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Drift-Wave Instabilities of a Compressional Mode in a High-β Plasma

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While the ordinary electrostatic drift mode is stabilized by either high-β effects or an admixture of cold plasma, a compressional drift mode is shown to be destabilized under these same circumstances. The condition of the instability is approximately given by $n_h/n_c < \beta\kappa_0^2\rho_i^2/2$, where n_h and n_c are the number densities of the hot and cold components, respectively; κ_0 is a measure of the density, temperature, or magnetic field gradients; and ρ_i is the ion Larmor radius.

The ordinary electrostatic drift-wave instability is known to be stabilized in a high-β plasma ($\beta > 0.13$) by Landau damping of ions drifting as a result of a magnetic field gradient in the same direction as that of electron diamagnetic drift.[1,2] This instability can also be shown to be stabilized by a fractional mixture $[n_c/n_h \gtrsim (m_e/m_i)^{1/2}]$ of cold electrons that short-circuit the parallel (to the ambient magnetic field) electric field which is needed to maintain the drift wave.[3]

On the other hand, drift waves associated with compressional modes (modes which produce changes in the parallel component of the magnetic field) have a tendency to destabilize at a larger value of β because the transit-time damping[4] which is proportional to β plays the role of Landau damping in the electrostatic mode.

Mikhailovskii and Fridman[5] have considered drift waves in magnetosonic modes (coupled modes of an acoustic wave and a compressional wave) and have shown in fact a wider range of unstable regions in the value of β. However, these modes are again strongly modified by an admixture of cold plasma because of the disappearance of the ion acoustic wave. Stabilization of the modes can be shown to occur when $n_c \sim n_h$. Therefore, most of the drift-wave instabilities presented in the past are stabilized in a high-β plasma with an admixture of cold plasma.

We will show here that when the cold-plasma density exceeds a threshold, however, the compressional Alfvén wave is destabilized either by inversed transit-time damping of ions or by inversed Landau damping associated with resonant particles drifting as a result of a magnetic field gradient.

We consider a nonuniform and high-β plasma embedded in a straight magnetic field $B_0(y)\hat{e}_z$. The nonuniformity is taken in the y direction. In the low-frequency ($\omega \ll \omega_{ci}$) and the long-wavelength ($k_\perp v_{Ti} \ll \omega_{ci}$) limit, the dispersion relation for the magnetosonic mode can be written as[5,6]

$$c^2k^2/\omega^2 - (\epsilon_{yy} + \epsilon_{yz}^2/\epsilon_{zz}) = 0. \tag{1}$$

If we consider an admixture of cold electrons with $n_c \sim n_h$, the ϵ_{zz} component becomes very large and the dispersion relation can be reduced to

$$c^2k^2/\omega^2 - \epsilon_{yy} = 0, \tag{1a}$$

where ϵ_{yy} can be derived using the ordinary WKB approximation,[5]

$$\epsilon_{yy} = -\sum_{\text{species}} \frac{\omega_p^2}{\omega} \sum_{n=0,1} \int \frac{v_\perp^2[J_n'(k_\perp v_\perp/\omega_c)]^2}{\omega - n\omega_c - k_\parallel v_\parallel + k_\perp v_d}\left(\frac{m}{T} + \frac{k_\perp}{\omega\omega_c}\frac{\partial}{\partial y}\right) f_0\, dv, \tag{2}$$

where $dv \equiv 2\pi v_\perp dv_\perp dv_\parallel$, v_d $[=(v_\perp^2/2\omega_c)(\partial \ln B_0/\partial y)]$ is the ∇B_0 drift speed, and the other notations are standard. We consider the velocity distribution function f_0 to represent an isotropic Maxwellian distribution with both density and temperature being functions of y. Substituting Eq. (2) into Eq. (1a) and expanding the derivatives of the Bessel functions, J_n', to a suitable order, we find the following dispersion relation for $n_c \gg n_h$[7]:

$$1-\frac{\omega^2}{k^2 v_A{}^2}+\sum_{\text{species}}\frac{\beta_j k_\perp{}^2}{2k^2}\{[\omega+\omega_{*j}(1-\tfrac{3}{2}\eta_j)]I_{1j}+\omega_{*j}\eta_j I_{2j}\}=0, \tag{3}$$

where

$$\eta_j=\partial \ln T_j/\partial \ln n_j, \tag{4}$$

$$I_{1j}=\frac{1}{\sqrt{\pi}}\int_0^\infty\int_{-\infty}^\infty \frac{\epsilon^2\exp(-\epsilon-\xi^2)}{\omega-\sqrt{2}k_\parallel v_{Tj}\xi+\omega_{Gj}\epsilon}d\epsilon d\xi, \tag{5}$$

$$I_{2j}=\frac{1}{\sqrt{\pi}}\int_0^\infty\int_{-\infty}^\infty \frac{\epsilon^2(\epsilon+\xi^2)\exp(-\epsilon-\xi^2)}{\omega-\sqrt{2}k_\parallel v_{Tj}\xi+\omega_{Gj}\epsilon}d\epsilon d\xi, \tag{6}$$

$$v_A{}^2=B_0{}^2/\mu_0 m_i n_c, \tag{7}$$

$$\omega_{*j}=\frac{k_\perp v_{Tj}{}^2}{\omega_{cj}}\frac{\partial \ln n_j}{\partial y}\equiv\frac{k_\perp v_{Tj}{}^2\kappa}{\omega_{cj}}, \tag{8}$$

and

$$\omega_{Gj}=\frac{k_\perp v_{Tj}{}^2}{\omega_{cj}}\frac{\partial \ln B_0}{\partial y}\equiv\frac{k_\perp v_{Tj}{}^2\kappa_B}{\omega_{cj}}. \tag{9}$$

Note that ω_* and ω_G as well as ω_c include signs. Also note that because of the small β the cold-plasma contribution appears only in the second term in Eq. (3). The pressure-balance condition requires that

$$\omega_{Gj}+(\beta_i+\beta_e)(1+\eta_1+\eta_e)\omega_{*j}=0. \tag{10}$$

In the absence of nonuniformity, Eq. (3) can be identified as the dispersion relation of the compressional Alfvén wave with transit-time damping represented by the imaginary part of the ξ integral of Eq. (5). It must be noted that in the presence of diamagnetic drift, ω_*, there are ranges in the frequency ω in which the signs of the coefficient of I_1 or I_2 are reversed, indicating the inverse transit-time damping due to the diamagnetic drift. In the absence of the cold component, the second term of Eq. (3) vanishes; thus previous authors have concluded that the compressional Alfvén wave is unaffected by the drift wave.[5,8] However, it can be shown that in the absence of the second term, there is no instability because of the absence of a mode (rather than that of an excitation source), except for an unrealistic case of $\eta<0$.[9] The cold plasma reduces the Alfvén speed without modifying the transit-time damping and creates a mode to which the inverse transit-time damping can couple and make the mode grow.

The instability can be excited in various ranges in the angle of propagation.

Case (A): $\omega+\omega_G \ll k_\parallel v_{Ti}$.—In this case if $T_i\sim T_e$ the inversed *ion* transit-time damping excites the instability, where

$$I_{1i}\sim -2i(\tfrac{1}{2}\pi)^{1/2}|k_\parallel v_{Ti}|^{-1},\quad I_{2i}\sim -6i(\tfrac{1}{2}\pi)^{1/2}|k_\parallel v_{Ti}|^{-1},$$

and the growth rate γ is given by

$$\gamma=(\tfrac{1}{2}\pi)^{1/2}\tfrac{1}{2}k_\perp{}^2 v_A{}^2(\beta_i/|k_\parallel v_{Ti}|)[-\omega_{*i}(1+\tfrac{3}{2}\eta_i)/|kv_A|-1] \tag{11}$$

at $\omega=|kv_A|$. If we take $k_\perp$ to be in the direction of the ion diamagnetic drift, then $k_\perp<0$, or $\omega_{*i}<0$, hence the instability occurs for the wave propagating in this direction. The condition of instability is obtained from Eq. (11) to be

$$\frac{n_c}{n_h}>\frac{1}{\beta_i}\frac{2}{\rho_i{}^2\kappa^2(1+\tfrac{3}{2}\eta_i)^2}. \tag{12}$$

The maximum growth is reached when $kv_A\sim k_\parallel v_{Ti}$ and $\gamma_m\sim\beta_i\omega_{*i}$ if $\eta_i=0$, or $\gamma_m\sim\beta_i\omega_{*i}\eta_i$ if $\eta_i\gg 1$.

Case (B): $k_\parallel v_{Ti}\ll\omega+\omega_G\ll k_\parallel v_{Te}$.—In this case the ion term in Eq. (3) becomes

$$-\left(\frac{k_\perp}{k}\right)^2\frac{\beta_i}{2}\left\{\frac{2\omega_{*i}\eta_i}{|\omega_{Gi}|}+\left[\frac{\omega}{|\omega_{Gi}|}\left(1+\frac{\omega_{*i}\eta_i}{|\omega_{Gi}|}\right)+\frac{\omega_{*i}}{|\omega_{Gi}|}(1-\eta_i)\right]I'\right\}, \tag{13}$$

where

$$I' = \int_0^\infty \frac{\epsilon^2 e^{-\epsilon}}{\epsilon - \omega/\omega_G} d\epsilon. \tag{14}$$

If $\eta_i < 1$, the ion Landau damping due to the spread in ∇B_0 drift [ϵ integral in Eq. (14)] tends to stabilize the instability generated by the inverse transit-time damping by *electrons* that originate from the ξ integral. The stabilization mechanism is similar to that for the case of the electrostatic drift mode. The instability is possible for a relatively low-β plasma for which

$$|\omega_{*e}|(1+\tfrac{3}{2}\eta_e)/|\omega_{Gi}| \gg 1$$

is satisfied, where $|\omega_{*e}|$ is the electron diamagnetic drift frequency. Then the condition of instability is given by

$$\frac{n_c}{n_h} > \frac{2T_e/T_i}{\rho_i^2\kappa^2(1+\frac{3}{2}\eta_e)^2}. \tag{15}$$

If $\eta_i > 1$, the inversed Landau damping of ions can excite the instability. When $\eta_i \gg 1$, the growth rate γ is expressed as

$$\gamma = \frac{\pi}{4}|kv_A|\left(\frac{k_\perp}{k}\right)^2 \frac{\beta_i\eta_i\omega_{*i}}{|\omega_{Gi}|}\left(1-\left|\frac{kv_A}{\omega_{Gi}}\right|\right)\left(\frac{kv_A}{\omega_{Gi}}\right)^2 \exp\left[-\left(\frac{kv_A}{\omega_{Gi}}\right)^2\right]. \tag{16}$$

Hence the condition of instability is $|kv_A| < |\omega_{Gi}|$ or

$$n_c/n_h > 2/\beta_i\rho_i^2\kappa_B^2, \tag{17}$$

and the maximum growth rate γ_m is approximately given by

$$\gamma_m \sim \beta_i\eta_i\omega_{*i}.$$

Case (C): $k_\parallel = 0$. —The instability exists also for a flute mode, $k_\parallel = 0$; however, the result is effectively the same as in case (B) for $\eta > 1$. If $0 < \eta < 1$, there is no instability for this case.

Although the mechanism of excitation is different in all of the above three cases, the condition of instability and the growth rate are similar and are generally written as

$$n_h/n_c < \beta_i\kappa_0^2\rho_i^2/2,$$

where κ_0 is a measure of the density, the temperature, or the magnetic field gradient, and the growth rate $\gamma \sim \beta_i\eta\omega_{*i}$.

A plasma for which the present study of instability is immediately applicable is that of the magnetosphere, especially within the plasmapause, where low-energy protons ($T_i \sim 10$ keV) having $\beta_i \sim 0.1$ to 1 are intermixed with cold plasma ($T \sim 0.3$ eV) whose density is 10^2 to 10^3 times that of the low-energy protons. Although the growth rate obtained here is proportional to $k_\perp$, if we consider the effect of finite Larmor radius, it will have a maximum value at $k_\perp \sim 1/\rho_i$. The period of the excited oscillation then becomes typically 1 ~ 10 min, which agrees with recent observations of relatively regular oscillations of the compressional mode in the magnetosphere.[10] The author wishes to thank Mrs. C. G. Maclennan for the proof reading.

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$$-\left[1+\frac{\omega_{*i}}{\omega}(1+\eta_i)-\frac{2\omega_{Gi}}{\omega}-\frac{2\omega_{Gi}\omega_{*i}}{\omega^2}(1+2\eta_i)\right]\frac{\omega^2\mu_0 m_i n_h}{B_0^2},$$

should be added to Eq. (3) as the contribution of the hot component. Conditions for instability become more complicated, but the essential feature still remains the same. The instability is possible only for $\beta \gtrsim 1$.

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rf Surface Impedance in the Presence of Magnetic-Field-Induced Surface States

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The contribution of magnetic surface states to the radio-frequency impedance of metals is considered in the low and high magnetic field limits. A peak in dX/dH is predicted to occur at a field for which the length of a bounce along the skipping trajectory of the first quantum state is comparable to the mean free path. The model is compared with experiments on Cu.

At a sufficiently smooth surface, conduction electrons are specularly reflected; and in a magnetic field they become bound to the surface in quantized states. At microwave frequencies surface-bound electrons carry a component of the skin-layer currents, and are responsible for the surface-state resonance spectra.[1] It is likewise expected that surface electrons should make a notable contribution to the rf (MHz range) impedance of high-purity, metal single crystals at low temperatures. Koch[2] first suggested that surface electrons might account for the distinctive dX/dH (reactance-derivative) signals observed at low magnetic fields by Cochran and Shiffman[3] in Ga and by Gantmakher[4] in Bi. Gantmakher, Fal'kovskii, and Tsoi[5] also gave an argument involving surface electrons to account for the dX/dH signal observed in K. Neither of the models proposed adequately explained the experimentally observed dX/dH peak. In this communication we present a new model for the contribution of surface electrons to be rf skin-layer currents, which accounts for our observations in copper.

The original model[2] led to a minimum in dX/dH at a field where the first quantized skipping trajectory becomes entirely confined in the rf skin depth δ, i.e., when $z_1 \sim \delta$. Here z_n is the maximum trajectory depth for the nth surface level. This model predicts a linear dependence of the dX/dH peak position on frequency in conflict with experiment. The model also fails to account for the peak position shift observed with changes in temperature.

Gantmakher, Fal'kovskii, and Tsoi[5] arrive at the relation $R\delta \approx l^2$ for the peak position H_0. R is the cyclotron radius and is related to the magnetic field and Fermi-surface radius K by $R = \hbar K/eH$. l is the mean free path (mfp). Neither the frequency dependence $H_0 \propto \omega^{-1/3}$ nor the mfp relation $H_0 \propto l^{-2}$ is observed experimentally.

The present model is related to that proposed by Koch[2] but takes into account the crucial effects of the finite mfp. According to the ineffectiveness concept, the anomalous-skin-effect (ASE) currents in the absence of a magnetic field are carried by electrons moving within an angular range δ/l to the surface. Electrons such as in Fig. 1, moving more steeply away from the

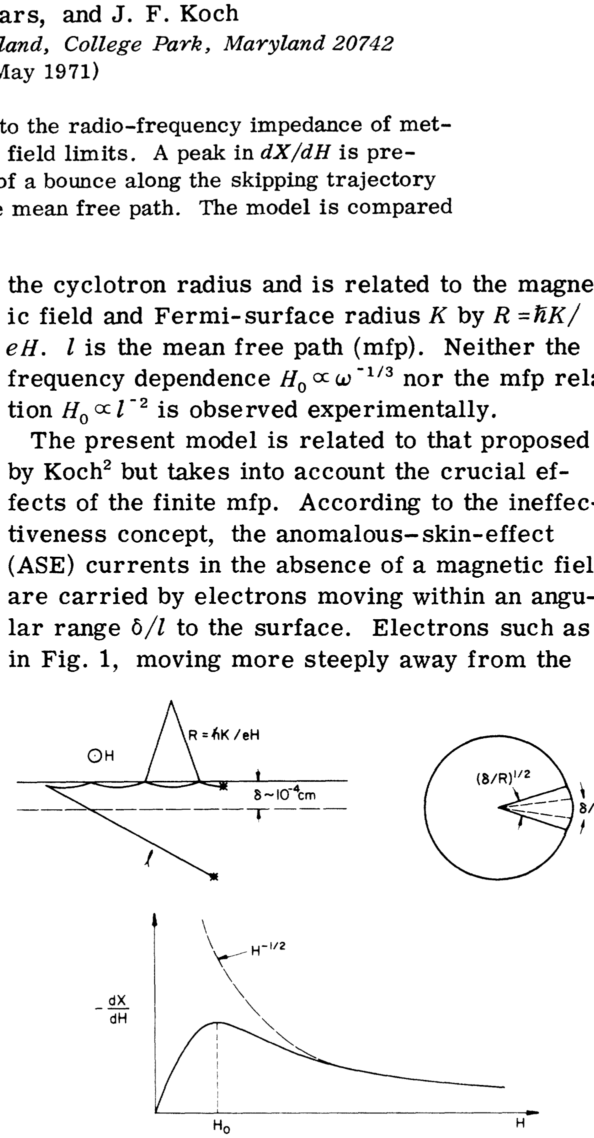

FIG. 1. A field H, applied parallel to the metal surface, converts ineffective electrons into surface bound states that effectively contribute to the skin-layer currents. On the Fermi surface, effective electrons are contained within the angular range $(\delta/R)^{1/2}$. As a result dX/dH is expected to diverge at low fields. For finite mfp the divergence is terminated and a peak results at H_0.