Test of Lepton Quantum Numbers by Muon Capture*

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It is shown that the $\Delta Q=2$ semileptonic process can be tested by experiments on muonic atoms. The Δ resonances in nuclear ground states will lead to a $\mu^- \to e^+$ reaction presently observable if the $\Delta Q=2$, $\Delta S=0$ interaction is within about two orders of magnitude of the ordinary strangeness-conserving interactions. The consequences for various lepton schemes is discussed.

The assignment of leptonic quantum numbers is not unique.[1,2] There are several schemes which are consistent with the observed weak processes and also with the processes which have proved to be inhibited. In the scheme most widely accepted, a lepton number +1 is assigned to e^-, ν_e, μ^-, and ν_μ with their antiparticles having lepton number -1. Consistency with the inhibited processes, such as neutrinoless decays of muons into electrons, can be obtained with an additional additive (Scheme A) or multiplicative (Scheme A') muon quantum number.

An alternative scheme in which e^-, ν_e, μ^+, and $\bar{\nu}_\mu$ are the leptons (Scheme B) was proposed earlier[3] and is also consistent with present experiments. The neutrinos ν_e and ν_μ are assigned helicity -1, as in Scheme A, so that there are still two distinguishable neutrinos. Thus no additional muon quantum number enters, and the only invariance principle involved in accounting for the inhibited processes is lepton conservation (or, alternatively, the two-component theory). The $\Delta Q=1$ semileptonic reactions cannot be used to distinguish between Schemes A and B, since these reactions will involve neutrinos, and the helicity choice in Scheme B will prohibit the same processes as does the muon number in Scheme A. Schemes A and B are only distinguishable if the current $\mu^\pm \leftrightarrow e^\mp$ exists. Purely leptonic tests are not possible, although they can distinguish between Schemes A and A', and lepton-proton reactions cannot test $\Delta Q=2$ interactions in the first order. Thus it is now generally accepted that Schemes A and B are equivalent in practice.[1]

It is the purpose of this note to point out that experiments on μ^- capture in muonic atoms provide a practical test of these $\Delta Q=2$ weak currents. It will be shown that an experiment in progress searching for the semileptonic reaction[4]

$$\mu^- + (Z,N) \to e^+ + (Z-2, N+2) \tag{1}$$

will provide rather definitive information, and that even the earlier experiments[5] which placed an upper limit on this reaction contain useful information.

If the nucleus is simply a composite system of neutrons and protons, Reaction (1) cannot take place in the first order (weak) in any of the schemes. However, it has been shown that nuclei contain baryon resonance components with a probability of about 1% for the least massive resonances.[6,7] This has been exploited by Primakoff and Rosen[8] in the study of double β decay. The isospin-$\frac{3}{2}$ resonances within the nucleus provide a mechanism for the first-order $\Delta Q=2$ current (if it exists). Here we consider only the $\Delta(1236)$ resonance, which should be most important. We assume an effective Hamiltonian

$$H_{\mathrm{eff}}^{\Delta Q=2} = G_\Delta[(1-\hat{\nu}\cdot\vec{\sigma})/2]\tau^{(2)+}\sum_{i=1}^{A} I_i^{(2)-}, \tag{2}$$

of the form of the usual $\Delta Q=1$ effective Hamiltonian except that $\tau^{(2)+}$ changes a μ^- state to an e^+ state and $I_i^{(2)-}$ is a $\Delta Q=-2$, rank-two, baryon-isospin step-down operator. Here $\hat{\nu}$ is a unit vector in the direction of the lepton momentum, and $\vec{\sigma}$ is the Pauli spin operator. Note that there are no $\Delta Q=2$ first-order currents in the quark model. The effective Hamiltonian (2) can be derived from a π-core model of the baryons.[9,10] This model gives radiative widths of baryon resonances consistent with experiment and with quark-model calculations.[10] In this model the $\Delta Q=2$ baryon current arises from a basic $\pi^+ \leftrightarrow \pi^-$ current, as illustrated in Fig. 1.[11]

The basic nuclear mechanism underlying Reaction (1) is $\mu^- + p + p \to e^+ + n + n$, as illustrated in Fig. 2. The two blobs represent the strong interactions $(\text{nucleon}+\text{nucleon})^{I=1} \leftrightarrow [\text{nucleon}+\Delta(1236)]^{I=1}$. This is taken into account by using a coupled-channel two-baryon wave function, e.g.,

$$\Psi(I=1, I_Z=1) = \Psi(pp) + a_\Delta^{I=1}\Psi([p\Delta]I=1, I_Z=1) \tag{3}$$

to represent an $I=1$ nuclear pair within the nucleus. In perturbation theory with a one-meson-ex-

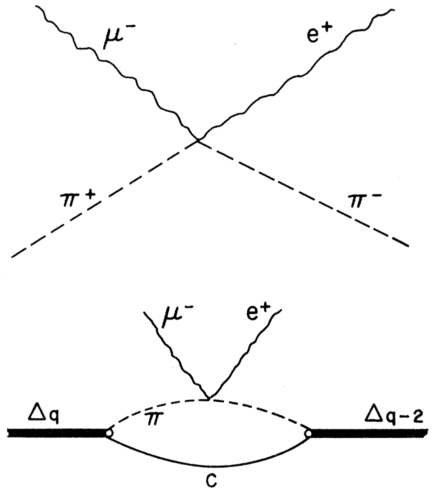

FIG. 1. The $\Delta Q=2, \Delta S=0$ interaction derived from the π-core model.

change coupling potential, one finds that[12]

$$(a_\Delta^{I=1})^2 \approx 0.005\text{-}0.01.$$

All nuclear correlations are assumed to be included via this parameter. Using the wave function (3), the $\mu^- \to e^+$ process is calculated as a perturbation with the Hamiltonian (2).

Assuming the closure approximation, one can use the work of Primakoff[13] to calculate the ratio of Reaction (1) to the usual μ capture $\mu^- + (Z,N) \to N + (Z+1, N-1)$:

$$\mathcal{R}^{\text{theor}} = \frac{\mu^- \to e^+}{\mu^- \to \nu} = \frac{P_{2p} G_\Delta^2 P_{2p\to 2n} (a_\Delta^{I=1})^4}{P_p[G_V^2 + 3G_A^2 + G_P^2 - 2G_P G_A] P_{p\to n}}, \tag{4}$$

where $P_p = Z/A$ and $P_{2p} = Z(Z-1)/A(A-1)$ are the probabilities of finding a proton and a proton pair, respectively, and $P_{p\to n}$ and $P_{2p\to 2n}$ are the probabilities associated with a proton turning into a neutron and two protons turning into two neutrons, respectively. The coupling constants in the denominator of (4) are the usual spin-averaged combination, and G_Δ is defined in Eq. (2). The estimate of Ref. 13, $P_{p\to n} = 1 - 3(A-Z)/2A$, is used here. We also use this in the calculation of $P_{2p\to 2n}$, thereby accounting for the exclusion principle, but assuming that any additional correlation effects are included in a_Δ^I. Thus Eq. (4) becomes

$$\mathcal{R}^{\text{theor}} = \frac{Z(Z+1)}{A(A+1)}(3Z-A-3) \times (a_\Delta^{I=1})^4 \frac{G_\Delta^2}{G_V^2 + 3G_A^2 + G_P^2 - 2G_P G_A}. \tag{5}$$

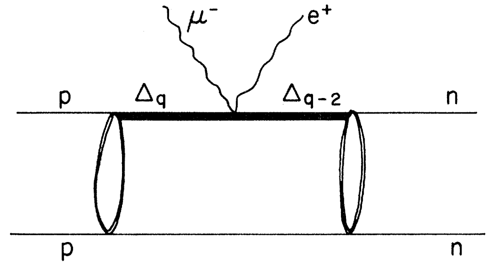

FIG. 2. Mechanism for the $\mu^- \to e^+$ process on a proton pair in the nucleus.

Evaluating this for μ capture on Cu, with a $\Delta(1236)$ probability of $(a_\Delta^{I=1})^2 = 0.0075$, one finds that

$$\mathcal{R}^{\text{theor}} \approx 1.6 \times 10^{-5} G^2 / [G_V^2 + 3G_A^2 + G_P^2 - 2G_A G_P] \approx 2.6 \times 10^{-6} (G_\Delta / G_V)^2. \tag{6}$$

The earlier experimental upper limit is[5,2]

$$\mathcal{R}^{\text{exp}} \lesssim 2.2 \times 10^{-7}, \tag{7a}$$

while the experiment in progress proposes that it will detect the $\mu^- \to e^+$ process unless

$$\mathcal{R}^{\text{exp}}(\text{proposed}) \lesssim 1 \times 10^{-10}. \tag{7b}$$

Thus the present experimental upper limit on the $\mu^- \to e^+$ decay suggests that the $\Delta Q = 2$ strangeness-conserving current cannot be as strong as the $\Delta Q = 1$ current ($G_\Delta < G_V$). This result is marginal because of the uncertainty in the value of a_Δ. If the new experiment achieves the proposed accuracy (7b), a negative result will rule out the possibility that $G_\Delta \cong G_V$, and thus Scheme B will lose much of its attraction. Note that this result follows only by taking into account the baryon resonances in the nuclear ground state.

The negative results would be consistent with a superweak $\Delta Q = 2, \Delta S = 0$ interaction of the magnitude of the $\Delta Q = 0, \Delta S = 2$ superweak interaction suggested to explain CP-invariance violations.[14] The possibility of such a current has been explored for double β decay.[15] However, in this case the second-order weak interaction allowed in Scheme B would be dominant. A positive experimental result at the level of $G_\Delta \approx G_V$ not only selects Scheme B, but demonstrates the existence of nonoctet currents which would require extensive modification of the present models of weak interactions. If the experiment detects the $\mu^- \to e^+$ process at the lower limit, i.e., $\mathcal{R}^{\text{exp}} \approx 1 \times 10^{-10}$, the result would suggest either that Scheme B is valid with a coupling $G_\Delta / G_V \approx 10^{-2}$ or that lepton conservation is violated with a strength of about 10^{-2} (in contrast to the estimated[8] upper limit of 10^{-4} for the lepton-conservation-violating interaction in double β decay).

Note added in proof.—A preliminary analysis of the experiment in progress[4] indicates that the ratio $\mathcal{R}^{\text{exp}}$ will be tested to a value between 1×10^{-9} and 1×10^{-10}.

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Possible Time-Reversal Noninvariance in Nuclear Forces*

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A model suggested by Sudarshan which relates strong, weak, and electromagnetic interactions through vector and axial-vector currents is adapted to nucleon-nucleon scattering. Strong time-reversal violation is predicted in n-p scattering above 100 MeV through the same mechanism assumed to cause CP-invariance violation in weak hadronic decays. Where a T-invariance violation has been sought for and not found, i.e., in p-p scattering up to 635 MeV and low-energy n-p scattering, the model predicts very little T-invariance violation.

Sudarshan has suggested a model for strong, weak, and electromagnetic interactions which violates CP invariance in strong interactions.[1] Assuming that CPT invariance holds, T invariance is likewise violated. At first sight, it would seem unreasonable to suggest such a model when all experiments in strong interactions to date indicate very little time-reversal invariance violation (TRV), consistent with zero. However, it is possible that experimentalists have been looking in the wrong places. We have extended Sudarshan's model, in a manner to be described shortly, in order to get accurate predictions for nucleon-nucleon scattering and find that in p-p scattering, T invariance is only slightly violated from 0 to 635 MeV, and in n-p scattering, T-invariance violation is likewise slight below 100 MeV. The only place strong violation occurs is in n-p scattering above 100 MeV, but here there are no experimental measurements. If the mod-