in Fig. 3 are based on this assumption. The inclusion of contributions from the $N=6$ shell made little difference on the calculated shapes. The DWBA calculations to the 0^+ transitions are generally good although the shallow first minimum observed in the ground and first excited 0^+ states in the reaction $^{154}\mathrm{Gd}(p,t)^{152}\mathrm{Gd}$ is not reporduced by the calculations. On the other hand, the DWBA calculations show a filling in of this minimum with increasing excitation energy. Such a shallow first minimum in the $^{152}\mathrm{Gd}$ 0^+ data is not a feature of the (p,t) $L=0$ transitions in the higher-mass Gd nuclei (cf. Fig. 3). Use of the triton-potential from Jaskola *et al.*,[22] which was chosen to fit the (d,t) g.s. angular distribution on $^{160}\mathrm{Gd}$, considerably worsened the agreement with the (p,t) data. Other parameter choices were not attempted. It should be noted that in the reaction $^{160}\mathrm{Gd}(p,t)$-$^{158}\mathrm{Gd}$, the position of the first minimum in the angular distribution to the 1452-keV 0^+ β vibration is shifted inward by about 5° with respect to the g.s. transition, and this shift is qualitatively reproduced by the calculations. A similar shift in the position of the first minimum for (p,t) reactions populating excited 0^+ states has been reported in the actinides.[5]

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Isospin Splitting of the Giant Dipole Resonance, and Muon Capture

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The correspondence between the upper fragment of the split electromagnetic giant dipole resonance and the parent dipole mode excited by muon capture is investigated for nuclei with a neutron excess. Results already found for light $(N=Z)$ nuclei are extended to heavier $(N>Z)$ nuclei, suggesting new types of experiments to demonstrate the isospin origin of this splitting.

There is considerable interest at present in isospin splitting of the electromagnetic giant dipole resonance (gdr) of nuclei with extra neutrons; in such nuclei, two dipole modes with different values of the isospin quantum number are expected to appear in rather well-separated energy regions.[1] Numerous recent photonuclear experiments tend to confirm the reality and the isospin origin of such a splitting.[2] However, none of them, except maybe one,[3] gives a clear-cut indication of the analog character of the upper fragment. On the other hand, considering a nucleus with $N-Z=2T=-2T_3$, isospin sum rules involve reduced matrix elements which are identical for transitions $(T\to T+1,\ \Delta T_3=0)$ and $(T\to T+1,\ \Delta T_3=-1)$. Foldy and Walecka[4] used this analogy to investigate the dipole transitions induced through muon absorption by light $(N=Z)$ nuclei, starting from experimental data on photonuclear reactions. The object of this note is to investigate the FW scheme for nuclei with excess neutrons.

This extension would connect in a quantitative manner the dipole mode excited by muon capture and the upper fragment of the gdr. Such a correspondence suggests the use of muon capture experiments to demonstrate the analog character of this fragment and therefore the isospin origin of the splitting of the gdr.

The steps taken by FW are summed up below. We essentially follow their notation and introduce the following expressions:

$$|M_{V,A,P}|^2=\overline{\sum_a}\sum_b\left(\frac{\nu_{ab}}{\nu_\mu}\right)^2\int\frac{d\hat{\nu}}{4\pi}|\langle b|\sum_{i=1}^A\tau_{i-}O_{V,A,P}{}^{(i)}\exp(-i\vec{\nu}_{ab}\cdot\vec{x}_i)|a\rangle|^2,$$

where

$$O_V{}^{(i)}=1_i,\quad O_A{}^{(i)}=\vec{\sigma}_i/\sqrt{3},\quad O_P{}^{(i)}=\vec{\sigma}_i\cdot\hat{\nu};$$

$|a\rangle$ and $|b\rangle$ are initial and final states of the nuclear system, respectively; and $\nu_{ab}/\nu_\mu=\nu_{ab}/m_\mu c^2$, the energy of the outgoing neutrino for the nuclear transition over the rest mass of the muon. $|M_{V,A,P}|_L{}^2$ is the form obtained from $|M_{V,A,P}|^2$ on replacing $\exp(-i\vec{\nu}_{ab}\cdot\vec{x}_i)$ by the Lth multipole expansion term $\omega_L(i)$;

$$\sigma_{-1}=\int_0^\infty\frac{\sigma_\gamma(E)}{E}\,dE,$$

where $\sigma_\gamma(E)$ is the photonuclear absorption cross section of a proton of energy E.

The procedure is then as follows: (a) Use photoabsorption data to extract the corresponding (unretarded, angular momentum $L=1$) matrix element $|M_V|_{\text{U.D.}}{}^2$, where U.D. is the unretarded dipole. (b) Obtain the dipole part of $|M_V|^2$ through multiplication by the elastic form factor $F_{\text{el}}(\nu)$. For the nuclei under study, it is important to recall the relative predominance of $|M_V|_{L=1}{}^2$ compared to $|M_V|_{L\neq1}{}^2$. This justifies the FW evaluation of other multipole contributions by the shell model. (c) Assume the equality of $|M_V|^2$, $|M_A|^2$, and $|M_P|^2$ defined above. These equalities are exact in the supermultiplet scheme and not much perturbed in more refined particle-hole calculations.[5] Recall that for the light nuclei under study, $|M_{V,A,P}|^2$ summed over $J^\pi=0^-,\ 1^-,\ 2^-$ is equal to $|M_{V,A,P}|_{L=1}{}^2$ up to a high accuracy.

Extraction of $|M_V|_{U.D.}{}^2$ *from photonulcear data.*—Here, $|M_V|_{\text{U.D.}}{}^2$ is related to the upper component of the gdr. The equations below are useful guides to estimate the relevant quantities[1,6]:

$$(E_{T+1})/E_T=\tilde{U}(T+1)/T, \tag{1}$$

$$\{(\sigma_{-1})_{T+1}\}/(\sigma_{-1})=(T+1)^{-1}(1-3T/2A^{2/3}), \tag{1'}$$

where E_T, E_{T+1} are the respective energies of the centroids of the T and $T+1$ components of the gdr; $\tilde{U}$ is simply related to the symmetry energy; σ_{-1} is defined above.

For light nuclei, with small neutron excess, the relative strength of the $T+1$ fragment is large enough to be measured directly in order to get $|M_V|_{\text{U.D.}}{}^2$, exactly as in the FW case. In sufficiently heavy nuclei, the measurement of the upper fragment may be difficult; then $(\sigma_{-1})_T$ is well-known experimentally and can yield $(\sigma_{-1})_{T+1}$ through the sum rule of Eq. (1) and hence the matrix element $|M_V|_{\text{U.D.}}{}^2$.

Relation between $|M_V|_{U.D.}{}^2$ *and* $|M_V|_{L=1}{}^2$.—This problem has already been studied in a former publication.[7] The multiplicative factor is in principle more complicated, but is in practice equal to the elastic form factor up to momentum transfers higher than those involved in muon capture.

The relative importance of the various multipole terms is estimated in the frame of a pure shell model in Table I. Two factors decrease the relative importance of dipole states (compared to quadrupoles for instance): the increase of retardation effects with the nuclear radius, and the quenching due to excess neutrons which affects more strongly the dipole transitions. Up to $A\approx100$, once-forbidden transitions are dominant and $L=3$ transitions contribute less than 10%.

TABLE I. Relative percentage of multipole transitions in a pure shell model.

J	Ca^{40}	Ca^{48}	Sr^{88}	Ce^{140}	Pb^{208}
0^+	7	9	15	18	29
1^-	76	73	51	34	8
2^+	13	16	27	38	54
3^-	3	2	6	9	7
4^+	~0	~0	~0	1	2

Investigation of the equality between the $|M_{V,A,P}|^2$.—The isospin relation $2\tau_{i-} = -[T^-, \tau_{i3}]$ leads to

$$|M_{V,A,P}|_L{}^2 = \tfrac{1}{4}\overline{\sum_a}\sum_b\left(\frac{\nu_{ab}}{\nu_\mu}\right)^2\Big|\sum_{b'}\langle b|\sum_{i=1}^A O_{V,A,P}{}^{(i)}\tau_{i-}|b'\rangle\langle b'|\sum_{i=1}^A \tau_{i3}\omega_L{}^{(i)}|a\rangle - \sum_{b''}\langle b|\sum_{i=1}^A \tau_{i3}\omega_L{}^{(i)}|b''\rangle\langle b''|\sum_{i=1}^A O_{V,A,P}{}^{(i)}\tau_{i-}|a\rangle\Big|^2. \tag{2}$$

FW, on the basis of supermiltiplet theory applied to $N=Z$ nuclei, assume the second term of the right-hand side of Eq. (2) to be zero and arrive at

$$|M_{V,A,P}|_L{}^2 = \tfrac{1}{2}\overline{\sum_a}\sum_{b'}\left(\frac{\nu_{ab}}{\nu_\mu}\right)^2\int\frac{d\hat{v}}{4\pi}|\langle b'|\sum_{i=1}^A \tau_{i-}\omega_L{}^{(i)}|a\rangle|^2(T+1). \tag{3}$$

Such a relation is found to be still valid for $N>Z$ nuclei with *either* a doubly closed shell in protons, *or* a doubly empty shell in neutrons, provided we neglect the spin-orbit coupling which has been shown to create only a small effect.[8] The derivation of Eq. (3) is based on the cancelation of the second term of the right-hand side of Eq. (2) (allowed β^- transition matrix element), and also on summations of LRT type.[9] Inequalities between the $|M_{V,A,P}|_{L=1}{}^2$ come from non-LRT transitions—for instance, the transition $1p_{3/2}\to 1p_{1/2}$ in C^{12}, giving for the second term of the right-hand side of Eq. (2) zero for $|M_V|_{L=1}{}^2$ and nonzero for $|M_A|_{L=1}{}^2$. The quantities $|M_{V,A,P}|_{0^-,1^-,2^-}{}^2$ can then be investigated for the various transitions which occur in a given nucleus. For LRT transitions,

$$|M_{V,A,P}|_{0^-,1^-,2^-}{}^2 = |M_{V,A,P}|_{L=1}{}^2 + |M_{V,A,P}|_{L=3(2^-)}{}^2;$$

for non-LRT transitions,

$$|M_{V,A,P}|_{0^-,1^-,2^-}{}^2 = |M_{V,A,P}|_{L=1}{}^2 + |M_{V,A,P}|_{L=3(2^-)}{}^2 + [L, L'=1,3],$$

where $|M_{V,A,P}|_{L=3(2^-)}{}^2$ involves a sum over a truncated set of states $|b'\rangle$, and $[L, L'=1,3]$ is an interference term. Thus, the modification of the equalities between $|M_{V,A,P}|_{0^-,1^-,2^-}{}^2$ comes from the slowly increasing contribution of $L=3$ and, in the case of non-LRT transitions, from the $L=3$ contributions and also the interference terms which destroy the meaning of L. Table II shows the evolution with increasing A.

A calculation of $|M_{V,A,P}|_{0^-,1^-,2^-}{}^2$ in Sr^{88}, using a residual two-nucleon potential of Soper type, yields the same features as the various hole-particle calculations done in $N=Z$ nuclei: The axial-vector strength is quite concentrated in the highest energy state and the polar vector strength shared between the second and third state; despite the redistribution of strength, we still have a near equality, $|M_V|^2:|M_A|^2:|M_P|^2 = 1:1.03:1.06$.

In conclusion, up to $A\approx 100$, the upper fragment of the gdr still is correlated in a significant way to the dipole excitation induced by muon capture; the excitation of such a collective mode and its identification (γ decay, neutron escape, etc.) at

TABLE II. Comparison of the expressions $|M_{V,A,P}|_{0^-,1^-,2^-}{}^2$ over several nuclei. Note that Ca^{40}, Ca^{48}, and Ce^{140} involve only LRT transitions, so that subtraction of the $L=3$ contribution yields exact equality; in Sr^{88}, the only non-LRT transition $(2p_{3/2}-1g_{7/2})$ is $L=3$ and only gives a small correction; Pb^{208} presents an $L=1$ non-LRT transition $(1h_{11/2}-1i_{11/2})$ which strongly modifies the equality and introduces large interference terms.

	$\lvert M_{V,A,P}\rvert_{0^-,1^-,2^-}{}^2$				
	V	A	P	$\lvert M_A\rvert_{L=3(2^-)}{}^2$	$\lvert M_P\rvert_{L=3(2^-)}{}^2$
Ca^{40}	3.083	3.109	3.130	0.026	0.046
Ca^{48}	1.428	1.442	1.456	0.015	0.026
Sr^{88}	2.358	2.449	2.523	0.091	0.164
Ce^{140}	1.772	1.900	2.003	0.129	0.231
Pb^{208}	0.268	1.309	1.366	0.069	0.125

the expected energy should bring a neat proof of the isospin nature of the splitting. The relative contribution of the dipole part may become too small to consider other multipoles as corrections and then evaluate total muon capture rate as FW did; however, comparison of results with experimental total-capture-rate data for several nuclei can bring some interesting information on the shell-model estimates of other multipole contributions; this aspect, as well as several points only touched upon in this note, shall be investigated in a forthcoming publication.

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Measurements of Polarization in $\pi^- p$ Elastic Scattering at Large Angles*

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We have made measurements of polarization in π^-p elastic scattering, with emphasis over the backward region, at 1.60 to 2.28 GeV/c. The results indicate the absence of u-channel dominance in the backward region, as was observed in the case of π^+p scattering. Comparisons have been made with predictions of various phase-shift analyses which show that the agreement is generally very poor in the backward region.

Recent polarization measurements[1,2] of π^+p elastic scattering at large angles in the region of pion incident momenta below 2.75 GeV/c revealed the following interesting phenomena: (1) large changes in the sign of polarization, with respect to incident momenta, in the backward region, (2) a dip[2] in the polarization at constant $u\sim-0.65$ (GeV/c)^2, and (3) poor agreement in the backward region with predictions of the existing phase-shift analyses.[3]

In order to continue our study of these problems, we have measured polarization in π^-p elastic scattering in the energy range between 1.60 and 2.28 GeV/c, with emphasis on the backward region. Previous measurements[4-6] in a similar energy range have covered mainly the forward