

$T_>$ collective dipole strength decreases rapidly and since prominent single-particle analog states occur¹⁴ at energies above the main ($T_<$) GDR which complicates the identification of the collective $T_>$ strength.

The approximate constancy of \bar{V} over a large mass region proves that the major part of the displacement energy between the different isospin components of the collective dipole state is well described by the assumption of an average symmetry energy which is about 60% of the s.p. value. The average of all values in Table I is 58 ± 5 MeV. A small residual dependence of \bar{V} on T , evident in Table I, is well fitted by $\bar{V} = 67(1 - 3.9T/A)$ MeV. Terms of this order in T/A , which have been neglected in Ref. 7 but may be obtained from Ref. 11, cannot account for the sign of the observed deviation.

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Energy Displacement of Dipole Isodoublets*

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The difference in energy between the two isospin components of the giant dipole resonance in a nucleus with nonvanishing neutron excess is studied. An estimate is presented where this difference is expressed as a function of the mass number and the ground-state isospin of the nucleus.

The isovector nature of the interaction of an electric dipole field with a nucleus implies the familiar isospin selection rule $\Delta T = \pm 1, 0$ with no zero-to-zero transitions.¹ Since the isospin in the nuclear ground state has normally its minimum possible value, i.e., $T = (N - Z)/2$, the electric-dipole absorption of photons by a nucleus can only lead to the formation of states with isospin T or $T + 1$ with the first possibility disappearing when $T = 0$. Estimates of the fraction of the total absorption strength associated with each isospin group are already available² with the general conclusion being that the $\Delta T = 0$ transitions are normally (for $T \geq 1$) stronger. This is relat-

ed to the fact that there are in general more (particle-hole) configurations leading to $T \rightarrow T$ transitions, with this number becoming larger when the number of excess neutrons increases.

The characteristic structure of the two (i.e., T and $T + 1$) spectra is also of some interest. This has been studied³ in some detail theoretically for nuclei in the $A \approx 90$ region. Predictably, the dipole strength was seen to concentrate near the high-energy end of each group. The states with isospin T (which contained the normal giant dipole resonance) were in general located at energies lower than those of the states with isospin $T + 1$ (which are also analogs of levels in a neigh-

boring nucleus). This displacement in energy is a result of the competition between (a) the symmetry energy, which tends to move states with higher isospin to higher energies, and (b) the difference in the particle-hole interactions, which (since there are more states in the T -lower group) works in the opposite direction.

Rather than continuing the study of individual cases we will present in this note a simple and rather general estimate of the average displacement energy between the centers of strength of the two groups.^{4,5} This clearly corresponds also to the energy difference between the giant members of each spectrum. Denoting these two energies by E_T and E_{T+1} , respectively, we can follow a schematic approach⁶ and write

$$\begin{aligned} E_{T+1} &= \epsilon + U + g|b_{T+1}|^2, \\ E_T &= \epsilon - T^{-1}U + g|b_T|^2. \end{aligned} \quad (1)$$

Here ϵ represents the average single-particle excitation energy while U is the symmetry energy as expressed, e.g., by the Lane potential.⁷ The last term in each expression corresponds to the particle-hole interaction, which was assumed to be proportional to a coupling constant g and the (reduced) dipole strength in each case. The strengths $|b_{T+1}|^2$ and $|b_T|^2$ were estimated in Ref. 2 with the results

$$|b_{T+1}|^2 \simeq S_0[1 - \alpha_1], \quad |b_T|^2 \simeq S_0[1 + T^{-1}\alpha_1], \quad (2)$$

where

$$S_0 = NZ/2\gamma A \simeq A/8\gamma = A^{4/3}/8\gamma_0 \quad (3)$$

and

$$\alpha_1 S_0 \simeq (T/10)r_0^2 A^{2/3}. \quad (4)$$

The quantity S_0 is essentially (a shell-model estimate of) the total dipole transition strength, γ is the radial oscillator parameter given by $\gamma = \gamma_0 A^{-1/3}$, and r_0 is the parameter appearing in the expression for the nuclear radius, $R = r_0 A^{1/3}$. If we now introduce the definitions

$$E_D = \epsilon + gS_0, \quad U_D = U - g\alpha_1 S_0, \quad (5)$$

we can rewrite the expressions in Eq. (1) in the form

$$E_{T+1} = E_D + U_D, \quad E_T = E_D - T^{-1}U_D, \quad (6)$$

or alternatively

$$\begin{aligned} E_D &= \frac{1}{T+1}E_{T+1} + \frac{T}{T+1}E_T, \\ E_{T+1} - E_T &= \frac{T+1}{T}U_D. \end{aligned} \quad (7)$$

Our remaining problem is the evaluation of the quantity U_D which is essentially a symmetry energy for dipole states, i.e., includes the effect of the particle-hole interactions. Since the symmetry energy U is known to be⁸

$$U = V_1 T/A, \quad (8)$$

with

$$V_1 \simeq 100 \text{ MeV}, \quad (9)$$

and since the quantity $\alpha_1 S_0$ is already given by Eq. (4), we can determine U_D if we first determine the coupling constant g . We will obtain this parameter by requiring that E_D have the general behavior familiar from the giant dipole resonance.⁹ We first find the mass dependence of g by noting that g times the square of a (single-particle) dipole matrix element represents in this picture a nuclear two-body interaction matrix element which varies with the mass number as A^{-1} . Since the dipole matrix element is proportional to the nuclear radius,¹⁰ i.e., $A^{1/3}$, we obtain for the mass dependence of g

$$g = g_0 A^{-5/3}, \quad (10)$$

where g_0 is a mass-independent constant to be determined later on. From this expression and the result of Eq. (3) we then obtain for the dipole shift

$$gS_0 = \Delta_0 A^{-1/3}, \quad \Delta_0 = g_0/8\gamma_0. \quad (11)$$

Remembering that the single-particle excitation energy is also inversely proportional to the nuclear radius, i.e.,

$$\epsilon = \epsilon_0 A^{-1/3}, \quad (12)$$

we obtain

$$E_D = (\epsilon_0 + \Delta_0)A^{-1/3}, \quad (13)$$

i.e., the correct mass dependence for the dipole energy.⁹ The mass dependence of g , Eq. (10) can now be introduced in Eq. (5) which together with Eqs. (4) and (8) yields

$$U_D = V_1 \frac{T}{A} - g_0 \frac{T}{10} r_0^2 A^{-1} = (V_1 - \Delta_1) \frac{T}{A}, \quad (14)$$

where

$$\Delta_1 = 0.8\gamma_0 r_0^2 \Delta_0. \quad (15)$$

We notice that the correction due to the particle-hole interactions has the same T/A dependence as the symmetry energy. In order to obtain rough numerical results we can set $0.8\gamma_0 r_0^2 \simeq 1$ (with $\gamma_0 \simeq 1 \text{ fm}^{-2}$ and $r_0 \simeq 1.2 \text{ fm}$) and let $\epsilon_0 \simeq 40$

MeV. If we then require $E_D \approx 80A^{-1/3}$ we find from Eq. (13) $\Delta_0 \approx 40$ MeV and therefore also $\Delta_1 \approx 40$ MeV. With Eq. (9) kept in mind we finally arrive at the result

$$U_D \approx [(100 - 40) \text{ MeV}]T/A = (60 \text{ MeV})T/A. \quad (16)$$

We summarize our results as follows. In the discussion of the isospin splitting of single-proton levels we start with the observation that the single-proton energy is equal to that of the corresponding neutron plus the Coulomb energy minus the symmetry energy. The latter measures the difference in the nuclear interactions between a neutron and a proton with the nuclear core. The energy difference between the two isospin components of the proton state is then given (apart from the geometrical factor) by the symmetry energy. In the dipole problem, however, we are comparing the energy of neutron-proton-hole states with that of proton-proton-hole and neutron-neutron-hole states in order to find the appropriate symmetry energy. Together with the particle-shell interactions this involves also the difference in the particle-hole interactions which, as we found, reduces the effective symmetry energy. The separation of the two isospin components is then given by a familiar expression [Eq. (7)] but with "symmetry energy" appropriately evaluated for the dipole problem. Our approach has been rather simple-minded but it should be expected to describe general trends and to provide a meaningful comparison with ex-

perimental data.¹¹

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ft Asymmetry in Mirror Gamow-Teller β Decay: Binding-Energy Effects

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It is shown that the systematic asymmetry observed between positron and negatron emitters in mirror Gamow-Teller β decay is not due to binding-energy differences between the respective β -transforming nucleons; it must be due either to a fundamental weak interaction effect or to a nuclear structure effect of some type not yet quantitatively discussed.

Mirror Gamow-Teller β decay takes place from analog $T=1$ states, such as ${}^8\text{Li}$ and ${}^8\text{B}$, leading to a common $T=0$ final state, or from analog $T=\frac{3}{2}$ states, such as ${}^9\text{Li}$ and ${}^9\text{C}$, then leading to analog $T=\frac{1}{2}$ final states. Contrary to simple ideas, the reduced speeds of the mirror positron and negatron transitions, $(ft)^+$ and $(ft)^-$, respectively, are systematically different¹⁻⁷; the asymmetry parameter $\delta = (ft)^+ / (ft)^- - 1$ is typically

+0.15 or so, as is shown in Tables I and II.

This surprising asymmetry may be due either to a new weak-interaction effect such as a second-class current¹¹ of some type¹² or to a failure of exact symmetry in the nuclear structure. It is critically important to distinguish between these fundamental and trivial explanations. Of the trivial effects so far considered quantitatively, the only one of significant magnitude is that