where the prime indicates that the product is taken over $m = 1$ to n, except for $m = k$ and $m = l$.

The identity is easily verified for $n = 1$ and 2. Now, assume that the identity holds for $n - 2$. Then, relying on properties (i), (ii), and (iii), it is seen that D can be factored by a symmetric homogeneous polynomial

$$\prod_{k<l}^{(n)} (P_k^2 - P_l^2)^2$$

of degree $2n(n-1)$. On the other hand, Eq. (13) shows the degree of D to be $n(n-1)+4$. Hence, D must be zero for n, and the identity holds.

[1] I. Kay and H. E. Moses, J. Appl. Phys. 27, 1503 (1956).
[2] C. S. Gardner, J. M. Greene, M. D. Kruskal, and R. M. Miura, Phys. Rev. Lett. 19, 1095 (1967).
[3] J. Rubinstein, J. Math. Phys. 11, 258 (1970).

Energy Loss of H, D, and ^{4}He Ions Channeled Through Thin Single Crystals of Silicon*

G. Della Mea, A. V. Drigo, S. Lo Russo, and P. Mazzoldi
Istituto di Fisica dell'Università, Unità del Gruppo Nazionale di Struttura della Materia, Padova, Italy

and

G. G. Bentini
Laboratorio per la Chimica e Tecnologia dei Materiali e Componenti per l'Elettronica, Consiglio Nazionale dell Ricerche, Bologna, Italy
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The energy loss of ions channeled through the ⟨111⟩ Si channel is studied in the energy range 0.9 to 5.0 MeV. The energy dependence of the ratio between channeling and random stopping power above 3 MeV shows an increase which can be interpreted in terms of core-electron excitation. The velocity dependence of the channeling stopping power is also studied.

In recent years many experiments have been performed on the energy loss of light ions channeled through semiconductor single crystals in the energy range above 3 MeV.[1-3] The results have been discussed in terms of localized and nonlocalized contributions to the electronic stopping power.[2] Appleton, Erginsoy, and Gibson[1] used their results to extract the local density of valence electrons, sampled by the well-channeled protons along the Si ⟨110⟩ axial direction. The value they obtained in this way was about 4.

In the present work we extend the energy-loss measurements to the lower energy region, using H^+, D^+, and $^4He^+$ ions. Our aim is to check (a) the channeling energy loss in the energy range where the incident ions interact only with the weakly bounded valence electrons; (b) the energy threshold for the core-electron contribution to the stopping power; (c) the mass dependence of the channeling energy loss at low energy; and (d) the energy dependence of the ratio between channeling and random stopping power. In this Letter we present some preliminary results concerning the Si ⟨111⟩ axial direction.

The incident beam, obtained from the 5.5-MeV Van de Graaff accelerator of Laboratori Nazionali Legnaro, was collimated by annular collimators of various sizes and arranged at given distances. The minimum hole was 0.3 mm in diameter and the maximum divergence of the beam was always kept better than 0.1°. The thickness of the targets ranged from 1.5 to 32 μm. The thickness of the samples was carefully checked by the energy loss of the transmitted particles in random conditions, using the tabulated stopping power.[4]

In the case of ^{4}He ions, corrections to these values, due to a reduction in the effective charge of the particles, could also be considered, as suggested by Bloom and Sauter.[5] Since, however, only a few experimental data are available at present on this point, we preferred to follow Williamson's treatment which, in addition, yielded a measured value for the thickness of the sample, which was constant with varying energy.

The crystals were clamped on a three-axis goniometer and the energy of the transmitted beams was analyzed by means of a large silicon solid-state detector placed 4 mm behind the sample. The acceptance angle of the detector was about 100°.

The normal spectrum we obtain is equivalent to that expected for an amorphous silicon sample of the same thickness, while the channeling spectrum shows both a peak at higher energy and the contribution of the beam dechanneled fraction. The latter contribution depends on the thickness of the target and on the incident energy. The true most-probable energy loss of the best-channeled particles was taken as the peak of a Gaussian curve fitted to the high-energy side of the spectrum.

The results are discussed in terms of the experimental effective stopping number $\overline{B}$. The effective stopping number B is defined from Bethe's treatment of energy loss, which applies the Born approximation to the collisions between the incident particle and the atomic electrons. Bethe's expression for the stopping power dE/dx can be written as

$$\frac{dE}{dx} = -\frac{C}{E}\ln(bE) = -\frac{B(E)}{E}, \qquad (1)$$

where C and b depend on the stopping material. The experimental value $\overline{B}$ is calculated by

$$\overline{B} = \frac{1}{\Delta x}\int_{E_f}^{E_i} E\, dE = \frac{E_i^2 - E_f^2}{2\Delta x},$$

where E_i is the energy of the ion incident on the crystal, E_f is the energy of the ion emergent from the crystal, and Δx the crystal thickness. From Bethe's theory, $\overline{B}/AZ^2$ is a universal function of velocity (A is the mass number and Z the nuclear charge of the incident particle).

As mentioned before, we took into account the effective charge of particles in the analysis of $^4He^+$ random spectra, through Williamson's data. The same correction has been used for the analysis of the channeling data. This is equivalent to assuming that the charge state of the particles is unaffected by the channeling condition. However, in our opinion, this is still an open problem.

Figure 1 shows the function $\overline{B}/AZ^{*2}$ vs $\overline{E}/A$, where $Z^* = \gamma Z$ is the effective charge, $\gamma = \tanh[137 \times (\pi/2)^{1/2} V/CZ]$ is the effective-charge parameter,[4] and $\overline{E} = \frac{1}{2}(E_i + E_f)$. $\overline{B}/AZ^{*2}$ values have also been calculated from the data in Ref. 1, using the results reported in their Table III. The experimental uncertainties shown in Fig. 1 are mainly due to the Gaussian fit of the energy spectra.

The comparison of the energy losses of H^+, D^+, and $^4He^+$ shows that, within the experimental uncertainties, velocity is the important parameter in the slowing down of channeled ions at low velocities.

It is well known that both valence and core electrons contribute to the stopping power. Excitation of core electrons, however, shows a dependence on the impact parameter.

The adiabaticity criterion allows one to estimate the maximum distance ρ_{max} at which excitation of a shell is still possible, for a given shell electron binding energy ΔE and for a given ion velocity v, through

$$\rho_{max} = \hbar v/\Delta E.$$

Assuming for silicon L-shell electrons $\Delta E = 120$

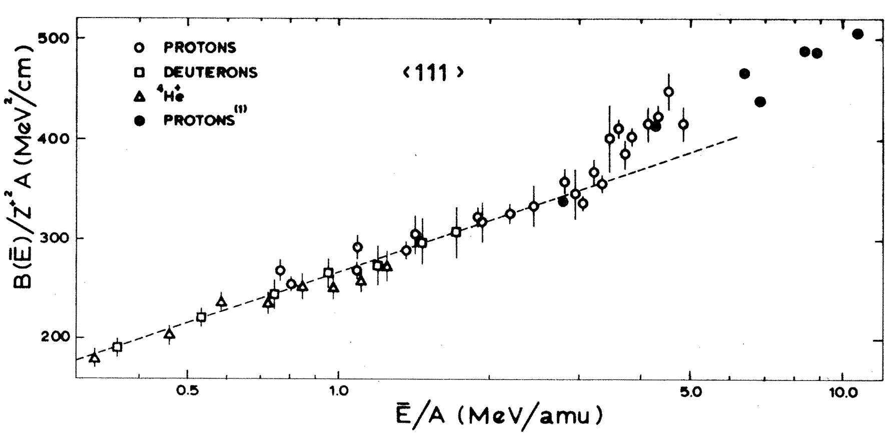

FIG. 1. Plot of universal stopping number $\overline{B}/AZ^{*2}$ of protons, deuterons, and ^4He ions incident along the Si ⟨111⟩ axis, showing the A and Z dependence of channeling for light ions. The dashed line represents a linear best fit to the data below 3 MeV/amu energy.

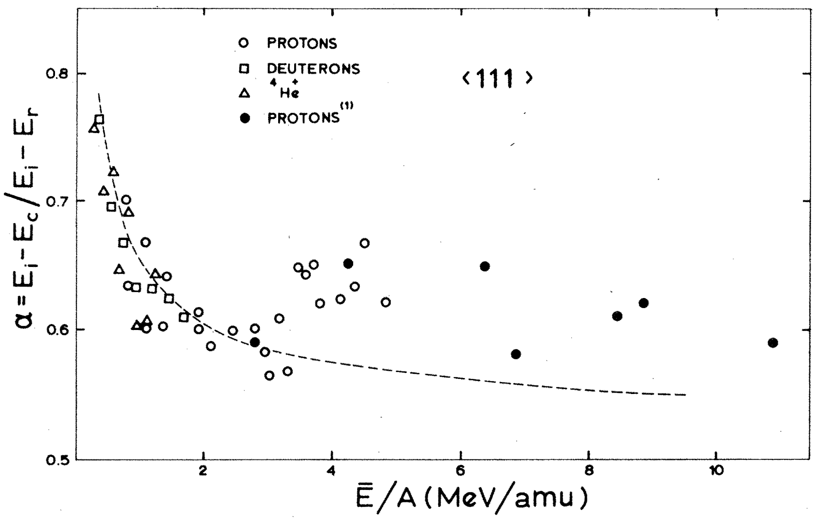

FIG. 2. The ratio α between channeling and random stopping power as a function of energy. The dashed line represents the α ratio calculated from the best fit of Fig. 1 and the random tabulated data.

eV and $\rho_{max} = 1.3$ Å, the ⟨111⟩ channel radius, we obtain from Eq. (2) the minimum ion energy at which the contribution of the L shell starts. The calculated value is about 3 MeV/amu.

The trend of $\bar{B}/AZ^{*2}$ vs $\bar{E}/A$ (Fig. 1) shows a change at about 3.1 MeV/amu, supporting the hypothesis of a threshold contribution of core electrons to the stopping power. Hence at energies below 3 MeV/amu the stopping power might be due only to the interaction with the valence electrons.

A linear best fit of our data gives for C and b in Eq. (1) the values $C = 75.2 \pm 3.0$ MeV^2/cm and $\ln b = 3.54 \pm 0.55$.

For protons at ~3 MeV energy the ⟨111⟩ stopping power is greater than the ⟨110⟩ stopping power found by Appleton, Erginsoy, and Gibson.[1] This difference is thought to be due to a higher value of local electron density sampled by the ion inside the ⟨111⟩ channel. However, the analysis of the data in terms of close single collisions and collective excitations[1,6,7] appears difficult.

The ratio $\alpha = (E_i - E_c)/(E_i - E_R)$ between channeling and random stopping power versus $\bar{E}/A$ is shown in Fig. 2. E_c is the energy of the ion emergent from the crystal in channeling condition and E_R that for random condition. The dashed line represents the α values calculated by the ratio $(dE/dx)_c(dE/dx)_R^{-1}$ obtained using the best fit of our channeling data below 3 MeV/amu and the tabulated random data.[4] Figure 2 shows that α is a strongly varying function of energy at low energy. The trend of $\alpha(E)$ above 3 MeV/amu is obviously correlated with the change of the slope of $B(E)$ as shown in Fig. 1.

Further work is in progress to perform similar measurements in other axial and planar directions. Our final aim is to study how the excitation of the core electrons depends on energy and how it relates to the channel radius. We would like also to test the validity of the analysis of the channeling stopping power in terms of close single collisions and distant collective excitations in narrow channels.

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[1] B. R. Appleton, C. Erginsoy, and W. M. Gibson, Phys. Rev. 161, 330 (1967).

[2] G. J. Clark, D. V. Morgan, and J. M. Poate, in *Atomic Collision Phenomena in Solids*, edited by D. W. Palmer, M. W. Thompson, and P. D. Townsend (North-Holland, Amsterdam, 1970), p. 388.

[3] A. R. Sattler and G. Dearnaley, Phys. Rev. 161, 244 (1967).

[4] C. F. Williamson, J. F. Boujot, and J. Picard, Commissariat à l'Energie Atomique Report No. R-3042, 1966 (unpublished); L. C. Northcliffe and R. F. Schilling, Nuclear Data, Sect. A 7, 233 (1970).

[5] S. D. Bloom and G. D. Sauter, Phys. Rev. Lett. 26, 607 (1971).

[6] D. Pines, *Elementary Excitations in Solids* (Benjamin, New York, 1963), p. 148.

[7] W. Brandt and J. Reinheimer, Phys. Rev. B 2, 3104 (1970).