MeV. If we then require $E_D \simeq 80A^{-1/3}$ we find from Eq. (13) $\Delta_0 \simeq 40$ MeV and therefore also $\Delta_1 \simeq 40$ MeV. With Eq. (9) kept in mind we finally arrive at the result

$$U_D \simeq [(100-40)\ \text{MeV}]T/A = (60\ \text{MeV})T/A. \quad (16)$$

We summarize our results as follows. In the discussion of the isospin splitting of single-proton levels we start with the observation that the single-proton energy is equal to that of the corresponding neutron plus the Coulomb energy minus the symmetry energy. The latter measures the difference in the nuclear interactions between a neutron and a proton with the nuclear core. The energy difference between the two isospin components of the proton state is then given (apart from the geometrical factor) by the symmetry energy. In the dipole problem, however, we are comparing the energy of neutron–proton-hole states with that of proton–proton-hole and neutron–neutron-hole states in order to find the appropriate symmetry energy. Together with the particle-shell interactions this involves also the difference in the particle-hole interactions which, as we found, reduces the effective symmetry energy. The separation of the two isospin components is then given by a familiar expression [Eq. (7)] but with "symmetry energy" appropriately evaluated for the dipole problem. Our approach has been rather simple-minded but it should be expected to describe general trends and to provide a meaningful comparison with experimental data.[11]

We wish to thank Professor P. Paul for several stimulating discussions.

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ft Asymmetry in Mirror Gamow-Teller β Decay: Binding-Energy Effects

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It is shown that the systematic asymmetry observed between positron and negatron emitters in mirror Gamow-Teller β decay is not due to binding-energy differences between the respective β-transforming nucleons; it must be due either to a fundamental weak interaction effect or to a nuclear structure effect of some type not yet quantitatively discussed.

Mirror Gamow-Teller β decay takes place from analog $T=1$ states, such as ^{8}Li and ^{8}B, leading to a common $T=0$ final state, or from analog $T=\frac{3}{2}$ states, such as ^{9}Li and ^{9}C, then leading to analog $T=\frac{1}{2}$ final states. Contrary to simple ideas, the reduced speeds of the mirror positron and negatron transitions, $(ft)^+$ and $(ft)^-$, respectively, are systematically different[1-7]; the asymmetry parameter $\delta=(ft)^+/(ft)^- - 1$ is typically $+0.15$ or so, as is shown in Tables I and II.

This surprising asymmetry may be due either to a new weak-interaction effect such as a second-class current[11] of some type[12] or to a failure of exact symmetry in the nuclear structure. It is critically important to distinguish between these fundamental and trivial explanations. Of the trivial effects so far considered quantitatively, the only one of significant magnitude is that

TABLE I. Mirror asymmetry in the $1p$ shell.

A	logft (expt)	logft (theor)	δ_0 (A)	δ_0 (B)	δ (A)	δ (B)	$\langle\delta\rangle^a$ (A)	$\langle\delta\rangle^a$ (B)	δ (expt)
8	5.48	5.26	0.132	0.075	0.040	0.048	...	...	0.107 ± 0.011^b
9	5.19	5.12	0.086	0.040	−0.050	−0.016	−0.014	−0.002	0.188 ± 0.030^c
9^{*d}	5.13	5.10	0.081	0.039	0.062	0.026			
12	4.10	4.11	0.074	0.047	0.146	0.098	...	...	0.115 ± 0.009^c
12^{*e}	5.10	4.80	0.075	0.044	0.063	0.048	...	...	-0.117 ± 0.041^f
13	4.06	3.93	0.049	0.030	0.047	0.031	0.043	0.027	0.166 ± 0.026^g
13^{*h}	4.48	4.68	0.052	0.029	0.013	−0.001			

$^a\langle\delta\rangle$ takes into account branches to excited states.
^{b}Ref. 6.
^{c}Ref. 1 with small corrections.
$^d E_x = 2.43$, 2.33 MeV in ^{9}Be, ^{9}B, respectively.
$^e E_x = 4.44$ MeV.
^{f}This figure derives from ^{12}N/^{12} branching-ratio ratios of 1.72 ± 0.15(Ref. 8) and 1.84 ± 0.10 (Ref. 9).
^{g}Ref. 4.
$^h E_x = 3.68$, 3.51 MeV in ^{13}C, ^{13}N, respectively.

due to the binding-energy difference between the proton that makes the β transition in the positron emission and its mirror neutron that accomplishes the negatron emission.[13] This binding-energy difference entrains different overlap integrals with the particles into which β transformation takes place, and hence generates a finite δ (of plus a few percent for $A = 12$, the only system so far treated[13,14]). This note demonstrates that this effect cannot, in fact, systematically account for the experimental asymmetry and so eliminates what appears to be the most important of the trivial explanations.

A defect of the earlier calculations[13,14] of the binding-energy effect is that they are single-particle computations in which the ground state of the parent nucleus is taken as the unique parent. In fact, a whole spectrum of parent states is operative in the $A-1$ system: It is only by taking this into account that the actual ft values can be understood; the finer effect that concerns us here must certainly have regard for it. For the $1p$-shell cases (Table I), excellent wave functions are available from Cohen and Kurath[15] and they have been used here. For the $(2s, 1d)$-shell cases (Table II), similarly detailed wave func-

TABLE II. Mirror asymmetry in the $(2s, 1d)$ shell.

A	$\delta_0(1d_{5/2})$	$\delta_0(2s_{1/2})$	δ (expt)	δ (expt)$/\delta_0{}^a$
17	0.023	0.072	0.15 ± 0.03^b	4.3 ± 0.9
18	0.005	0.013	-0.008 ± 0.015^c	$...^d$
20	0.033	0.082	0.054 ± 0.023^e	1.2 ± 0.5
24	0.030	0.070	-0.03 ± 0.06^f	-0.8 ± 1.8
25	0.029	0.060	0.207 ± 0.065^g	5.6 ± 1.8
28	0.040	0.084	0.25 ± 0.05^f	4.9 ± 1.0
30	0.004	0.008	0.02 ± 0.05^f	$...^d$

$^a\delta_0$ here weights $\delta_0(1d_{5/2})$ and $\delta_0(2s_{1/2})$ in the ratio 3:1 appropriate to the respective numbers of particles in the shells; this is not inconsistent with the Oak Ridge wave functions (Ref. 10).
^{b}Ref. 5.
^{c}Ref. 3.
$^d A = 18$ and 30 are not quoted because small δ values may be expected for them, as observed: They involve two successive positron emissions rather than the positron-negatron comparison of all the rest.
^{e}Ref. 4.
^{f}Ref. 1.
^{g}Ref. 7.

tions are not everywhere available, and a Monte Carlo approach has been used instead. For both shells we conclude that this binding-energy effect cannot systematically account for the experimental asymmetry.

The single-nucleon wave function associated with each parent state falls off asymptotically outside the nucleus as prescribed by the separation energy with respect to that parent state. In the $1p$ shell two approaches have been made to the problem of the single-nucleon wave functions *inside* the nucleus. In the first approach, method A, the wave functions were generated in standard[16] Woods-Saxon potentials with $r_0 = 1.36$ fm, $a = 0.55$ fm, including a Thomas spin-orbit term of strength $9(\hbar/m_\pi c)^2$ MeV (plus a Coulomb term as needed); the various separation energies were induced by variation of the depth of the central potential. The second approach, method B, respects the fact that the real situation is a complicated one of many coupled channels[17]; the effect of this is largely to remove, inside the nucleus, the differences between the individual nucleon motions associated with the various parent states but at the same time to allow these differences to develop as the asymptotic region outside the nucleus is approached. Prakash and Austern[18] have suggested using a constant central potential plus a δ function of adjustable strength somewhere toward the surface to induce the correct asymptotic behavior. The present work adopts the gentler stratagem of using a Woods-Saxon potential of the above form but of fixed depth plus a potential of Thomas form adjustable in strength to induce the correct asymptotic behavior. For the fixed central potential a depth of 49.7 MeV has been used, the mean of the values found necessary for it using method A. Method B is probably closer to the truth but the results of both methods are presented here. For the $(2s, 1d)$-shell cases only method A, which probably exaggerates the calculated effect, has been used.

In the many-parent description, the β-transition amplitude for mirror transitions with N equivalent nucleons, $J_i \to J_f$, where the parent J_π combines with $j_1 \to j_2$ as $\vec{J}_\pi + \vec{j}_1 = \vec{j}_i$ and $\vec{J}_\pi + \vec{j}_2 = \vec{J}_f$, is given by

$$[\Lambda^\pm]^{1/2} = N[3(2J_f+1)(2-1/T_i)]^{1/2} \sum_\pi (-)^{J_\pi + J_f + j_1 + j_2 - 1/2} \times [(2j_1+1)(2j_2+1)]^{1/2} \begin{Bmatrix} j_1 & j_2 & 1 \\ j_f & J_i & J_\pi \end{Bmatrix} \begin{Bmatrix} j_1 & j_2 & 1 \\ \frac{1}{2} & \frac{1}{2} & l \end{Bmatrix} \langle f[\![\pi j_2\rangle \langle i[\![\pi j_1\rangle \Omega_{\pi j_1 j_2}{}^\pm,$$

where $\log(ft)^\pm = 3.61 - \log\Lambda^\pm$. Here the $\Omega_{\pi j_1 j_2}{}^\pm$ are the single-nucleon overlap integrals discussed above. Then, $\delta = \Lambda^-/\Lambda^+ - 1$.

For the $1p$ shell the Cohen-Kurath fractional parentage coefficients[19] yield the δ values of Table I; the logft values deriving from the same wave functions there illustrate the excellence of the theoretical account. δ_0 is the δ value appropriate to the ground state of $A-1$ as unique parent (using the nominal j_1, j_2 of jj coupling). We see that the binding-energy effect fails systematically to reproduce the experimental δ values.[20] Using method B, $\langle\delta_{\text{theor}}\rangle = 0.042$ vs $\langle\delta_{\text{expt}}\rangle = 0.144$[21]; method A gives $\langle\delta_{\text{theor}}\rangle = 0.054$.

For the $(2s, 1d)$-shell cases of Table II the following Monte Carlo treatment has been used: A reasonable distribution of the fractional parentage coefficients was adopted[19]; a distribution for the $6j$ symbols, etc. was defined relating to the situations operative in practice in both shells; similarly, $\Omega_{\pi j_1 j_2}{}^\pm$ values were computed (using method A) for parentage excitations that reproduce, statistically, those encountered in practice. The results of this computation[22] are shown in Fig. 1 where the probability of a certain value of δ is displayed as a function of δ/δ_0; also shown are the δ/δ_0 values computed explicitly for the $1p$-shell cases from Table I. These explicit computations fall well into accord with the Monte Carlo computations. The experimental cases of Table II may now confidently be compared with

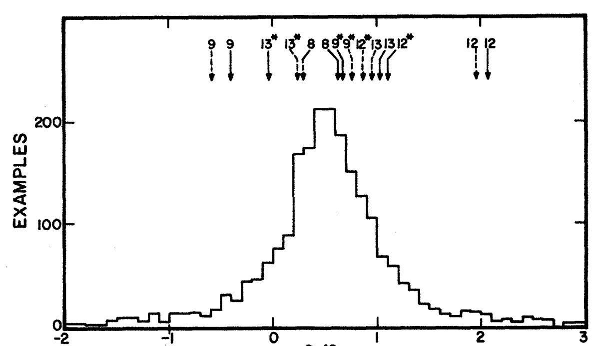

FIG. 1. Results of the Monte Carlo computation of the asymmetry δ; δ_0 regards the ground state of $A-1$ as unique parent. The arrows show the δ/δ_0 values from Table I computed from the explicit detailed wave functions for method A (dashed arrows) and method B (solid arrows).

the Monte Carlo computation of Fig. 1. We see that the experimental δ/δ_0 values ($\langle\delta/\delta_0\rangle = 3.0$) are exceedingly unlikely to be due to binding-energy effects, and also that δ/δ_0 values of about -2, symmetrically disposed to the left of the maximum, should in that case be equally likely, which they are not.[23] The evidence of the ($2s, 1d$) shell therefore conclusively rejects the possibility, both in respect to magnitude and, in a related fashion, to sign, that the binding-energy effect can be responsible for the experimental asymmetry.

The $1p$ shell and the ($2s, 1d$) shell therefore give the same answer: The experimental asymmetry is not due to the binding-energy effect on the β-transforming nucleon. Other trivial possibilities remain to be considered. The present computation has concerned itself only with the overlap integrals associated with the β-transforming nucleons. But, in the β transition, the other $A-1$ nucleons are not wholly unaffected and their changes affect the transition rate. A crude estimate[24] of this effect has been made in a spherical basis in the $1p$ shell and it there seems incapable by a large factor of explaining the empirical effect. The trivial factor awaiting serious quantitative evaluation is the possibility of changes in configurational mix across the multiplets: For example, a change of deformation will affect the β-transition moment to the power A and so may be a potent effect. It would certainly be surprising if any such effect should have about the same numerical magnitude over the wide range of A covered by the experimental data, and also that it should be of constant sign. It must also be remarked that any attempt to explain the β-asymmetry effect must respect the rather good analog symmetry seen across light isobaric multiplets[25] and also the excellent conserved vector current evidence from the pure Fermi transitions which display a high degree of mutual consistency.[26]

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[21]^{18}12* is so glaring an exception to the systematics that we withold it from the present averaging while emphasizing the great importance of its redetermination (which is under way).

[22]Further details will be given elsewhere but it may be remarked here that the resulting distribution of ft values reproduces very well that found in practice across the two shells.

[23]$A = 24$ has $\log ft = 6.1$. The Monte Carlo computation showed that, as expected, large ft values tend to entrain large (positive or negative) values of δ/δ_0 and this effect may be operative here.

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Structure of the Electromagnetic Field in a Spatially Dispersive Medium*

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The exact mode expansion is derived for the electromagnetic field in a spatially dispersive model dielectric occupying the volume $0 \leq z \leq d$. The dispersion relations for the transverse as well as the longitudinal waves are deduced and the nature of the modes is briefly discussed.

Electrodynamics of spatially dispersive media, i.e., of media whose response to an incident electromagnetic field is spatially nonlocal, has attracted a great deal of attention since Pekar[1] predicted some rather remarkable phenomena associated with spatial dispersion. The close connection between this subject and the theory of excitons is, of course, well known.[1-4]

In spite of the great deal of interest in this subject, some rather basic questions in this domain have as yet not been solved. One of them concerns the exact mode expansion of an electromagnetic field in a spatially dispersive medium that does not occupy the whole infinite space. It is often assumed that in any volume occupied by a spatially dispersive medium the electromagnetic field may be expanded in terms of plane waves whose (generally complex) propagation vectors are identical with those appropriate to a field in a spatially dispersive medium occupying all space. That this assumption is questionable is clear if one recalls that in a half-space, even in the absence of any material medium, plane waves may be propagated that cannot be physically realized in the whole empty space. These are the so-called evanescent waves, well known in the theory of total internal reflection[5] and in connection with other interaction problems.[6]

In the present paper we derive an exact mode expansion for the field in a spatially dispersive model dielectric that occupies the volume $-\infty \leq x \leq \infty$, $-\infty \leq y \leq \infty$, $0 \leq z \leq d$. The exact dispersion relations for both transverse and longitudinal modes are found, and the nature of the expansion is briefly discussed. This mode expansion has a bearing on many aspects of the electrodynamics of spatially dispersive media and on the theory of excitons. We will show in another publication that our expansion leads readily to the exact solution of the problem of refraction and reflection on a half-space filled with a spatially dispersive medium and that the solution provides complete resolution of a long-standing controversy about the so-called additional boundary conditions,[4,7-9] generally believed to be necessary for the solution of this problem.

Consider first an electromagnetic field in a spatially dispersive medium occupying the whole infinite space. For the sake of simplicity we assume the medium to be homogeneous and nonmagnetic. The constitutive relation which couples the electric vector $\vec{E}$ and the electric displacement $\vec{D}$ may be expressed in the form

$$\hat{\vec{D}}(\vec{k}, \omega) = \hat{\epsilon}(\vec{k}, \omega)\hat{\vec{E}}(\vec{k}, \omega), \tag{1}$$

where the circumflex denotes a four-dimensional Fourier transform [with kernel $e^{i(\vec{k}\cdot\vec{r}-\omega t)}$]. Following Hopfield and Thomas,[4] we restrict our discussion, for the sake of simplicity, to a medium for which the dielectric constant $\hat{\epsilon}(\vec{k}, \omega)$ is of the form

$$\hat{\epsilon}(\vec{k}, \omega) = \epsilon_0(\omega) + \alpha_e \omega_e^2 [\omega_e^2 - \omega^2 + (\hbar\omega_e/m_e^*)k^2 - i\omega\Gamma_e]^{-1}. \tag{2}$$

Here, $\epsilon_0(\omega)$ is the wave-vector–independent background dielectric constant associated with all transi-