Limiting Distributions for Target Fragmentation in the Reactions $\pi^{\pm}+p\to\pi^{-}$ +Anything

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We study the inclusive reactions $\pi^-+p\to\pi^-$ + anything at 8, 18.5, and 24.8 GeV/c and $\pi^++p\to\pi^-$ + anything at 18.5 GeV/c. Distributions of the particle density $\omega(d\sigma/dp_{\parallel})/(\sigma_T)_\infty$ at $p_{\parallel}\lesssim 1$ GeV/c for the "inclusive" π^- indicate an approach to limiting behavior for π^+p at current accelerator energies. Invariant cross sections for π^-p decrease significantly in magnitude as the incident energy increases. The decrease is consistent with an energy dependence similar to the form $A+Bs^{-1/2}$ suggested by Regge phenomenology, approaching the π^+p limiting distribution as a limit.

Inclusive reactions of the type

$$a+b\to c+\text{anything} \qquad (1)$$

are being studied intensively[1-4] as a source of information on the mechanism of multiparticle production at high energies. A major source of interest has been the possibility of limiting behavior as a function of energy in these inclusive reactions. It has been predicted[5-8] that the differential cross section for production of the "inclusive particle" c should, at high energies, approach a limit independent of the energy or of the initial particles. For example, Feynman[5] has predicted limiting behavior for the function $f(x,p_\perp,s)=(2E/\pi s^{1/2})d^2\sigma/dx\,d(p_\perp^2)$, where $s^{1/2}$ is the total center-of-mass (c.m.) energy for the interaction, and where the variables describing the inclusive particle of mass μ are $p_\perp$, the transverse momentum; the scaling variable $x=2p_l/s^{1/2}$, where p_l is the c.m. longitudinal momentum; and $E=[(s/4)x^2+p_\perp^2+\mu^2]^{1/2}$, the c.m. energy. The function $f(x,p_\perp,s)$ is predicted to scale, i.e., to become independent of s, as s becomes large. Other models[6] predict limiting distributions in different reference frames, such as target or projectile frames.

Recently, Mueller[9] has used the techniques of Regge phenomenology to relate the inclusive Reaction (1) to forward elastic three-body amplitudes of the type $a+b+\bar{c}\to a+b+\bar{c}$. Chan *et al.*[10,11] have applied this method in the "single-Regge limit" where $s_{b\bar{c}}$, the square of the mass for one of the two-body systems, is held fixed while s becomes large. Assuming that the amplitude for the three-body reaction is dominated by the usual Regge singularities, they make predictions about the behavior of the invariant cross section for production of the "inclusive particle" c in reactions of type (1). In particular, they predict that the invariant cross section for c will have the form

$$d\sigma/(d^3p/\omega)\equiv f(s,p_{\parallel}^b,p_\perp^2)=A+Bs^{-1/2}, \qquad (2)$$

where $p_{\parallel}^b$, $p_\perp$, and ω are the longitudinal momentum, transverse momentum, and energy, respectively, of particle c in the rest frame of particle b. The function $A(p_{\parallel}^b,p_\perp^2)$ represents contributions from Pomeranchuk exchange while $B(p_{\parallel}^b,p_\perp^2)$ represents contributions from other meson trajectories.

The reactions of particular interest to us are

$$\pi^-+p\to\pi^-+\text{anything} \qquad (3)$$

and

$$\pi^++p\to\pi^-+\text{anything}. \qquad (4)$$

Chan *et al.* consider target fragmentation yielding a low-momentum π^- in the laboratory. From the duality hypothesis, they suggest that when the quantum numbers of the $ab\bar{c}$ system are exotic, the Pomeranchuk-exchange contribution may dominate at relatively low energies leading to a limiting behavior for the distribution of c in Reaction (1). Ellis *et al.*[12] have suggested that not only $ab\bar{c}$ but also ab must be exotic if limiting distributions are to be observed at low energies. If Chan *et al.* are correct, we might expect the distribution of π^- from Reaction (4) to exhibit limiting behavior at current accelerator energies, while for Reaction (3) we might observe an $s^{-1/2}$ dependence for the invariant cross section approaching that for (4) as a limit.

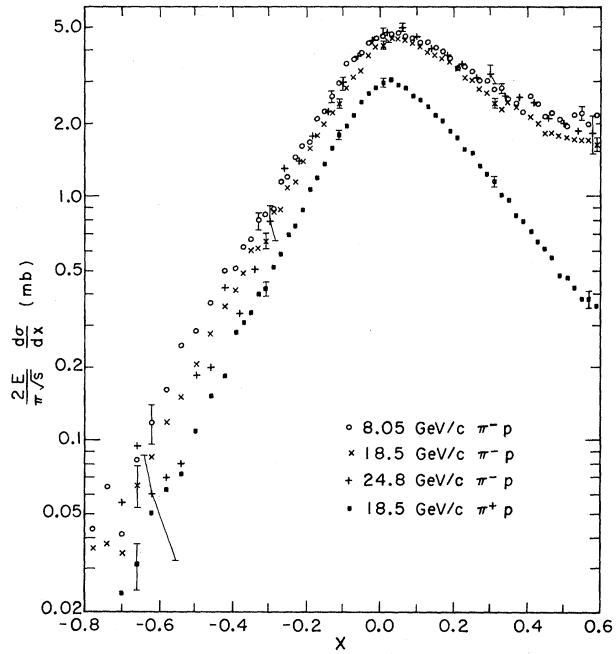

FIG. 1. Distributions of $(2E/\pi s^{1/2})d\sigma/dx$ for $\pi^{\pm}+p\rightarrow\pi^-$ + anything. Representative statistical error bars are shown.

We report here a comparison of data for Reaction (3) at 8.05 GeV/c ($s^{1/2}=4.00$ GeV), 18.5 GeV/c ($s^{1/2}=5.98$ GeV), and 24.8 GeV/c ($s^{1/2}=6.89$ GeV) with data for Reaction (4) at 18.5 GeV/c. Our study is based on samples of about 5×10^4, 10^5, and 10^5 negative tracks for, respectively, 8.05-GeV/c π^-p and 18.5-GeV/c π^-p and π^+p obtained by the University of Notre Dame,[1] and a subsample of about 12 000 tracks from a University of Wisconsin π^-p experiment[13] at 24.8 GeV/c. Invariant cross sections have been determined independently in each case.[14]

In Fig. 1 are shown distributions of $(2E/\pi s^{1/2})d\sigma/dx$ integrated over all $p_\perp$. The three distributions for Reaction (3) are quite similar in shape and magnitude near $x=0$ and are significantly larger in magnitude than the distribution for Reaction (4). The similarities may be seen more clearly in Fig. 2 where ratios of $(2E/\pi s^{1/2})d\sigma/dx$ in the different samples of Reaction (3) are shown. The ratios are seen to be consistent with unity for $-0.2\lesssim x\lesssim+0.4$. Thus the data appear consistent with a limiting distribution for this quantity[15] near $x=0$ in Reaction (3). However, for negative x in Reaction (3) we observe a systematic decrease of invariant cross section with increasing incident momentum as evidenced by ratios greater than 1 in Fig. 2(a) and less than 1 in Fig. 2(b) for $x\lesssim-0.4$. This region of x corresponds to relatively small $p_{\parallel}$ in the laboratory system, the

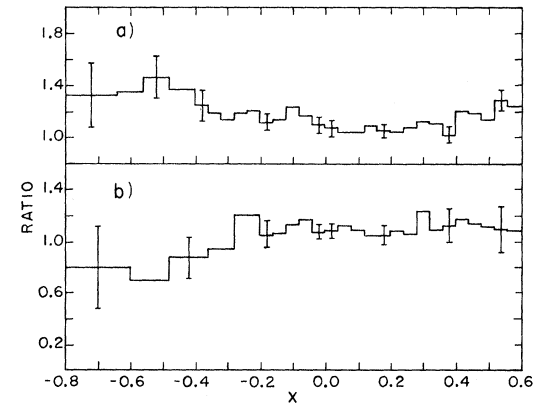

FIG. 2. Ratios of $(2E/\pi s^{1/2})d\sigma/dx$ as a function of x for $\pi^-+p\rightarrow\pi^-$+anything: (a) (8-GeV/c π^-p)/(18.5-GeV/c π^-p); (b) (24.8-GeV/c π^-p)/(18.5-GeV/c π^-p). Representative statistical error bars are shown.

region for which predictions have been made from Regge phenomenology.

In order to test these predictions we have followed the procedure suggested by Chan *et al.* of normalizing the invariant cross sections by the corresponding asymptotic total cross sections[16] $(\sigma_T)_\infty$. In Fig. 3 are shown distributions of the particle density $\omega(d\sigma/dp_{\parallel})/(\sigma_T)_\infty$ for π^- from Reactions (3) and (4) with laboratory $p_{\parallel}<3$ GeV/c. To improve our statistics we have integrated over all transverse momenta.[17] The distributions for Reaction (3) all lie significantly above that for Reaction (4). In addition we observe a decrease in invariant cross section with increas-

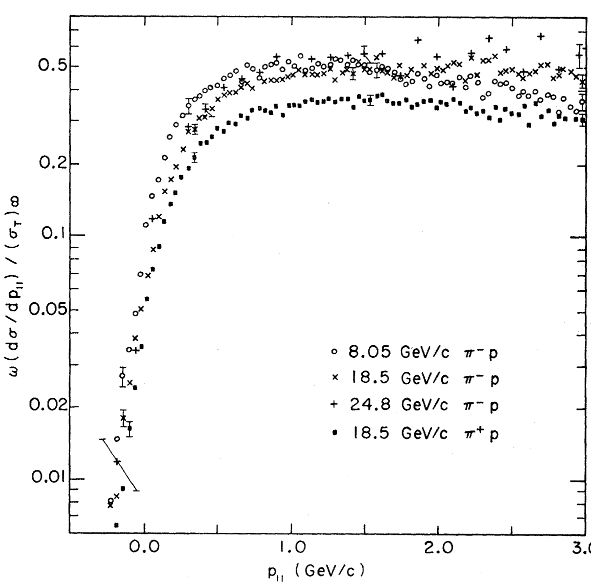

FIG. 3. Distributions of $\omega(d\sigma/dp_{\parallel})/(\sigma_T)_\infty$ as a function of laboratory $p_{\parallel}$ for $\pi^{\pm}+p\rightarrow\pi^-$+anything. Representative statistical error bars are shown.

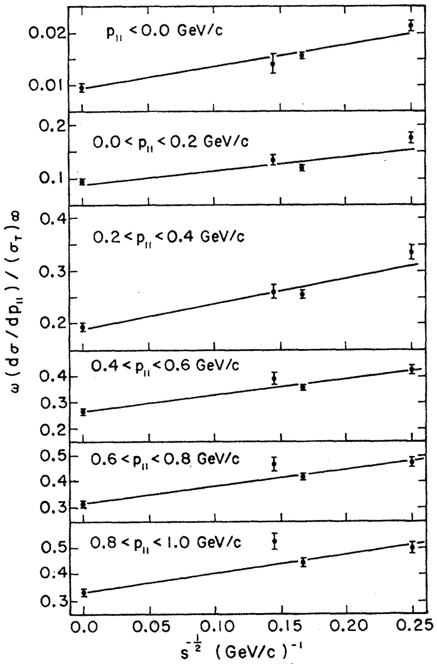

FIG. 4. Straight-line fits of the function $A+Bs^{-1/2}$ to values of $\omega(d\sigma/dp_{\parallel})/(\sigma_T)_\infty$ in the reactions $\pi^-+p\to\pi^-$ +anything at 8, 18.5, and 24.8 GeV/c and $\pi^++p\to\pi^-$ +anything at 18.5 GeV/c (plotted at $s^{-1/2}=0$) for different ranges of laboratory $p_{\parallel}$.

ing incident momentum for fixed small values of $p_{\parallel}$ in Reaction (3), consistent with the predictions of Chan *et al.*

The prediction of a limiting distribution for target fragmentation at relatively low energies in Reaction (4) is confirmed by a comparison of our π^+p data at 18.5 GeV/c with published data[2] at 7 GeV/c. The values[18] of integrated cross sections for $p_{\parallel}<0.5$ GeV/c divided by $(\sigma_T)_\infty$ are in agreement (0.23 ± 0.02 and 0.230 ± 0.008 at 7 and 18.5 GeV/c, respectively), and the shapes of the distributions are quite similar[19] for small $p_{\parallel}$.

To compare the observed behavior for Reactions (3) and (4) with the suggested form of relation (2) we have calculated particle densities for each of the four samples of data in various regions of $p_{\parallel}$ and have fitted straight lines of the form $A+Bs^{-1/2}$ to these data with $s^{-1/2}$ taken as 0.0 for the π^+p data, consistent with the assumption that a limiting distribution has been reached for Reaction (4). The fits are shown in Fig. 4. The trend of the data agrees[20] with the predicted $s^{-1/2}$ dependence for Reaction (3), approaching the same limiting distribution as Reaction (4). This confirmation of the qualitative predictions[21] of Regge phenomenology for target fragmentation in Reactions (3) and (4), together with the approach to a limiting distribution near $x=0$ for Reaction (3) alone, indicates the need for a comprehensive model providing quantitative predictions for the whole range of observations.

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[14]Invariant cross sections for the Notre Dame data were obtained by normalizing the number of observed four-prong events to independently determined four-prong topological cross sections. Invariant cross sections for the University of Wisconsin data were determined by normalizing the observed number of events to the total cross section after correction for scanning losses, etc.

[15]It has been previously suggested (see Ref. 4) that $d\sigma/dx$, rather than $(2E/\pi s^{1/2})d\sigma/dx$, is approaching a limiting distribution. We note that near $x=0$, $d\sigma/dx$ increases significantly with increasing energy ($d\sigma/dx=89.9\pm4.1$, 127.8 ± 3.9, and 161 ± 6 mb in the region $-0.02\leq x\leq+0.02$ at 8, 18.5, and 24.8 GeV/c, respectively), and thus shows no tendency to approach a limit as s becomes large.

[16]Asymptotic total cross sections of 23.4 and 24.9 mb have been assumed for π^+p and π^-p, respectively.

[17]The transverse-momentum distributions are quite similar for small $p_{\parallel}$ in the different data samples.

[18]For comparison, corresponding values in Reaction (3) are 0.396 ± 0.013, 0.302 ± 0.009, and 0.319 ± 0.015, respectively, at 8, 18.5, and 24.8 GeV/c. The errors here and in the figures include only statistical errors on the distributions and on the cross-section determinations. Some allowance should be made for systematic uncertainties of approximately a few percent in the evaluations of cross sections, where somewhat different procedures (see Ref. 14) had to be used with the different samples.

[19]The shapes differ at larger $p_{\parallel}$, the 18.5-GeV/c distribution falling less rapidly with increasing $p_{\parallel}$ than the 7-GeV/c distribution.

[20]The fit to the data for $p_{\parallel} < 0$, where target fragmentation is most surely expected to dominate, has a χ^2 probability of 8.2% with no allowance for systematic effects such as differences in normalization of cross sections.

[21]As $p_{\parallel}$ is varied, systematic effects can be observed in Figs. 3 and 4 which indicate the limitations of the Regge model in its present form. For higher incident momenta the particle densities decrease less rapidly as $p_{\parallel}$ increases. This is consistent with the presence of significant contributions at $p_{\parallel} \gtrsim 0.6$ GeV/c from sources other than target fragmentation which become larger at higher energies.

Two-Component Poincaré-Invariant Equations for Massive Charged Leptons*

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An alternative to Dirac's factorization of the Klein-Gordon equation is developed; the resulting two-component, $m \neq 0$ equations are proved Poincaré invariant. The particles interact *chirally* and *minimally* with the electromagnetic field ($g=2$). Our equations yield factorizations of Kramers's equations and a conserved chiral current (without γ_5 projection) to implement in a natural way the Feynman-Gell-Mann approach to weak interactions. Besides possessing sharp chirality, the particles possess a new dichotomic quantum number.

The theory of spin-$\frac{1}{2}$ particles was constructed by Dirac by means of a factorization of the Klein-Gordon equation over the field of four-component spinors. Wigner's subsequent analysis of the irreducible representations (irreps) of the Poincaré group provided a secure general foundation and extension of Dirac's construction. It is a familiar result of this analysis that a two-component spin-$\frac{1}{2}$ irrep (particle) can be characterized only by sharp CP and not P or C separately. The neutrinos ($m=0$) provide a well-known physical example.

It is commonly believed that for mass $m \neq 0$ Dirac's construction is unique, and that, in particular, a two-component particle having $m \neq 0$ cannot possess a first-order Poincaré-invariant wave equation. We will show that this belief is incorrect; we shall explicitly construct Poincaré-invariant first-order wave equations for two-component massive charged particles, characterized by chirality and a second new quantum number.

Let us note that our construction in no way contradicts Dirac's work. Dirac explicitly states that the uniqueness of his construction hinged on a fundamental assumption: that the matrices entering the factorization are to be *independent of space-time*, that is, *they are to represent independent new degrees of freedom*. This assumption is omitted in our construction.

That the factorizing matrices now involve space-time complicates the proof of invariance; accordingly we first applied these equations to an external Coulomb field[2] which—by singling out a particular point and a particular Lorentz frame—makes invariance questions irrelevant. We found precisely the usual Dirac-Coulomb levels, with this distinction: The new quantum number splits the (degenerate) spectrum into two *nondegenerate* spectra (omitting spatial degeneracy).

The Feynman-Gell-Mann theory of the Fermi interaction[3] began from the iterated Dirac equation in the presence of arbitrary electromagnetic