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### Collisional Diffusion in the dc Octopole\*

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(Received 28 July 1971)

The collisional diffusion coefficient is measured in the dc octopole plasma. All measurements indicate that the process is classical diffusion for the bulk of the plasma. The value of the coefficient agrees with the theoretical value within the experimental error. Also anomalous transport is observed in the average minimum-*B* region which becomes large near the minimum-length flux line.

It has previously been reported<sup>1</sup> that the plasma loss rate in the collisional regime in the dc octopole is consistent with the classical diffusion rate. In this note more detailed measurements of the transport rate are described.

A detailed description of the machine may be found in an earlier publication.<sup>1</sup> The device is an axisymmetric magnetic well with only a poloidal magnetic field. The plasma, usually hydrogen, is injected from a coaxial plasma gun with an initial density of about  $10^{11}$  cm<sup>-3</sup> (3-cm interferometer). Electron and ion temperatures during the measurement time are both about 0.5 eV. For the hydrogen ion this corresponds to about 18 gyroradii from the separatrix to the stability limit. Normal background pressure is  $1 \times 10^{-7}$  Torr. After injection of the plasma, the influx of neutral gas from the gun increases the pressure to  $1 \times 10^{-5}$  Torr in 0.1 sec.

The density distribution is measured by Langmuir probes. The density is proportional to the reciprocal of time at every point on the profile as shown in Fig. 1. This indicates that the transport coefficient is proportional to density.

The diffusion coefficient may be calculated from the spatial distribution of the density times the time as follows: The diffusion equation in the

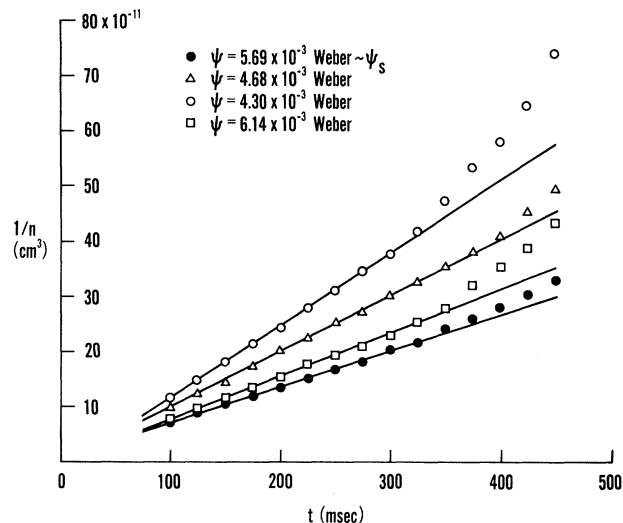


FIG. 1. Reciprocal of plasma density as a function of time. The plasma is a 0.5-eV hydrogen plasma. Plots are given for probe positions both inside and outside the separatrix. The separatrix is at  $\Psi_s = 5.69 \times 10^{-3}$  Wb and the values of  $\Psi$  increase toward the octopole rings.

flux-function coordinates is given by

$$\frac{\partial n}{\partial t} \oint \frac{d\chi}{B^2} - \frac{\partial}{\partial \Psi} \left( \oint DR^2 d\chi \frac{\partial n}{\partial \Psi} \right) = 0, \quad (1)$$

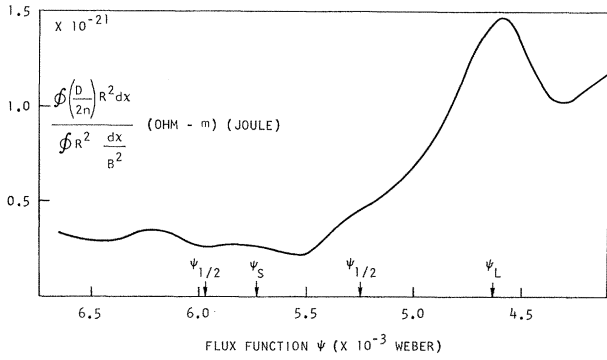


FIG. 2. Experimental value of  $\oint (D/2n)R^2 d\chi / \oint R^2 (d\chi/B^2)$  plotted as a function of  $\Psi$ . The separatrix and the minimum-length line are denoted by  $\Psi_S$  and  $\Psi_L$ , respectively. If the diffusion is classical, the quantity  $\oint (D/2n) \times R^2 d\chi / \oint R^2 (d\chi/B^2) = \eta(T_e + T_i)/2$ .

where  $\Psi$  is the flux function,  $\chi$  is the magnetic potential,  $n$  is the density,  $D$  is the diffusion coefficient,  $B$  is the magnetic field strength, and  $R$  is the major radius. By using  $\partial(nt)/\partial t = 0$ , we obtain

$$\frac{\oint (D/2n)R^2 d\chi}{\oint R^2 (d\chi/B^2)} = \frac{\int_{\Psi_S}^{\Psi_L} \phi nt (d\chi/B^2) d\Psi}{\oint R^2 (d\chi/B^2) [\partial(nt)^2 / \partial \Psi]}. \quad (2)$$

The right-hand side of Eq. (2) is evaluated from the experimental data. If the diffusion is classical, then one obtains

$$\frac{\oint (DR^2/2n)d\chi}{\oint R^2 (d\chi/B^2)} = \frac{\eta(T_e + T_i)}{2}, \quad (3)$$

where  $\eta$  is the electric resistivity of the plasma.

Experimental results are shown in Figs. 2 and 3 for the hydrogen plasma. The right-hand side of Eq. (2) is plotted as a function of  $\Psi$  in Fig. 2. If the transport is classical everywhere, the experimental curve should be a horizontal straight line, as the temperature gradients are negligible. Since the magnetic field is quite nonuniform in space, this measurement represents a sensitive test for the  $B$  dependence of the coefficient. The figure shows that the classical dependence is indeed observed for the bulk of the plasma, since the  $\Psi$  surfaces denoted by  $\Psi_{1/2}$  enclose half the plasma. An anomalous transport is also observed in the average well region, which becomes large near the flux line where  $\phi d\chi/B$  is a minimum. The dependence of the coefficient upon  $B$  was also tested by changing the magnitude of the octopole field (i.e., by changing the magnitude of the ring current). Here again the scaling is seen to go as the inverse square of the field (Fig. 3). As a further test, the above experiments were repeated for a helium plasma. The magnitude of the diffu-

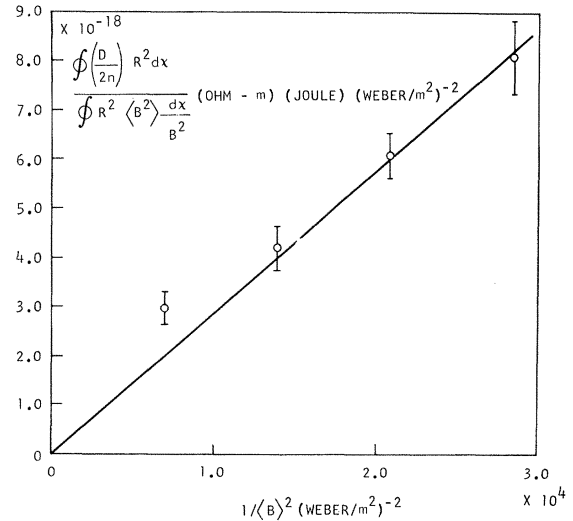


FIG. 3. The quantity  $\oint (D/2n)R^2 d\chi / \oint R^2 \langle B \rangle^2 (d\chi/B^2)$  is plotted as a function of  $1/\langle B \rangle^2$ , where  $\langle B \rangle \equiv \mu_0 I / 2R_0$ ,  $R_0$  is the radius of the multipole axis (1.43 m), and  $I$  is the sum of the octopole-ring currents.

sion coefficient was found to be independent of ion mass within experimental error. This is, of course, also consistent with classical diffusion.

The theoretical value for the diffusion coefficient may be calculated by using Spitzer's<sup>2</sup> value for  $\eta$ ,

$$\eta = 1.29 \times 10^4 \frac{\ln \Lambda}{T_e^{3/2} (\text{°K})} \Omega \text{ cm}, \quad (4)$$

and by assuming  $T_e = T_i$ . This is reasonable because the energy relaxation time between the ions and electrons is much shorter than that of the time between the ions and the neutral gas. If the electron temperature of 0.5 eV measured by a Langmuir probe is used, the experimental value for the diffusion coefficient is 1.5 times that of the theoretical value. Since the temperature measurements by a Langmuir probe probably have a considerable error, the discrepancy may be considered as within the errors. Nevertheless, it is possible that the diffusion rate is actually larger than the classical value. There are several effects which may increase the diffusion coefficient. For example, the particle orbits deviate from a flux line by  $\rho \Delta R / R$ , where  $\rho$  is the gyroradius and  $\Delta R$  is the excursion in the major radius. This shift of the guiding center tends to increase the diffusion rate. The enhancement factor may be roughly  $1 + (\Delta R / R)^2$ . Also the resistivity may be corrected because the electron dielectric constant is not very large compared with unity.<sup>3</sup>

The anomalous transport coefficient has the

following characteristics: (1) It is proportional to density, indicating that the process is collisional; (2) it is weakly dependent on the magnetic field strength; (3) it is small in the minimum- $B$  region; (4) it is peaked near the minimum-length flux line; and (5) it is not associated with low-frequency plasma turbulence. These characteristics are indicative of the collisional diffusion due to a slight asymmetry in the magnetic configuration.<sup>4</sup>

The guiding-center orbits are perturbed when a magnetic configuration has a small asymmetry. The shift of the orbits  $\Delta\Psi$  is given by

$$\Delta\Psi = \Delta J / (\partial J / \partial \Psi), \quad (5)$$

where  $J$  is a longitudinal invariant and  $\Delta J$  is a variation of  $J$  due to the asymmetry. Since  $\Delta\Psi$  has a spread for the particle ensemble, collisions will lead to a diffusion. The diffusion rate is independent of the strength of the magnetic field and depends only on the degree of the asymmetry. The rate is larger than the classical rate if the spread of the orbit becomes larger than the gyro-radius. For the minimum- $B$  region, the polarity of  $\partial J / \partial \Psi$  is the same for all particles. Thus, the spread is small for a given asymmetry. On the other hand, there are particles with opposite sign of  $\partial J / \partial \Psi$  in the average minimum- $B$  region, making the spread large. Therefore, an average minimum- $B$  configuration is much more susceptible to this type of diffusion.

Near the minimum-length line, the orbit shifts become large for particles with small magnetic moment  $\mu$ . By expanding  $\partial J / \partial \Psi$  in  $\mu$ , we have

$$\frac{\partial J}{\partial \Psi} = v \frac{\partial}{\partial \Psi} \oint \frac{d\chi}{B} + O(\mu^2). \quad (6)$$

The shift integrated over  $\mu$  diverges on the minimum-length line, since the first-order term with respect to  $\mu$  is absent. The orbit spread is then given by

$$\begin{aligned} (\Delta\Psi)^2 &= 2\Delta J / (\partial^2 J / \partial \Psi^2) \\ &= \frac{2\Delta \oint (d\chi/B)}{(\partial^2 / \partial \Psi^2) [\oint (d\chi/B)]}. \end{aligned} \quad (7)$$

It is not possible to calculate the transport coefficient without knowing the actual size and the wavelength of the magnetic field asymmetry. It requires only a small amount (roughly 1%) of asymmetry to explain the experimental data because the minimum of the integral  $\oint d\chi/B$  is very broad, i.e.,  $\partial^2 J / \partial \Psi^2$  is small.

The asymmetry could also be caused by an electric field produced by the instability. However, the absence of the low-frequency fluctuation and the lack of dependence on the magnetic field strength seems to suggest the magnetic field errors as the cause of asymmetry.

In summary, the collisional diffusion coefficient is measured in the dc octopole plasma. The spatial distribution and the time dependence of the density, the dependence of the coefficient on ion mass, and the numerical value all indicate that the process is classical diffusion within the experimental error for the bulk of the plasma. The inaccuracy of the electron temperature measurements is the main source of the error. Also, we observed an anomalous diffusion in the average minimum- $B$  region which becomes large near the minimum-length field line. This is ascribed to the effect of a small asymmetry present in the magnetic field.

We wish to thank Torkil H. Jensen for his stimulating discussions.

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\*Work supported by the U. S. Atomic Energy Commission, Contract No. AT(04-3)-167, Project Agreement No. 38.

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