cant. It follows, since the impedance $Z=\rho(3\hbar k_F/4m)$ is rather less[5] than the zero sound value Z_0, that the disagreement between the magnitudes of experimental and theoretical values of R_K noted by Gavoret[6] is not improved if an independent QP model is assumed. Rice did, however, note that the pressure dependence of R_K^{-1} closely follows the variation of $Z \propto k_F^4$ rather than Z_0. Nevertheless, there seems to be no *a priori* justification for neglecting Fermi-liquid effects and the consequent existence of zero sound modes in a calculation of Kapitza conductance since direct measurement of the acoustic impedance at hypersonic frequencies confirms the zero sound value although there are small discrepancies.[12]

We would, however, remark that the calculation of Bekarevich and Khalatnikov strictly applies to energy flow due to a single mode ω. For heat flow due to a thermal distribution of modes there is a temperature gradient in the liquid ^{3}He which is not included in their calculation. This could have consequences for the experimental determination of ΔT as discussed recently by Budd and Vannimenus.[13]

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Nonlinear Damping of Electron Plasma Waves by Induced Decay Into Ion Waves

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Enhanced damping of an electron plasma wave due to induced nonlinear decay into a second electron wave plus an ion wave is demonstrated experimentally. The particular ion distribution allows decay into both positive and negative energy ion waves. The threshold for decay into the positive energy waves is shown to be consistent with theory.

There has been considerable recent interest in the development of weak electrostatic turbulence in a plasma. Theoretical work[1,2] has mainly preceded experimental verification, but demonstrations have been made of several of the various nonlinear processes involved in the transfer of wave energy within the fluctuation spectrum. For instance, several features of nonlinear Landau damping have been verified,[3] and the interaction of transverse and longitudinal waves observed.[4] This Letter describes experiments which demonstrate, in a one-dimensional, collisionless plasma, the nonlinear damping of a finite-amplitude monochromatic electron wave caused by its decay into an ion wave and a second electron wave, a process first considered by Oraevski and Sagdeev[5] and observed in a beam-plasma system by Bakai, Kornilov, and Krivoruchko.[6]

Our measurements are made in a thermally ionized sodium plasma column: Na^+ ions and electrons are produced at a single tungsten plate, 25 mm diam and temperature $T=2500°$K, in a uniform strong magnetic field (2–4 kG). Electrons are reflected by a cold tantalum planar electrode, at floating potential, 80 cm from the hot plate. The collisional mean free paths for both charge species are long compared with the column length at the low densities used ($n\approx 3\times 10^{7}$ cm^{-3}), and the axial density variation, $n^{-1}\times dn/dx$, is less than 1%. Most quiescent conditions are obtained when the sheath at the ionizing surface is slightly electron rich.

The unperturbed electron and ion velocity distributions are determined by the self-consistent equilibrium potential distribution for a low-density, single-ended Q machine[7]; i.e., the electrons have essentially a full one-dimensional Maxwellian velocity distribution (truncated at energies $\gtrsim 10kT$) with the same temperature as the hot plate, and no average drift, while the ions have a half-Maxwellian distribution at the same temperature but truncated to zero for velocities *below* some value $v_c \sim 2v_i$, where $v_i=(2kT/M)^{1/2}$, M is the ion mass, and k is Boltzmann's constant.[8] As a result, the linear propagation and damping of electron waves for phase velocities v_φ in the range $2.4\leq v_\varphi/v_e\leq 4$ [$v_e=(2kT/m)^{1/2}$ and m is the electron mass] is essentially that for a complete Maxwellian distribution at the same temperature as the hot plate, T, as we have confirmed experimentally.[9]

The ion wave dispersion can be found from the appropriate expression for plasma permittivity:

$$\epsilon = 1-\left(\frac{\omega_{pe}^2}{k^2v_e^2}\right)Z'\left(-\infty,\frac{\omega}{kv_e}\right) - \frac{\omega_{pi}^2}{k^2v_i^2}\left(\frac{2Z'(\nu,\omega/kv_i)}{\operatorname{erfc}\nu}\right)=0, \tag{1}$$

where Z' is the derivative with respect to ζ of the function

$$Z(\nu,\zeta)=\frac{1}{\sqrt{\pi}}\int_\nu^\infty \frac{\exp(-x^2)}{x-\zeta}dx,$$

ν is the normalized cutoff velocity v_c/v_i, and ζ is a normalized phase velocity.

Solutions of (1) show[10] the expected features of ion wave propagation in the beam-like situation: a fast wave, less heavily damped than for a thermal distribution, plus a slow wave, which is undamped since it has a phase velocity less than v_c, and for which $\partial(\omega^2\epsilon)/\partial\omega<0$; i.e., it has negative energy. Propagation of the fast (positive energy) wave has been observed up to $\omega\approx 4\omega_{pi}$ using conventional grid excitation, and verifies the above theory.

Because there are two modes of ion wave propagation, one should expect *two* resonant decay processes in each of which an initial electron wave $l_1(\omega_1,\vec{k}_1)$ can decay into a second electron wave $l_2(\omega_2,\vec{k}_2)$ and an ion wave $s^\pm(\omega_3^\pm,\vec{k}_3^\pm)$, provided that l_1 has an amplitude φ greater than some threshold value $\varphi_*^\pm$. The three waves must satisfy the laws of energy and momentum conservation for each process $l_1\to l_2+s^\pm$:

$$\omega_1=\omega_2+\omega_3^\pm, \tag{2}$$

$$\vec{k}_1=\vec{k}_2+\vec{k}_3^\pm. \tag{3}$$

For decay into s^+, $\omega_3^+>0$, and $\omega_1-\omega_2=+|\omega_3|$, so that ω_2 is below ω_1 (i.e., we should observe the second high-frequency wave l_2 as a red-shifted satellite of l_1 displaced by an amount $\Delta\omega=|\omega_3^+|$). For decay into s^-, since $\omega_3^-<0$, energy conservation requires that $\omega_1-\omega_2=-|\omega_3^-|$, so that ω_2 is above ω_1 (i.e., the satellite l_2 should be blue shifted with respect to l_1 by $\Delta\omega=|\omega_3^-|$).[2,11] The values of ω_2, $\omega_3^\pm$ which appear for any given input wave number k_1 are selected by (3), which for a one-dimensional system means that

$$|\vec{k}_3^\pm|=|\vec{k}_1|+|\vec{k}_2|\simeq 2|\vec{k}_1|,$$

such that $\epsilon(\omega_3^\pm,\vec{k}_3^\pm)=0$. Notice that for the same wave number $\vec{k}_3^\pm$ (see Fig. 1) $|\omega_3^+|>|\omega_3^-|$.

In the experiment a single electron wave l_1 propagating in the lowest radial eigenmode in the frequency range 30–60 MHz is launched by a conventional probe antenna and the resulting fluctuation spectrum detected by a second axially movable probe which is connected (via appropriate filters to minimize spurious instrumental effects) to both high- and low-frequency spectrum analyzers. Typical responses are shown inset in Fig. 1. For a given input wave l_1, whose wave vector is known to within ±3% from linear dispersion measurements,[9] the high-frequency spectrum analyzer shows the injected frequency and satellite lines separated from it by between 50 and 500 kHz. For large values of k_1 only a blue-shifted line is observed, consistent with preferential generation of the negative energy wave s^-, growth of s^+ being prevented by its heavy damping, whereas at longer wavelengths, for which both ion waves can propagate with small damping, both red and blue satellites are seen. At the same time, the low-frequency analyzer shows signals corresponding to $\omega_3^\pm$. In all cases the high-frequency separation agrees, within measurement accuracy, with the observed low-frequency signal.

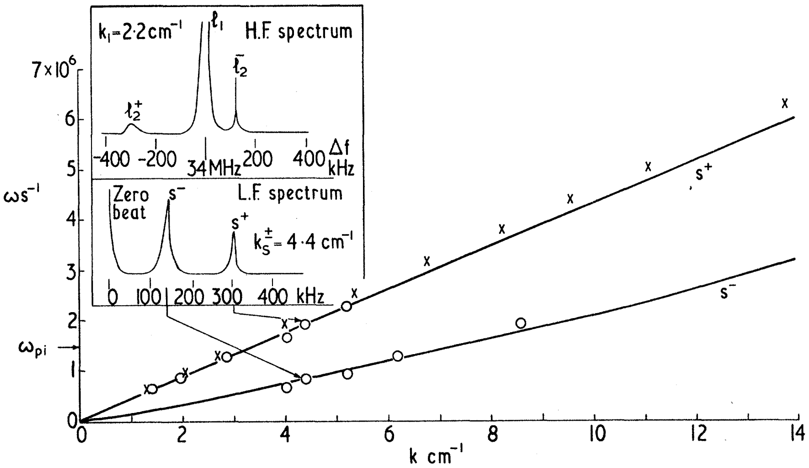

FIG. 1. Dispersion of ion waves: solid lines, theoretical ($\nu=2$); crosses, direct linear excitation; circles, excitation by nonlinear decay.

For sufficiently lightly damped ion waves it has been possible to measure their wavelength from the standing waves set up between the exciter and the partially reflecting cold plate. This confirms that $|\vec{k}_3^{\pm}|=2|\vec{k}_1|$.

The data plotted in Fig. 1 show the measured values of $\omega_3^{\pm}$ plotted against $|\vec{k}_3^{\pm}|=2|\vec{k}_1|$, together with independently measured values of $(\omega_3^{+}, |\vec{k}_3^{+}|)$ from conventional low-frequency interferometer measurements. The solid lines represent theoretical dispersion curves derived from Eq. (1) for $\nu=2.0$, $T=2500°$K.

The variation in the amplitudes of the positive and negative energy waves, s^+ and s^-, with the initial amplitude φ_{10} of l_1 is shown in Fig. 2, which clearly demonstrates both a threshold and a saturation effect. We attribute the latter not to a saturation of the ion-wave decay process discussed here, but to other nonlinear electron-wave decay processes[12] (in particular to the trapped-particle instability) which will be reported elsewhere. Absolute values of the initial amplitude of the electron wave have been estimated for amplitudes such that the trapped electrons cause spatially periodic Landau damping,[13] using the relation $k_{\text{osc}}=(k^2/\omega)(e\varphi_{10}/m)^{1/2}$ and extrapolating assuming constant coupling of the probes to the plasma. The threshold for decay is given by Dysthe and Franklin[14]:

$$|\varphi_*^{\pm}|^2=\frac{\gamma_2\gamma_3^{\pm}(k_2k_3^{\pm})^2(\partial\epsilon/\partial\omega)_2(\partial\epsilon/\partial\omega^{\pm})_3}{4\Gamma_{123}^{\;2}}, \qquad (4)$$

where

$$\Gamma_{123}=-\frac{3}{2}\frac{e}{m}\frac{k_1^{\;4}}{\omega_{pe}^{\;2}} \text{ for } k\lambda_D\lesssim 0.25,$$

γ_2 and $\gamma_3^{\pm}$ are the linear Landau damping coefficients for the electron wave l_2 and the ion waves $s^{\pm}$, and $(\partial\epsilon/\epsilon\omega)_2$ and $(\partial\epsilon/\partial\omega^{\pm})_3$ are found from Eq. (1). For the conditions relevant to Fig. 2 $e\varphi_*^{+}/kT=0.1$ and is considered to be in good agreement with the observed value 0.13. In calculating the threshold only measured quantities were used

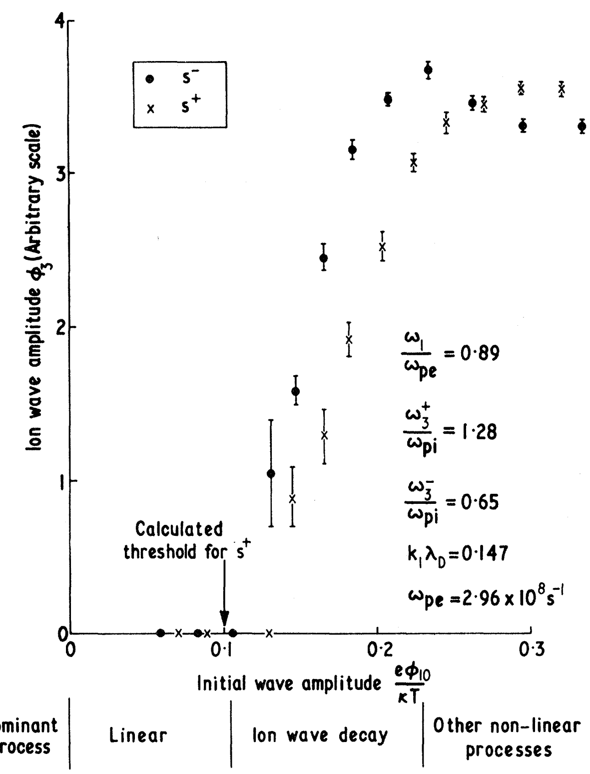

FIG. 2. Amplitudes of positive-energy ion wave s_3^{+} and negative-energy ion wave s_3^{-} as functions of the amplitude of the initial wave l_1 (circles, s^-; crosses, s^+).

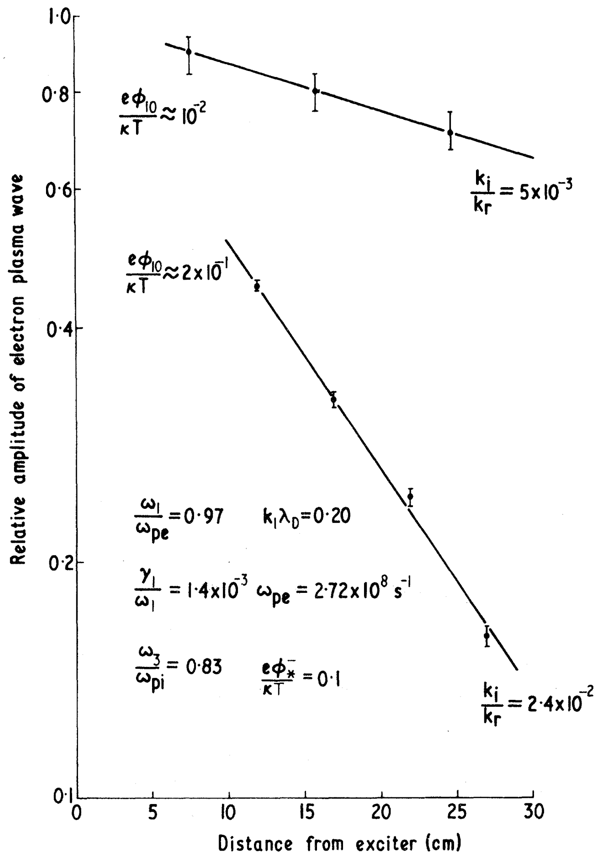

FIG. 3. Effect of initial amplitude φ_{10} on wave damping.

except for γ_2/ω_2 and $\gamma_3{}^+/\omega_3{}^+$, for which the theoretical values of 1.01×10^{-6} and 0.036, respectively, were taken. Since the phase velocity of the negative energy wave is less than the abrupt truncation velocity v_c assumed in the model of the ion distribution function, it is undamped, and therefore one would predict a "threshold" at zero amplitude. In reality, the wave appears to have some damping, resulting in a finite threshold.

Figure 3 shows the damping of an electron wave l_1 for two different initial amplitudes. The upper line shows the spatial attenuation of a wave whose initial amplitude $e\varphi_{10}/kT\approx10^{-2}$ is well below the threshold for the nonlinear effects mentioned above. The lower line shows the increased attenuation experienced by a similar wave but whose initial amplitude $e\varphi_{10}/kT\approx2\times10^{-1}$ is sufficiently large for induced decay to occur only into a negative-energy ion wave. This measurement was made using a very narrow-bandwidth correlation receiver so that the wave l_1 could be distinguished from its decay product l_2.

Although the results quoted here relate to the special case of lightly damped ion waves resulting from the "beam"-like ion distribution, a similar decay process should be seen for a normal Maxwellian ion distribution. In this case, of course, no negative energy ion wave would exist so that only one (red-shifted) electron wave satellite would be generated. Further, in a thermal plasma the ion waves would be much more heavily damped, with a consequent increase in the threshold.

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Dynamic Properties near the Nematic-Isotropic Transition of a Liquid Crystal*

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We have measured the temperature dependence of the flow birefringence in the isotropic phase of *p*-methoxybenzylidene *p*-*n*-butylaniline. We have observed a divergence as the phase transition is approached. We have determined the steady shear viscosity η_0 and the coupling coefficient μ between velocity shear and the order parameter as a function of temperature. We have also measured the real part of the shear impedance and compared the results with light-scattering data. Our results are well described by de Gennes's theory.

Recently de Gennes[1] has investigated the dynamics of fluctuations of the order parameter in the isotropic phase of a nematic liquid crystal and also discussed the flow birefringence, the spectrum of inelastically scattered light, and the attenuation of ultrasonic shear waves in terms of three viscosity coefficients η_0, μ, and ν. In this Letter we report precise measurements of flow birefringence in the isotropic phase of *p*-methoxybenzylidene *p*-*n*-butylaniline and determination of the temperature dependence of the viscosity coefficient μ, using our flow birefringence measurements together with the magnetic birefringence data given by Stinson and Litster.[2] We have measured the capillary viscosity η_0 on the same sample. The real part of the high-frequency shear impedance has also been determined from a shear-wave reflectance technique, described elsewhere.[3] To perform the flow birefringence experiments we used a high-sensitivity apparatus with photoelectric detection modulated by a rotating crystal. The wavelength of the light was 6328 Å. We regulated the temperature both inside and outside of the flow cell. The temperature gradient between the walls was smaller than 0.004°C. We used an internal rotor, 50 mm in diam, 70 mm in length, and with a 0.5-mm gap.

As shown earlier,[4,5] under a velocity gradient G the isotropic phase shows a positive birefringence Δn much larger than found in normal organic liquids. Δn is a linear function of G and the extinction angle is equal to 45° for all velocity gradients. As the phase transition $T_K = 45.5$°C is approached we find a divergent behavior of $\Delta n/G$ which is related to a divergence of the fluctuations of the order.

As shown by de Gennes, the order parameter for the isotropic phase is a symmetric traceless tensor $Q_{\alpha\beta}$,[6] proportional to the anisotropic part of the magnetic susceptibility tensor. The free-energy density may be written as

$$F = F_0 + \tfrac{1}{2}A\,Q_{\alpha\beta}Q_{\beta\alpha} + \tfrac{1}{3}B\,Q_{\alpha\beta}Q_{\beta\gamma}Q_{\gamma\alpha} + O(Q^4) - \tfrac{1}{2}\chi_{\alpha\beta}Q_{\beta\alpha}H_\alpha H_\beta, \qquad (1)$$

H being the magnetic field. The coefficient of the quadratic term, $A(T)$, is taken to be $A(T) = a(T - T_c^*)^\gamma$, where T_c^* is a temperature slightly below the transition point T_K, γ and a being unknown constants. One may obtain $A(T)$ from the temperature dependence of the magnetic birefringence using the equation[1]

$$n_\parallel - n_\perp = MH^2\Delta\chi/6A\bar{n}, \qquad (2)$$

where M is the proportionality coefficient between the dielectric and the order-parameter tensors and $\bar{n}$ the mean refraction index. $\Delta\chi$ is the anisotropy of diamagnetic susceptibility in the