Zone Oscillations in the Magnetoresistance of Tin

J. K. Hulbert and R. C. Young
Department of Physics, University of Birmingham, Birmingham B15 2TT, England
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A very high-frequency (2.458×10^8 G) quantum oscillation corresponding to the quantization of flux through the cross section of the Brillouin zone—Pippard's "zone oscillation" —is found when $\vec{H}$ is within 1° of the c axis. It is best seen at 4.2°K, where it constitutes 5% of the resistance, while its anomalously small effective mass causes it to be swamped by slower oscillations at 1°K. A second harmonic is seen for $\vec{H}$ within 0.04° of the c axis.

When magnetic breakdown couples electron orbits in a metal into a network, any theory of the energy states of the system must take account of the gauge of the magnetic vector potential. This problem was discussed by Pippard[1] who showed that if, at a point $\vec{r}$, an electron was switched from one orbit of gauge center $\vec{R}_1$ to another of gauge center $\vec{R}_2$ it was necessary to introduce a phase shift $\Delta=\frac{1}{2}\alpha\cdot\vec{r}\times(\vec{R}_2-\vec{R}_1)$, where $\vec{\alpha}=e\vec{H}/\hbar$. This phase shift is origin dependent, and so in a two-dimensional network the magnetic wave function of the coupled system will not in general have the same translational symmetry as the lattice of coupled orbits, except for discrete magnetic field values such that $\Delta/2\pi$ is integral. These field values are exactly those which would be given by quantizing the flux through the Brillouin zone,[2] and so Pippard predicted that a "zone oscillation" should occur in the properties of the two-dimensional network of coupled orbits.[3]

The purpose of this Letter is to report the first observation of this zone oscillation in the transverse magnetoresistance of a single crystal of tin. The appropriate two-dimensional network of coupled orbits, Fig. 1, occurs when $\vec{H}$ is near the c axis, and it is in this region that we see the zone oscillation.

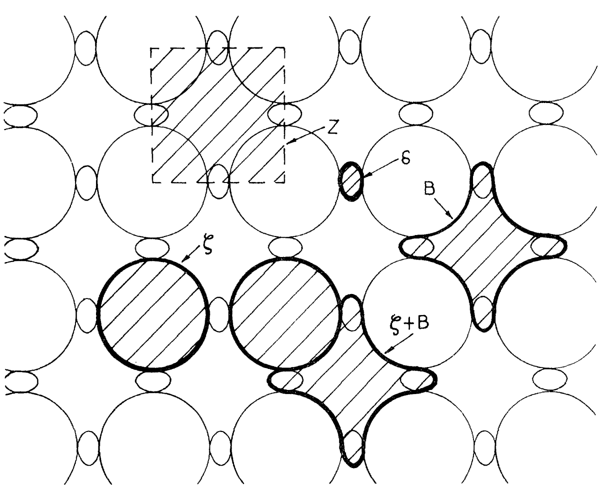

FIG. 1. The network of coupled orbits when $\vec{H}$ is along the c axis in tin, and breakdown couples orbits in the third and fourth zones. The orbits δ, ζ, and B are shown, as well as the combined orbit $\zeta+B$ which has an area (shaded) equal to $Z+2\delta$.

At 1°K the quantum oscillations which we observe are a slow frequency of 1.7×10^6 G and the fast frequencies 1.13×10^8 G and 1.33×10^8 G, in agreement with previous work,[4] corresponding to the extremal cross sections of the orbits δ, ζ, and B of Fig. 1. At 4.2°K the amplitude of these fast oscillations is so reduced by the Dingle factor as to be practically undetectable. At this temperature we find that if $\vec{H}$ is within about 1° of the c axis the dominant oscillation is the much faster frequency of 2.458×10^8G—the "zone oscillation." When $\vec{H}$ is nearer than 0.04° to the c axis there is even a second harmonic of the zone oscillation. Examples are shown in Fig. 2, where these very fast oscillations represent about 5% of the total resistance.

In all previous observations of quantum oscillations, except for the very low-frequency quantum interference effect of Stark and Friedberg,[5] the frequencies have corresponded, even for networks of coupled orbits, to the quantization of

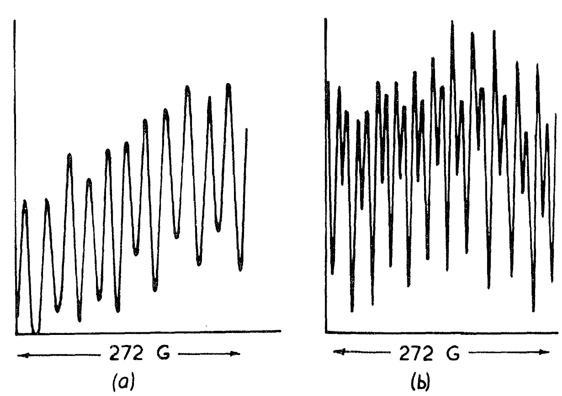

FIG. 2. Typical examples of zone oscillations at 4.2°K and 7 kG. (a) $\vec{H}$ within 1° of the c axis. (b) $\vec{H}$ within 0.04° of the c axis; second harmonic appears.

flux through the areas enclosed by actual electron orbits. The zone oscillation is an exception to this rule in that it represents the quantization of flux through an area which is not enclosed by a possible electron orbit, i.e., it is not possible to find a closed electron path which encloses an area equal to the cross section Z of the Brillouin zone.

The closed orbit nearest in area to Z, $\zeta + B$ of Fig. 1, has the area $Z+2\delta$, differing from Z by only 1.5%. To obtain the required accuracy to distinguish between these frequencies, we measured the magnetic field during the actual run using the NMR of copper. A Fourier analysis of our very fast oscillations gave the components shown in Table I. The principal frequency 2.458×10^8 G agrees well with the value of Z calculated from Craven's[6] lattice parameters for tin, 2.459×10^8 G. The other components identify with other combinations of Z and δ including the second harmonic $2Z$ and its satellites. Of these components, only $Z+2\delta$ represents a possible closed orbit.

TABLE I. The frequency components of a Fourier analysis of a 4.2°K sample, compared to predicted frequencies based on the lattice parameters given by Craven (Ref. 6).

Measured frequency (10^8 G)	Relative amplitude	Identification	Predicted frequency (10^8 G)
2.458	1	Z	2.459
2.476	0.59	$Z+\delta$	2.476
2.494	0.24	$Z+2\delta$	2.493
2.441	0.49	$Z-\delta$	2.442
2.424	0.22	$Z-2\delta$	2.425
4.917	0.53	$2Z$	4.918
4.935	0.27	$2Z+\delta$	4.935
4.953	0.09	$2Z+2\delta$	4.952
4.899	0.31	$2Z-\delta$	4.901
4.881	0.06	$2Z-2\delta$	4.884

The field and temperature dependences of the amplitude of quantum oscillations are governed by the Dingle[7] factor, $[\gamma T/\sinh(\gamma T)]\exp(-\gamma T^*)$, where $\gamma = 2km^*/eH$ and $m^* = (\hbar^2/2\pi)\partial A/\partial\epsilon$. Usually there is an approximate proportionality between the frequency and m^*, but the zone group of oscillations would be expected to show anomalously small values of m^*. In fact, Z itself is an area which is independent of electronic energy, i.e., to corresponds to zero effective mass, while its satellites should have m^* values of $0.10m_0$ and $0.20m_0$, much smaller than the values $0.6m_0$ and $0.8m_0$ for ζ and B respectively. It is exactly this property of anomalously small m^* which explains why the zone oscillations are dominant at 4.2°K but are swamped by slower oscillations at 1°K.

Similarly the observed change of amplitude with field is also anomalously small. If we assume that T^* will be the same for all frequencies on a network, it appears that the falloff with field is also described by a small m^*. Thus the usual "rules" that the faster oscillation will have the bigger temperature dependence, the steeper field dependence, and the smaller amplitude do not hold for the zone oscillations. The observed properties of these very fast oscillations are, at least qualitatively, consistent with the Dingle formula and small values of m^*. This implies that high harmonics of the zone oscillation should be readily observable although of course limited by the inhomogeneity of the magnetic field. However, since we see a large-amplitude second harmonic of the zone oscillation only when $\vec{H}$ is within 0.04° of the c axis, it is likely that considerably more accurate location will be required to find find the higher harmonics.

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[1] A. B. Pippard, Phil. Trans. Roy. Soc., Ser. A 256, 317 (1964).

[2] Integral values of $\Delta/2\pi$ actually lead to a frequency corresponding to half the zone, but it is not difficult to show that half-odd-integral values lead to the same structure of magnetic energy levels as integral values and hence to the zone frequency.

[3] The zone oscillation is also discussed by W. G. Chambers, Phys. Rev. 165, 799 (1968).

[4] R. C. Young, Phil. Mag. 18, 201 (1968); J. O. Strom-Olsen, Phys. Rev. 180, 726 (1969).

[5] R. W. Stark and C. B. Friedberg, Phys. Rev. Lett. 26, 556 (1971).

[6] J. E. Craven, Phys. Rev. 182, 693 (1969).

[7] R. B. Dingle, Proc. Roy. Soc., Ser. A 211, 517 (1952).