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Theorem for Energy-Weighted Averages of Spectroscopic Factors*

C. M. Shakin† and J. Da Providência
Laboratório de Física, Universidade de Coimbra, Coimbra, Portugal
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It is shown that for a system with strong correlations, the centroid of the spectroscopic amplitudes for particle removal is given by the single-particle energy (defined appropriately) times the occupation probability of the orbit.

If a particle is removed from a system very rapidly, or if the distorted-wave Born approximation is expected to apply to the description of the removal, then the states of the residual system are expected to be excited with a strength proportional to the spectroscopic factors $|\langle\Psi_n|a_i \times|\Psi_0\rangle|^2$. Here $|\Psi_0\rangle$ is the ground state of the target, $|\Psi_n\rangle$ is the state of the residual system, and a_i is the destruction operator for a particle in the orbit i. The energy-weighted average of the spectroscopic factors is a quantity of current theoretical interest,[1] particularly with respect to the interpretation of $(p,2p)$ and $(e,e'p)$ experiments at high energies.

We assume we have a system whose dynamics is governed by a Hamiltonian H containing two-body forces. (We will use the notation where $i,j,k,\cdots$ refer to occupied states, $m,n,p,\cdots$ to unoccupied states, and $\alpha,\beta,\gamma,\cdots$ to either.)

Consider the operator defined through the relation[1]

$$W_i=\{a_i^\dagger,[H,a_i]\}, \tag{1}$$

where the curly brackets mean an anticommutator. It is easily seen that

$$\langle\Psi_0|W_i|\Psi_0\rangle=\sum_n(E_n-E_0)|\langle\Psi_n|a_i|\Psi_0\rangle|^2 -\sum_n(E_n-E_0)|\langle\Psi_n|a_i^\dagger|\Psi_0\rangle|^2, \tag{2}$$

where $H|\Psi_0\rangle=E_0|\Psi_0\rangle$, $H|\Psi_n\rangle=E_n|\Psi_n\rangle$. It is well known that in the Hartree-Fock approximation $\langle\Psi_0|W_i|\Psi_0\rangle=\epsilon_i$, where ϵ_i is the Hartree-Fock single-particle energy. In general the operator W_i is given by

$$W_i=\langle i|t|i\rangle+\sum_{\alpha\beta}\langle i\alpha|v|i\beta\rangle_A a_\alpha^\dagger a_\beta, \tag{3}$$

where t is a kinetic-energy operator and

$$\langle i\alpha|v|i\beta\rangle_A=\langle i\alpha|v|i\beta\rangle-\langle i\alpha|v|\beta i\rangle.$$

It is clear that in a system with singular interactions the ground-state expectation value of W_i is infinite and Eq. (2) is not useful. We can trace this difficulty to the use of the anticommutator. If we denote the correlated ground state by $|\Psi_0\rangle$, we note that the quantity $\langle\Psi_0|[H,a_i]a_i^\dagger \times|\Psi_0\rangle$ is the source of the divergence of $\langle\Psi_0|W_i \times|\Psi_0\rangle$. This is because we are introducing a particle in orbit i into the system in such a manner that its correlations (due to the hard core) with the other particles are absent. Thus we are led to the consideration of the operator

$$\Omega_i=a_i^\dagger[H,a_i], \tag{4}$$

where, as before, we have

$$C_i\equiv\langle\Psi_0|\Omega_i|\Psi_0\rangle=\sum_n(E_n-E_0)|\langle\Psi_n|a_i|\Psi_0\rangle|^2. \tag{5}$$

We may write, in general,

$$|\Psi_0\rangle=F|\Phi_0\rangle/\langle\Phi_0|F^\dagger F|\Phi_0\rangle^{1/2}, \tag{6}$$

$$F=1+F^{(2)}+F^{(3)}+\cdots,$$

where $F^{(n)}$ is an n-body operator. (We may also write $F=e^S$, with $S=S^{(2)}+S^{(3)}+\cdots$. The neglect of $S^{(n)}$, $n>2$, is equivalent to the neglect of three-

body and higher-order cluster effects in the theory of correlated systems.) Also,

$$\Omega_i = \sum_\beta t_{i,\beta} a_i^\dagger a_\beta + \tfrac{1}{2}\sum_{\beta\gamma\delta} \langle i\beta|v_{12}|\gamma\delta\rangle_A a_i^\dagger a_\beta^\dagger a_\delta a_\gamma = \sum_\beta t_{i,\beta} a_i^\dagger a_\beta + \tfrac{1}{2}\sum_{\alpha\beta\gamma\delta} \langle \alpha\beta|v_{12}^{(i)}|\gamma\delta\rangle_A a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma, \tag{7}$$

where we have introduced the quantity

$$v_{12}^{(i)} = |i(1)\rangle\langle i(1)|v_{12}. \tag{8}$$

The result for C_i may be obtained using a cluster-expansion technique,[2,3]

$$C_i = \sum_j t_{i,j}\gamma_{j,i} + \sum_{gjkl} \langle gj|h_{12}^{(i)}|kl\rangle_A \gamma_{k,g}\gamma_{l,j} + \cdots, \tag{9}$$

where

$$\gamma_{j,i} = \langle \Psi_0|a_i^\dagger a_j|\Psi_0\rangle, \tag{10}$$

and the sum in Eq. (9) runs over the occupied orbits of $|\Phi_0\rangle$.[4] In Eq. (10),

$$\langle gj|h_{12}^{(i)}|kl\rangle_A = \langle gj|(1+f_{12}^\dagger)v_{12}^{(i)}(1+f_{12})|kl\rangle_A, \tag{11}$$

where f_{12} is defined through the relation[2,3]

$$F^{(2)} = \tfrac{1}{2}\sum_{mnij} a_m^\dagger a_n^\dagger \langle mn|f_{12}|ij\rangle a_j a_i. \tag{12}$$

It was shown previously[2] that it is appropriate to identify $1+f_{12}$ with the wave operator for the Bethe-Goldstone equation, i.e., $f_{12} = -(Q/e)K_{12}$. Recalling the definition of $v_{12}^{(i)}$ we have[5]

$$\langle gj|h_{12}^{(i)}|kl\rangle_A = \langle ij|K_{12}|kl\rangle_A \delta_{gi}, \tag{13}$$

and

$$C_i = \sum_j t_{i,j}\gamma_{j,i} + \sum_{jkl} \langle ij|K_{12}|kl\rangle_A \gamma_{k,i}\gamma_{l,j} + \cdots. \tag{14}$$

Further we will assume we are in the representation in which $\gamma_{j,i}$ is diagonal so that (14) becomes, with $\gamma_{i,j} = \gamma_i\delta_{ij}$ and $t_{i,i} = \langle i|t|i\rangle$,

$$C_i = t_{i,i}\gamma_i + \sum_j \langle ij|K_{12}|ij\rangle_A \gamma_i\gamma_j + \cdots. \tag{15}$$

Now we introduce the eigenvalue of the Brueckner-Hartree-Fock equation as defined to include occupation factors:[6,7]

$$\epsilon_i^{\mathrm{BHF}} = t_{i,i} + \sum_j \langle ij|K|ij\rangle_A \gamma_j, \tag{16}$$

so that finally[8]

$$C_i = \epsilon_i^{\mathrm{BHF}}\gamma_i. \tag{17}$$

Note that in deriving Eq. (17) we have neglected the effects of three-body cluster terms which are expected to be small.

The quantity γ_i is the occupation probability of the orbit i and is expected to have a value somewhere between 0.7 and 0.9 in the light of recent studies of finite nuclei.[7] We also note the sum rule,

$$\sum_n |\langle \Psi_n|a_i|\Psi_0\rangle|^2 = \langle \Psi_0|a_i^\dagger a_i|\Psi_0\rangle = \gamma_i. \tag{18}$$

As the quantity γ_i may differ appreciably from unity, these results should be important for understanding the spectroscopic centroid observed in $(p,2p)$ and $(e,e'p)$ experiments.

We should point out that the above result does not apply for the slow (adiabatic) removal of a particle, in which case we require a theory of separation energies. One of us (C.M.S.) wishes to acknowledge discussions with D. Koltun who has also studied this problem.

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†Permanent address: Department of Physics, Case Western Reserve University, Cleveland, Ohio 44106.

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[4]Additional terms in Eq. (9) would describe the effects of clusters of higher order.

[5]In deriving Eq. (13) we have used the fact that $f_{12}^\dagger$ does not contribute to Eq. (11). This is true since $f_{12}^\dagger$ scatters particles g and j, of Eq. (11), out of the occupied states. Thus,

$$\sum_{m,n} \langle gj|f_{12}^\dagger|mn\rangle\langle mn|i(1)\rangle\langle i(1)|v_{12} = 0,$$

since the unoccupied (m,n) orbitals are orthogonal to the occupied orbital (i). Similar considerations are used to show that the one-body operator of Eq. (7) does not contribute a two-body cluster term in Eq. (9); this again depends on the fact that the orbit i is occupied.

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$$\epsilon_{k,l}=t_{k,l}+\sum_{j,j'}\langle kj|K|lj'\rangle\gamma_{j',j}$$

may be simultaneously brought to diagonal form. The exact result is $C_i=\sum_k \epsilon_{i,k}\gamma_{k,i}$; in Ref. 3 it is shown that the *matrix product* $\epsilon\gamma$ may be brought to diagonal form.

Reaction $^{12}C(^{14}N, ^{6}Li)^{20}Ne$ and the Structure of the Upper $K = 0^+$ Bands

K. Nagatani and M. J. Le Vine
Brookhaven National Laboratory, Upton, New York 11973*

and

T. A. Belote
Massachusetts Institute of Technology,† Cambridge, Massachusetts 02139

and

A. Arima
State University of New York,‡ Stony Brook, New York 11790
(Received 27 July 1971)

The reaction $^{12}C(^{14}N, ^{6}Li)^{20}Ne$ was studied at 60 MeV and forward-angle reaction data to states of ^{20}Ne were obtained. Different rotational bands were populated with different strengths consistent with cluster transfer processes. The previously questioned structure of the $K = 0^+$ bands starting near 7 MeV is discussed and a new classification for the bands is proposed.

Studies of transfer reactions using heavy ions have become increasingly of interest. In this Letter, the eight-nucleon transfer reaction leading to states in ^{20}Ne is reported. This nucleus is of special interest in the testing of many nuclear structure models and has been the subject of a great deal of study.[1,2]

The lower rotational bands of ^{20}Ne have been extensively studied. However, while the structure of the $K = 0^+$ bands starting near 7-MeV excitation has been the subject of much theoretical interest, little experimental data exist which provide evidence in determining their characteristics.

Because of the expected collective nature of these states they can best be investigated by cluster transfer reactions. The α transfer reactions, $^{16}O(^{6}Li, d)^{20}Ne$ and $^{16}O(^{7}Li, t)^{20}Ne$,[3] have provided vital information about these states. These reactions excite levels having four particles coupled to an ^{16}O core. More complicated states such as those having holes in the $1p$ shell are expected to be excited by the transfer of eight nucleons to ^{12}C. Therefore, the reaction $^{12}C(^{14}N, ^{6}Li)^{20}Ne$ was studied. A similar experiment has been recently reported by Marquardt *et al.*,[4] but little analysis was made by those authors.

A $^{14}N^{5+}$ beam was obtained from the Brookhaven tandem facility using a direct-extraction negative-ion source. The incident energy was 60.0 MeV, and a beam of about 1 μA charge current was utilized. A nominal 20-$\mu g/cm^2$ natural carbon target was used. The ^{6}Li spectra were obtained using a conventional ΔE-E solid-state counter telescope having an angular resolution of 0.5°. The particle identification and energy calibration was checked by observing recoil ^{6}Li from a ^{6}Li target. The overall laboratory energy resolution was about 300 keV. Spectra were obtained at laboratory angles of 11.0°, 15.0°, 20.0°, 25.0°, 30.0°, and 40.0°. The 20.0° spectrum is shown in Fig. 1 where the peaks are identified by their excitation energies and J^π's. An absolute cross-section scale accurate to about a factor of two was determined; for example, the center-of-mass cross section of the 1.63-MeV (2^+) level at 20.0° (see Fig. 1) was 11 μb/sr.

In the present reaction we expect the mechanism to be dominantly a direct process. Experimentally this assumption is supported by the observation of the smaller yield of ^{7}Li relative to ^{6}Li by about a factor of ten,[5] while a ratio of the decay strengths from the compound state in ^{26}Al is cal-