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Breaking Nambu-Goldstone Chiral Symmetries*

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We establish that the nonanalytic behavior of some matrix elements in perturbation theory about a chiral symmetry implies that they can be calculated exactly to leading order in chiral breaking. For other matrix elements, analytic to leading order, we establish the hypothesis of threshold dominance in the chiral limit. As an application of this idea we calculate, to leading order in chiral breaking, the seven baryon-octet mass differences in terms of one parameter.

In a previous paper[1] we have pointed out that if a Hamiltonian symmetry is realized by Nambu-Goldstone bosons, then the S matrix and matrix elements of currents are not analytic in the perturbation parameter λ. Some matrix elements, for example, approach the symmetric limit like $\lambda \ln\lambda$. The reason for this nonanalytic behavior is that the Goldstone theorem[2] requires massless mesons in the symmetry limit so that the strong interactions acquire a long-range component.

It is the purpose of this note to point out that for matrix elements that exhibit such nonanalytic behavior in leading order in chiral breaking, it is possible to determine exactly the magnitude of the leading term specified in terms of chiral-breaking parameters. This is to assert that for some matrix elements (particularly those which are derivatives with respect to momentum transfer) *the symmetry itself exactly determines the symmetry breaking to leading order*. It is precisely because we have nonanalytic behavior that we can establish such chiral-limit theorems. We recall that once explicit symmetry-breaking factors have been extracted from a matrix element, this nonanalytic behavior can be viewed as a consequence of dispersion integrals diverging at the thresholds for the production of the Nambu-Goldstone bosons in the chiral limit. However, the production thresholds for the Nambu-Goldstone bosons, the π, K, and η in the case of $SU(3)\otimes SU(3)$, are precisely the points controlled by current-algebra low-energy theorems. It is the two features, that in such matrix elements one may *prove* that the threshold dominates in the chiral limit and that one knows precisely the threshold behavior, that enable one to establish the exact behavior in symmetry breaking as one approaches the symmetry limit.

For other matrix elements, which are analytic to leading order, the characteristic feature is that once explicit symmetry-breaking factors have been extracted, the corresponding dispersion integrals are finite at threshold in the chiral limit. Again the absorptive parts of such matrix elements may be computed exactly in the chiral limit at the production thresholds for the ground-state mesons. Utilizing this exact knowledge of the absorptive part in the threshold region, we may perform the dispersion integration over the threshold region and examine if indeed the threshold region dominates. *In general we expect threshold dominance from the observation that the Nambu-Goldstone bosons, since they are massless in the symmetry limit, provide the longest-range force.* Hence for matrix elements analytic to leading order we advance the hypothesis of threshold dominance (since one cannot prove it as in the nonanalytic case) which may then be established by experimental comparisons.

We propose a theoretical program to examine systematically the symmetry breaking in matrix elements of experimental interest, incorporating these implications of the Goldstone-Nambu realization of chiral symmetry. For nonanalytic matrix elements one may prove exact chiral-limit theorems[3] and for those analytic to leading order one may estimate using threshold dominance.

As an example of the latter and as a first step in such a program we will calculate the seven baryon-octet mass differences to leading order in chiral breaking in terms of a single parameter. We assume that the SU(3)⊗SU(3) symmetry is realized by an octet of Nambu-Goldstone bosons, π, K, and η, and by supermultiplets of SU(3) without parity doubling.[4]

We begin by considering the matrix element of the vector current between baryon states,

$$\langle B^a(p_2)|V_\mu{}^b(0)|B^c(p_1)\rangle = \overline{U}(p_2)[\gamma_\mu h_1{}^{abc}(t) + i\sigma_{\mu\nu}q^\nu h_2{}^{abc}(t) + q_\mu h_3{}^{abc}(t)]U(p_1), \quad t=q^2,$$

and its divergence

$$-i\partial_\mu V_\mu{}^b(0) = D^{abc}(t) = (M_a - M_c)h_1{}^{abc}(t) + th_3{}^{abc}(t).$$

We will assume the divergence is a gentle operator so that $D^{abc}(t)$ obeys an unsubtracted dispersion relation so that

$$D^{abc}(0) = (M_a - M_c)h_1{}^{abc}(0) = (1/\pi)\int_{t_0}^{\infty}(dt/t)\,\mathrm{Im}D^{abc}(t). \tag{1}$$

The nonrenormalization theorem[1] requires $ih_1{}^{abc}(0) = f^{abc} + O(\lambda^2\ln\lambda)$, and we may set $b=K^+$ since this gives mass differences. Then $t_0 = (\mu_K + \mu_\pi)^2$, $(\mu_K + \mu_\eta)^2$ corresponding to the two-pseudoscalar production threshold. At such a production threshold for two mesons of momentum q_1 and q_2, unitarity implies

$$\mathrm{Im}D^{abc}(t) = \tfrac{1}{2}i\varphi_2(t)d^{bef}(t)M_{ac}{}^{ef}(t), \tag{2}$$

where the two-body phase space is

$$\varphi_2(t) = (t-t_0)^{1/2}(t-t_1)^{1/2}/8\pi t, \quad t_1 = (\mu_K - \mu_\pi)^2, (\mu_K - \mu_\eta)^2;$$
$$id^{bef}(t) = f^{bef}[(\mu_e{}^2 - \mu_f{}^2)f_+(t) + tf_-(t)]$$

is the matrix element of the divergence of the current between meson states; and

$$M_{ac}{}^{ef}(t) = \langle B^a(p_2)\overline{B}^c(-p_1)|M^e(q_1)M^f(q_2)\rangle$$

is the S-wave projection of the amplitude for $M_e(q_1) + M_f(q_2) \to B_a(p_2) + \overline{B}_c(-p_1)$. The nonrenormalization theorem[1] implies

$$id^{bef}(0) = f^{bef}(\mu_e{}^2 - \mu_f{}^2) + O(\lambda^3\ln\lambda).$$

In the chiral SU(3)⊗SU(3) limit, $M_{ac}{}^{ef}(t)$ can be computed exactly at the unphysical threshold $t\to 0$, $q_{1,2}\to 0$ by a standard current-algebra treatment. The contribution from baryon exchange (in pseudovector coupling) is the most singular term, and in the chiral limit for which $q_1{}^2 = q_2{}^2 = 0$ as $t\to 0$ one obtains

$$M_{ac}{}^{ef}(t) \to -(\pi\sqrt{t}\,g_A{}^2/f_ef_f)(\Gamma_{eam}\Gamma_{fme} + \Gamma_{fam}\Gamma_{emc}), \quad \Gamma_{abc} = \alpha d_{abc} + i(1-\alpha)f_{abc},$$

with $f=(1-\alpha)$, $f+d=1$ the f and d coupling of the axial-vector current to the baryons and $f_e \approx \mu_\pi/\sqrt{2}$ the meson decay constant; $g_A = 1.24$.

Putting these results together we obtain, for example, the difference

$$M_N - M_\Sigma = (1/\pi)\int_0^\infty (dt/t)\mathrm{Im}D_{N\Sigma}(t), \tag{3}$$

where the threshold goes to zero in the chiral limit; and in this same limit, as $t\to 0$,

$$\mathrm{Im}D_{N\Sigma}(t) = (\sqrt{t}\,g_A{}^2/384f^2)[(\mu_K{}^2 - \mu_\pi{}^2)(15 - 48\alpha + 28\alpha^2) + 3(\mu_K{}^2 - \mu_\eta{}^2)(-3 + 8\alpha + 4\alpha^2)], \tag{4}$$

where $f = f_\pi = f_K = f_\eta$ in the chiral limit. This expression for the absorptive part represents the exact two-meson contribution near threshold. We see that the integrand of (3) diverges like $1/\sqrt{t}$ as $t\to 0$ although the integral is finite there. The contribution from four mesons to the absorptive part behaves like $t^2\sqrt{t}$, and that from six mesons like $t^4\sqrt{t}$, and so the two-meson state dominates the threshold region.

Writing

$$M_N - M_\Sigma = (1/\pi)\int_0^{4M^2}(dt/t)\,\mathrm{Im}D_{N\Sigma}(t) + I_H, \tag{5}$$

where I_H represents the integral for $4M^2 \leq t \leq \infty$, we find using our exact behavior [Eq. (4)] in the threshold region $0 \leq t \leq 4M^2$ that with $\alpha = 0.66$, a value of $M \sim 370$ MeV reproduces the complete mass difference $M_\Sigma - M_N = 250$ MeV in sign and magnitude. Since this value of M represents a low energy on the scale of hadron masses we conclude that threshold dominance is acceptable in this application and that the high-energy contribution I_H may be neglected. In what follows we will consider M an arbitrary parameter so that our conclusions do not depend on the precise value of M.

Proceeding to keep only the threshold contribution we obtain

$$\frac{M_N - M_\Sigma}{M} = \frac{g_A^2}{96\pi f^2}\left[(\mu_K^2 - \mu_\pi^2)(15 - 48\alpha + 28\alpha^2) + 3(\mu_K^2 - \mu_\eta^2)(-3 + 8\alpha - 4\alpha^2)\right],$$

$$\frac{M_\Sigma - M_\Xi}{M} = \frac{g_A^2}{96\pi f^2}\left[(\mu_K^2 - \mu_\pi^2)(-15 + 12\alpha + 8\alpha^2) + 3(\mu_K^2 - \mu_\eta^2)(3 - 4\alpha)\right], \tag{6}$$

$$\frac{M_\Lambda - M_\Xi}{M} = \frac{g_A^2}{96\pi f^2}\left[(\mu_K^2 - \mu_\pi^2)(9 - 36\alpha + 24\alpha^2) + 3(\mu_K^2 - \mu_\eta^2)(3 - 4\alpha)\right].$$

If we assume that the meson mass matrix is octet enhanced, then the mesons obey a Gell-Mann–Okubo formula,[5] $\mu_K^2 - \mu_\pi^2 = -3(\mu_K^2 - \mu_\eta^2)$. On this assumption it follows from (6) that the baryons obey the Gell-Mann–Okubo formula

$$2(M_N + M_\Xi) = 3M_\Lambda + M_\Sigma. \tag{7}$$

We conclude that octet enhancement in the meson sector implies the same for the baryons.

If the baryons obey a Gell-Mann–Okubo formula, their mass differences can be parametrized in terms of a d and f coupling with $(f/d)_B = \frac{2}{3}(M_N - M_\Xi)/(M_\Lambda - M_\Sigma)$. Our second remark is that in the chiral limit and with threshold dominance this ratio is related to the axial-vector–baryon coupling ratio $(f/d)_A = (1-\alpha)/\alpha$. From (6) we obtain

$$\tfrac{3}{10}(f/d)_B = (f/d)_A[1 - 3(f/d)_A^2]^{-1}. \tag{8}$$

With $(f/d)_B^{\exp} = 3.3$ we obtain as a solution to (8) $\alpha = 0.69$ to be compared with $\alpha = 0.66 \pm 0.02$,[6] so the agreement is excellent.

Proceeding in the same way for the matrix elements of $-i\partial_\mu V_\mu^{\pi^+}(0)$ between baryon states we obtain the electromagnetic mass differences

$$\frac{M_p - M_n}{M} = \frac{g_A^2}{8\pi f^2}\left[(\mu_{K^+}^2 - \mu_{K^0}^2)\tfrac{1}{6}(3 - 4\alpha^2) - \frac{\delta\mu_{\pi^0\eta}^2}{\sqrt{3}}(3 - 4\alpha)\right],$$

$$\frac{M_{\Sigma^+} - M_{\Sigma^-}}{M} = \frac{g_A^2}{4\pi f^2}\left[(\mu_{K^+}^2 - \mu_{K^0}^2)\alpha(1-\alpha) - \frac{2\delta\mu_{\pi^0\eta}^2}{\sqrt{3}}\alpha(1-\alpha)\right],$$

$$\frac{M_{\Sigma^+} + M_{\Sigma^-} - 2M_{\Sigma^0}}{M} = \frac{-g_A^2}{4\pi f^2}(\mu_{\pi^+}^2 - \mu_{\pi^0}^2)(1-\alpha)^2, \tag{9}$$

$$\frac{M_{\Xi^0} - M_{\Xi^-}}{M} = \frac{g_A^2}{8\pi f^2}(\mu_{K^+}^2 - \mu_{K^0}^2)\tfrac{1}{6}(-3 + 12\alpha - 8\alpha^2) - \frac{\delta\mu_{\pi^0\eta}^2}{\sqrt{3}}(2\alpha - 1)(3 - 2\alpha),$$

where $\delta\mu_{\pi^0\eta}^2 = \langle\eta|i\partial_\mu V_\mu^{\pi^+}(0)|\pi^-\rangle$. There immediately follows from (9) the Coleman-Glashow formula,[7]

$$M_{\Xi^-} - M_{\Xi^0} = M_{\Sigma^-} - M_{\Sigma^+} + M_p - M_n. \tag{10}$$

If we assume octet enhancement in the meson sector, then $\delta\mu_{\pi^0\eta}^2 = \frac{1}{3}\sqrt{3}(\mu_{K^0}^2 - \mu_{K^+}^2)$ and $\mu_{\pi^+}^2 - \mu_{\pi^0}^2 = 0$. With

$$(f/d)_{\rm em} = (M_{\Sigma^+} - M_{\Sigma^-})/[(M_p - M_n) + (M_{\Xi^-} - M_{\Xi^0})]$$

we have from (6) and (9)

$$(f/d)_B = (f/d)_{em}, \tag{11}$$

$$\frac{\mu_{K^0}^2 - \mu_{K^+}^2}{\mu_K^2 - \mu_\pi^2} = \frac{\frac{1}{2}(M_n + M_{\Xi^0}) - \frac{1}{2}(M_p + M_{\Xi^-})}{\frac{1}{2}(M_\Xi + M_N) - M_\Sigma}, \tag{12}$$

$$M_{\Sigma^+} + M_{\Sigma^-} - 2M_{\Sigma^0} = 0. \tag{13}$$

Of our six relations, five [(7) and (10)–(13)] have been previously obtained in the context of the tadpole model[8] which implements octet enhancement. Here we have octet enhancement as a consequence of the longest-range force arising from two-pseudoscalar-meson exchange. We have gone beyond the tadpole model in the relation (8) which involves $(f/d)_A$ and $(f/d)_B$. The experimental success of (8) we take as special evidence that our particular approach, that of threshold dominance, gives the major part of the medium-strong symmetry breaking.

Our treatment of the electromagnetic mass splittings does not include an additional "nontadpole" contribution which must be included before making experimental comparisons. A unified treatment of the baryon and meson mass splittings including the "nontadpole" terms is under investigation.

We conclude that even for matrix elements which are analytic to leading order in symmetry breaking for which one cannot rigorously prove threshold dominance, the threshold states still provide the major contribution. This phenomenon can provide a theoretical understanding of the experimentally observed octet enhancement. Finally we remark that this point of view requires that self-consistent or bootstrap equations for chiral-symmetry–breaking parameters are generally transcendental rather than algebraic in character.

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Does Duality Hold in Current Interactions?

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Deep-inelastic electron-proton scattering is shown to be naturally interpreted in a field theory where the current is given by a bilinear form in some fields. It is proved that in such a theory the Compton-scattering amplitude is not saturated with the vector-dominance model and that the term which scales in the Bjorken limit does not obey Freund-Harari duality.

Purely hadronic amplitudes are generally accepted to obey duality between resonances and Reggeons apart from Pomeranchuk contribution (the Freund-Harari duality[1]). It is interesting to ask the same problem in the case of the current interaction.

Since the success of the Veneziano model, many attempts[2] have been made to include cur-