether one should compare the quantities σ in Eq. (1) with the density matrices in Eq. (2) at the same t, or same $t-t_{\mathrm{min}}$, is not *a priori* clear. The presence of the pion pole would favor the former; however, the fact that σ_T and σ_I (and the spin-flip parts of σ_U and σ_L) vanish for $\theta=0$, favors the latter. Both assumptions have been tried, and the results for

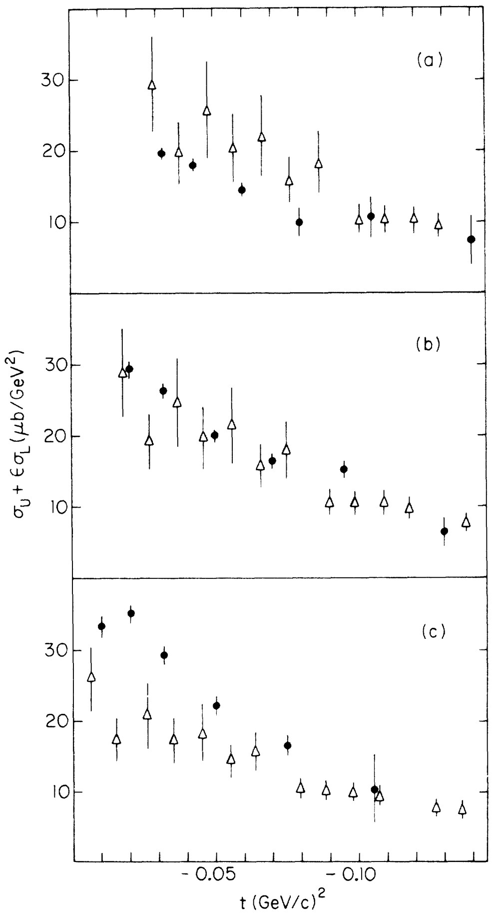

FIG. 1. Comparison of experimental data from Ref. 2 (circles) and VMD (triangles) for $\sigma_U + \epsilon\sigma_L$ vs t for (a) $K^2 = 0.75$, (b) 0.55, and (c) 0.26 (GeV/c)^2. The VMD results are obtained with the function $c_{\mathrm{c.m.}}$.

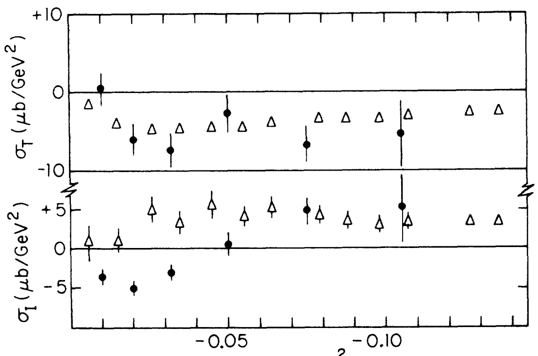

FIG. 2. Comparison of experimental data from Ref. 2 (circles) and VMD (triangles) for σ_T and σ_I vs t for $K^2 = 0.26$ (GeV/c)^2, using the function $c_{\mathrm{c.m.}}$.

the latter are presented here, being slightly better for the very small t values. For larger t, the difference becomes negligible.

(3) The function c can be obtained by assuming the matrix elements of the source density of the ρ^0 field to be smooth functions of K^2 in a certain reference frame. Sakurai, using the lab frame, finds[12] $c_{\mathrm{lab}} = (K^2/m_\rho^2)^{1/2} k_0^L(m_\rho^2)/k_0^L(K^2)$, where $k_0^L(m_\rho^2)$ and $k_0^L(K^2)$ are the lab energies of the ρ meson and the virtual photon, respectively. If one assumes smoothness in the c.m. system, the result becomes $c_{\mathrm{c.m.}} = (K^2/m_\rho^2)^{1/2} k_0(m_\rho^2)/k_0(K^2)$, with the corresponding c.m. energies.[13] A function c which has an intermediate behavior is $c_I = (K^2/m_\rho^2)^{1/2}$. We shall use these three functions.

(4) Of the additional terms, only the ρ-ω interference term can be of importance. We neglect it here, since at small t values $[|t| < 0.1$ (GeV/c)$^2]$ the correction is smaller than 20%, and becomes smaller for smaller $|t|$, as can be estimated from the ratio f_ρ/f_ω, and the measured ρ and ω production cross sections.[14] This is confirmed by measurements of the ratio of π^+ and π^- photoproduction,[15] and for one kinematical configuration in π^- electroproduction.[16] For larger t values, one may have to compare VMD predictions with the average of π^+ and π^- electroproduction data.

The results for $\sigma_U + \epsilon\sigma_L$ obtained in this way turn out to be the most successful for $c_{\mathrm{c.m.}}$, c_{lab} giving results roughly a factor of 2 too low. The predictions with c_I lie in between. The comparison is displayed in Fig. 1. The VMD prediction for the average of the 0° and 180° cross section of Ref. 1 $[K^2 \approx 0.4$ (GeV/c)$^2]$ is of the same quality as the one in Fig. 1(b).

The VMD predictions for $K^2 = 0.26$ (GeV/c)^2 for σ_T and σ_I are given in Fig. 2. Since σ_T is merely obtained from photoproduction data and the ρ-dominated form factor, the test is not too stringent. The prediction of σ_I, on the other hand, depends on the ρ-production data, and is in disagreement with experiment. The electroproduction data show a zero, whereas Reρ_{10} does not.[17] At $K^2 \approx 0.4$ (GeV/c)^2, there is also evidence for a zero from Ref. 1. At higher K^2, Ref. 2 no longer gives a zero. In that case, VMD predicts for all t the incorrect sign.

Next, we use data[2] at fixed $t - t_{\mathrm{min}} = -0.01$ (GeV/c)^2, to compare the K^2 dependence of all quantities σ with the VMD prediction. This dependence is governed by $F_\rho = m_\rho^2/(K^2 + m_\rho^2)$, and for $\sigma_U + \epsilon\sigma_L$ and σ_I there is an additional K^2 dependence through c. The data for σ_T are not accurate enough to allow a definite statement on the correctness of F_ρ. Although the t dependence of σ_I is not predicted correctly by VMD, its K^2 dependence may be correct. Assuming this, we can solve for c from the ratio $(\sigma_U+\epsilon\sigma_L)/\sigma_I$. For a value of $\rho_{11}/\rho_{00}=0.21$, it is possible to choose a value of $\mathrm{Re}\rho_{10}/\rho_{00}=0.066$ (which is inside the errors: 0.036 ± 0.048) for which, at all K^2, there exists a solution for c. The values found are again the most compatible with $c_{\mathrm{c.m.}}$. With the above values for the ratios of the density matrices and $c_{\mathrm{c.m.}}$, one can determine F_ρ from $\sigma_U+\epsilon\sigma_L$ and σ_I. The obtained values (F_{exp}) from $\sigma_U+\epsilon\sigma_L$ are compared in Table I with the VMD F_ρ. (The values from σ_I have larger errors.) There is a tendency for F_{exp} to lie above F_ρ for $K^2\lesssim0.6$ $(\mathrm{GeV}/c)^2$ and below F_ρ for higher K^2.[18] This statement, however, does depend on the values of the ρ density matrices.

Furthermore, one can perform another test which is independent of the chosen density matrices and independent of the ω-ρ interference terms. Taking again the data at $t-t_{\mathrm{min}}=-0.01$ $(\mathrm{GeV}/c)^2$, and the photoproduction limit thereof, one solves for $c^2\rho_{00}/\rho_{11}$ from $\sigma_U+\epsilon\sigma_L$. Then, the quantity

$$R=-\sigma_I(k/F_\rho^{\;2}c)(\rho_{11}/\rho_{00})^{1/2} \qquad (3)$$

should be constant with K^2. The values of R are listed in Table I, from which follows compatibility with a constant $R\approx3.9$. It is interesting to note that VMD predicts a value $R=-1.52\pm2.00$ from the density matrices. The difference in sign is due to the wrong sign prediction of VMD for σ_I, but the absolute value of R is not inconsistent.

In summary, one arrives at the conclusion that the t dependence of the interference term σ_I is badly described by VMD (like the interference term σ_T in photoproduction[5]), but that other quantities are in reasonable agreement with VMD. Deviations may exist which can only be clarified by more accurate vector-meson and electroproduction data, preferably at higher energies, where the uncertainties in the extrapolations in K^2 should be less severe.

TABLE I. K^2 dependence of the ρ-dominated form factor F_ρ, the experimental form factor F_{exp}, and R as defined in Eq. (3).

K^2 $(\mathrm{GeV}/c)^2$	$F_\rho^{\;2}$	$F_{\mathrm{exp}}^{\;2}$	R
0.200	0.555	0.680 ± 0.025	2.66 ± 0.95
0.360	0.383	0.571 ± 0.017	4.94 ± 1.05
0.475	0.325	0.383 ± 0.016	5.63 ± 1.68
0.625	0.234	0.233 ± 0.011	3.44 ± 1.79
0.675	0.216	0.206 ± 0.009	3.17 ± 1.72
0.825	0.172	0.112 ± 0.009	5.41 ± 2.77

Note added in proof. —After this work was completed, we received a preprint by Fraas and Schildknecht on a similar analysis. Using density matrices at $q_{1\mathrm{ab}}=15$ GeV/c and the function c_I, they obtain agreement with $\sigma_U+\epsilon\sigma_L$. That we do not find agreement using c_I is due to the large difference between their ratio ρ_{00}/ρ_{11} and ours.

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[1]C. N. Brown, C. R. Canizares, W. E. Cooper, A. M. Eisner, G. J. Feldman, C. A. Lichtenstein, L. Litt, W. Lockeretz, V. B. Montana, and F. M. Pipkin, Phys. Rev. Lett. 26, 987 (1971).

[2]C. Driver, K. Heinloth, K. Höhne, G. Hofmann, P. Karow, D. Schmidt, G. Specht, and J. Rathje, Phys. Lett. 35B, 77 (1971).

[3]C. Iso and H. Yoshii, Ann. Phys. (New York) 51, 490 (1969).

[4]C. Iso and D. Schildknecht, Nucl. Phys. B21, 242 (1970).

[5]For a general review of the present status of VMD, see J. J. Sakurai, in *Proceedings of the Fourth International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, September 1969,* edited by D. W. Braben and R. E. Rand (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970). For specifics on pion photoproduction, cf. D. Schildknecht, DESY Report No. 69/41.

[6]It is known that for $K^2>1$ $(\mathrm{GeV}/c)^2$, one may enter the deep inelastic region, where as far as total electroproduction cross sections are concerned, VMD disagrees with experiment. See M. Breidenbach, thesis, Massachusetts Institute of Technology (unpublished), and Massachusetts Institute of Technology–Laboratory of Nuclear Science Technical Report No. 2098-635.

[7]For convenience, we follow the notation of Ref. 2. Note that the angle φ differs 180° from the one in Ref. 1.

[8]The kinematical factor in front of $d\sigma/dt$ arises from the definition of Γ (Ref. 1) and the time reversal of the ρ production reaction.

[9]H. Burfeindt *et al.*, Phys. Lett. 33B, 509 (1970).

[10]C. F. Cho and J. J. Sakurai, Phys. Lett. 30B, 119 (1969).

[11]J. H. Scharenguivel *et al.*, Phys. Rev. Lett. 24, 332 (1970). We used these density matrices with the assumption $\mathrm{Tr}\rho=1$. This is not entirely correct, since also the s-wave π-π term is included in the normalization of the published density matrices. From model-dependent fits given in this reference one can verify that the above assumption is reasonable.

[12]J. J. Sakurai, Phys. Rev. Lett. 22, 981 (1969).

[13]$c_{\mathrm{c.m.}}$ would develop a pole at $K^2=W^2-m^2$. However, we are still far away from this value, W^2 being 4.84

GeV^2.

[14]See, e.g., B. Hyams *et al.*, Nucl. Phys. B7, 1 (1968); J. Matthews *et al.*, Phys. Rev. Lett. 26, 400 (1971).

[15]K. Lübelsmeyer, in *Proceedings of the Fourth International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, September 1969,* edited by D. W. Braben and R. E. Rand (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970).

[16]C. N. Brown *et al.*, private communication.

[17]$\mathrm{Re}\rho_{10}$ at higher energies (Ref. 11) does show some indication of a zero, but at smaller t values than observed in electroproduction.

[18]With this F_{exp} the maximum in $\sigma_U + \epsilon\sigma_L$ around $K^2 \approx 0.36\ (\mathrm{GeV}/c)^2$ is correctly reproduced, which is not possible with F_ρ and $c_{\mathrm{c.m.}}$ (but is with F_ρ and c_{lab} and c_I).

ERRATA

SELF-CONSISTENT FIELD THEORY OF RELATIVISTIC ELECTRON RINGS. George Schmidt [Phys. Rev. Lett. 26, 952 (1971)].

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PRODUCTION OF ELECTRON PAIRS FROM A ZERO-MASS STATE. Howard R. Reiss [Phys. Rev. Lett. 26, 1072 (1971)].

An omission from the printed text essentially reverses the meaning of the sentence which starts on page 1073, column 2, line 3. In this sentence, the phrase "sum over all processes with N_{min}" should read "sum over all processes with $N \geq N_{\mathrm{min}}$."

EXPERIMENTAL EVIDENCE FOR DYNAMIC NUCLEAR POLARIZATION BY COOLING OF ELECTRON SPIN-SPIN INTERACTIONS. M. Borghini and K. Scheffler [Phys. Rev. Lett. 26, 1362 (1971)].

The unnumbered equation following Eq. (3) should read

$$P \equiv P(x) = P_0\left[1 - \frac{tg(\omega)}{\langle\lambda\rangle + tg(\omega)}\lambda(x-\omega)\right].$$