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Scaling Law for Deep-Inelastic Hadronic Scattering*

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We point out that the dual-resonance model suggests a new scaling law for the inclusive reaction $A+B\to X+$anything at fixed production angle in the c.m. frame at high energies. The prediction is that $s^{-1}\ln(d^2\sigma/dqd\Omega)_{\text{c.m.}}$ approaches, up to a multiplicative constant, a universal function independent of s. The form of the function is simple and explicitly given. In the quasielastic limit this function approaches that predicted for wide-angle elastic scattering by the Veneziano model. Agreement with experiments is good.

Benecke *et al.*[1] and Feynman[2] have argued that scaling laws exist for high-energy inclusive reactions in certain kinematical regions, known as the pionization and the fragmentation regions. Mueller[3] has derived these properties from a new type of Regge analysis, and several authors have given explicit demonstrations of them in the dual-resonance model.[4-7] The scaling properties of inclusive reactions have also been studied within the framework of the multiperipheral model.[8] We wish to point out a new scaling law which holds in a region not heretofore considered, the deep-inelastic region.

We consider an inclusive reaction of the type

$$A+B\to X+\text{anything} \tag{1}$$

in the c.m. system for a fixed angle θ between the outgoing and the incident particle. Let the incident momentum be p and the outgoing momentum be q. It is convenient to introduce the variables

$$r=q/p,\quad z=\cos\theta. \tag{2}$$

These, together with $s=(p_A+p_B)^2$, completely specify the kinematics. For given s the complete phase space is shown in Fig. 1. The shaded strip near the phase-space boundary (hereafter referred to as "the strip"), has been discussed by the authors mentioned before. Its width is expected to shrink with increasing s. Our discussion applies to the complement of the strip, and specifically includes the quasielastic region ($r\to 1$). It bears a similar relation to the strip as wide-angle elastic scattering does to diffraction scattering. We shall refer to our region as the deep-inelastic region.

In the asymptotic limit $s\to\infty$ at fixed r and z, the relevant Lorentz scalars of the reaction can be written in the forms

$$\begin{aligned} s&\equiv(p_A+p_B)^2\to 4p^2,\\ t&\equiv(p_A-p_X)^2\to -\tfrac{1}{2}sr(1-z),\\ u&\equiv(p_B-p_X)^2\to -\tfrac{1}{2}sr(1+z),\\ M^2&\equiv(p_A+p_B-p_X)^2\to s(1-r).\end{aligned} \tag{3}$$

In the deep-inelastic region the limit $s\to\infty$ is taken before all other limits. For example, we always consider $M^2\to\infty$, and the quasielastic limit is defined as the limit $M^2\to\infty$, $r\to 1$, taken in that order. Similarly, whenever we consider $z\to 1$, the limit is taken after $s\to\infty$.

Unitarity relates the inclusive cross section $d^2\sigma/dqd\Omega$ to an absorptive part of a six-point function,[3] and we take the latter from the dual-

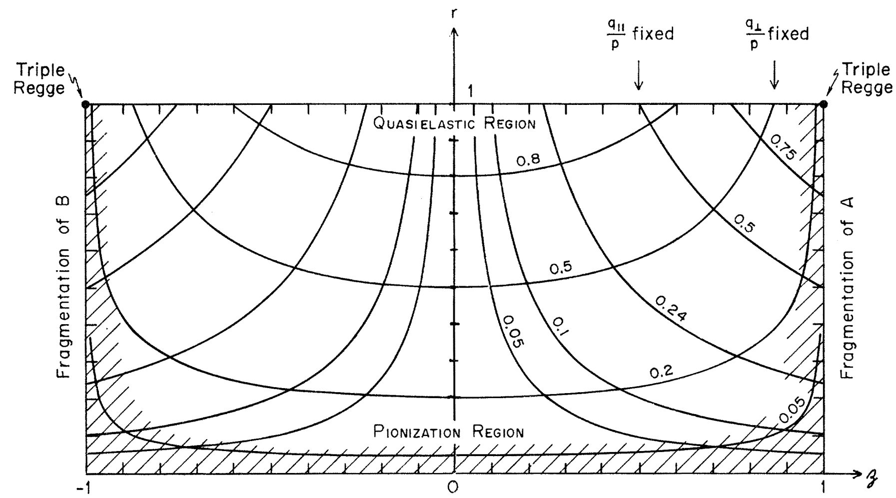

FIG. 1. Phase space for inclusive reaction at given s. Deep-inelastic region is entire rectangle except for the shaded strip, whose width decreases as s increases. Lines of constant $q_\parallel/p = rz$ and constant $q_\perp/p = r(1-z^2)^{1/2}$ are also shown.

resonance model, as given in Appendix B of Ref. 5. In the deep-inelastic region it takes the asymptotic form

$$d^2\sigma/dqd\Omega \to \sigma_0 \exp[bsG(r,z)], \tag{4}$$

where b is the universal Regge slope parameter in the dual-resonance model, in units of $(\mathrm{GeV}/c)^{-2}$, and

$$G(r,z) = (1+rz)\ln(1+rz) + (1-rz)\ln(1-rz) - (1+r)\ln(1+r) - (1-r)\ln(1-r), \tag{5}$$

and σ_0 is a function of s, r, and z that varies like a power of s. Its form depends on the details of the reaction (such as quantum numbers). The function $G(r,z)$, however, is universal. Let

$$F \equiv s^{-1}\ln(d^2\sigma/dqd\Omega) \to bG(r,z) + s^{-1}\ln\sigma_0, \tag{6}$$

which, for sufficiently large s, is independent of s for fixed r and z. This represents a new scaling law.

We emphasize that (4), and consequently (6), apply only to the deep-inelastic region. They do, however, approach limits that are qualitatively correct when one approaches the strip. For example, as $r \to 0$, one recovers the pionization limit:

$$\ln(d^2\sigma/dqd\Omega) \to -4bq^2\sin^2\theta. \tag{7}$$

As $|z| \to 1$, one recovers the fragmentation limit:

$$\ln(d^2\sigma/dqd\Omega) \to bt\ln[(1+r)/(1-r)], \tag{8}$$

which differs from the correct expression only in that bt should have been $a+bt$, where a is the Regge intercept.

The quasielastic limit $r \to 1$ is interesting, in that

$$d^2\sigma/dqd\Omega \to \exp\{bs[(1+z)\ln(1+z)/2 + (1-z)\ln(1-z)/2]\}, \tag{9}$$

which is precisely the wide-angle elastic differential cross section $\sigma_{el}(s,z)$ as predicted by the Veneziano four-point function.[9] In fact (6) may be stated in the form

$$\sigma_0^{-1}(d^2\sigma/dqd\Omega) \to \sigma_{el}(s,rz)/\sigma_{el}(s,r). \tag{10}$$

The quantity σ_0 in (4) is of course of no consequence if one goes to a sufficiently high energy. For comparison with existing data, however, we choose σ_0 to be a constant:

$$\sigma_0 = 1\ \mathrm{mb\ sr^{-1}}\ (\mathrm{GeV}/c)^{-1}, \tag{11}$$

whose value is consistent with the observed total

pp cross section. Thus, we compare our model with data by examining the ratio

$$b(s,r,z) \equiv s^{-1}\ln(d^2\sigma/dqd\Omega)_{\text{c.m.}}/G(r,z), \tag{12}$$

when $(d^2\sigma/dqd\Omega)_{\text{c.m.}}$ is the experimental cross section in the c.m. frame, in mb sr^{-1} $(\text{GeV}/c)^{-1}$. Scaling is tested by seeing whether $b(s,r,z)$ is independent of s. The dual-resonance form (5) of $G(r,z)$ is tested by the additional requirement that b be a constant, whose value would then be the slope parameter of some effective Regge trajectory.

We use the data of Anderson *et al.*[10] at 30 and 20 GeV/c, and those of Akerlof *et al.*[11] at 12.5 GeV/c. To exclude the strip, we keep only values of $|z|<0.7$, in all the data. The results are shown in Fig. 2, which is self-explanatory. The extensive data of Allaby *et al.*[12] at 19.2 GeV/c unfortunately fall entirely in the strip, and hence cannot be used for our purpose.

The validity of scaling, and the specific form of $G(r,z)$, seem to be borne out. The general feature is that $s^{-1}\ln(d^2\sigma/dqd\Omega)$ approaches the universal function $G(r,z)$ as $s\to\infty$, $z\to 0$, $r\to 1$. For fixed r, the rate of approach is faster the larger is s, and the smaller z. This is illustrated most clearly in the π^+ data in Fig. 2.

As an example of the quality of the fit, we compare two data points in the 30-GeV/c proton spectra:

r	z	G	b
0.17	-0.24	-0.029	0.32
0.57	0.36	-0.3	0.33

A detailed analysis of the rates of approach would involve a discussion of relevant Regge intercepts, which is beyond the scope of this note.

The effective slope parameter b seems to be different for different outgoing particles:

$$b \approx \begin{cases} 0.3 \text{ for } X=p, \\ 0.4 \text{ for } X=\pi^+, \\ 0.5 \text{ for } X=\pi^-, \\ 0.5 \text{ for } X=K^+,K^-. \end{cases} \tag{13}$$

A theoretical interpretation of these numbers clearly lies beyond the validity of the dual-reso-

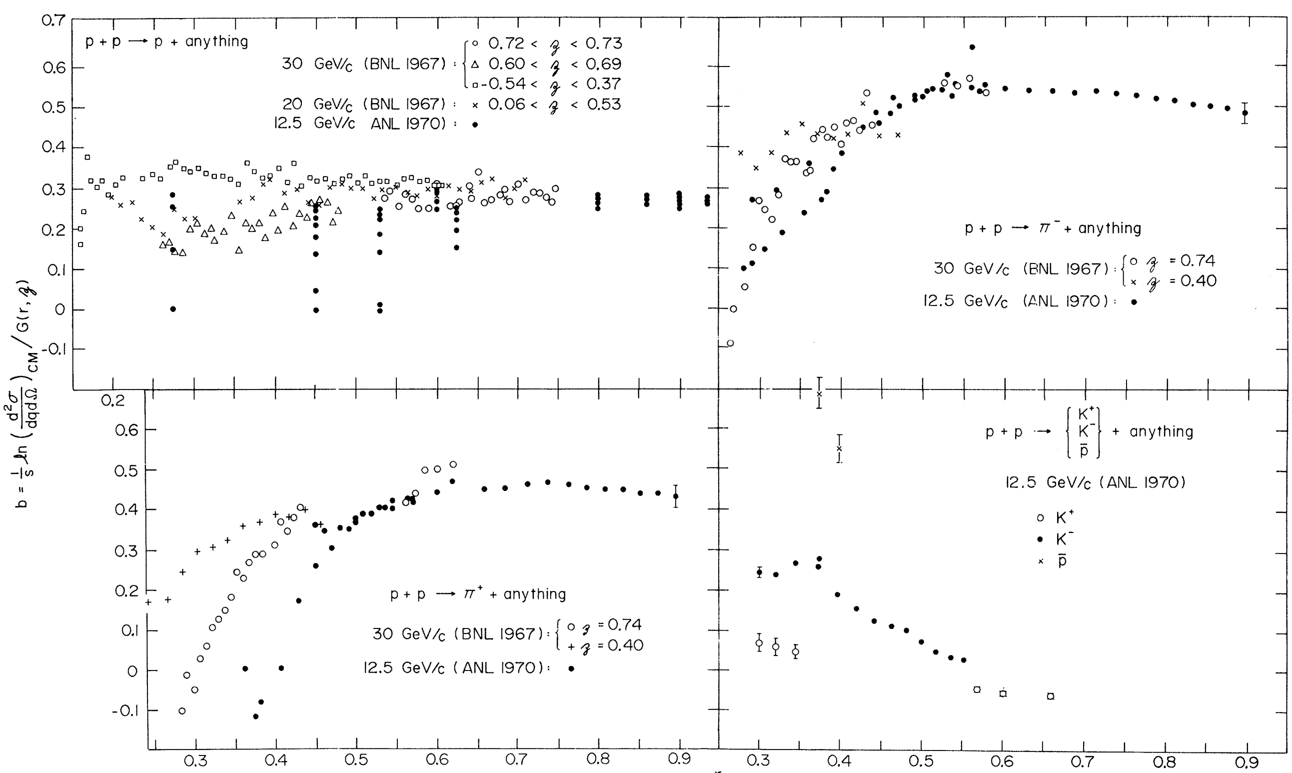

FIG. 2. Plot of experimental $s^{-1}\ln(d^2\sigma/dqd\Omega)_{\text{c.m.}}$ divided by known function $G(r,z)$, versus r for different values of z. The experimental differential cross section $d^2\sigma/dqd\Omega$ is in units of mb/(sr GeV/c), and s is in GeV^2. Data at 30 and 20 GeV/c are taken from Ref. 9, and those at 12.5 GeV/c are taken from Ref. 10.

nance model, for they undoubtedly have something to do with the Pomeranchuk singularity. Although the latter is specifically left out of the dual-resonance model, its effect may be partly included, for unitarity is used in arriving at (6). We have, however, no interpretation to offer at present.

Our findings bear on some recent speculations by Chen and Harte[13] and by Berman and Jacob.[14] They extended the Wu-Yang conjecture[15]

$$\sigma_{ep\to ep} \approx (\sigma_{\mathrm{el}})^{1/2} \tag{14}$$

to the inclusive reaction $pp \to p+$anything, and suggested that in part of the quasielastic region, namely, that for which M^2/t remains fixed as $r \to 1$,

$$d^2\sigma/dqd\Omega \approx (\sigma_{\mathrm{el}})^{1/2}R, \tag{15}$$

where R is a slowly varying function related to the cross section for $e+p \to e+$anything. This conjecture is not borne out by our result, which resembles more closely (15) with $(\sigma_{\mathrm{el}})^{1/2}$ replaced by σ_{el}.

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Flat Proton Spectrum and Its Implications*

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The longitudinal momentum distribution of the final detected proton in pp inclusive reactions is flat for energies ranging from 20 to 1500 GeV. We show how this can be understood in the diffractive model, and how this information provides a way for calculating the average multiplicity and the pion longitudinal momentum distributions in pp inclusive reactions. No arbitrary parameter is used. The agreement with experiment is remarkably good.

Proton spectrum.—It has been known for some time that up to 30 GeV/c, the c.m. longitudinal momentum distribution $d\sigma/dp_{\parallel}^*$ of the proton in the inclusive reaction $p+p\to p+$anything is independent of $p_{\parallel}^*$.[1-3] It is dramatically different from the pion spectra and has always been somewhat of a mystery. Recent results[4] from the CERN intersecting-storage-rings experiment show that this property persists up to 1500 GeV/c; moreover, the height of the proton spectrum is consistent with the invariant distribution being "limiting."[5] We show below how these characteristics can be understood within the framework of the diffractive model in which the multiparticle production amplitude can be described by the exchange of a single Pomeranchukon between two clusters.[6,7]

The mean transverse momentum $\langle k_\perp^2\rangle^{1/2}$ of the secondary pions in hadron-hadron collisions is known to be not too sensitive to the nature of the