In particular, excellent agreement is found for $W_2$, where both the shell-model eigenfunctions and the experimental data are well established. The empirical value of $W_3$ is not so well established, but the value we obtain here appears to substantiate Barker's estimate. The empirical value of $W_1$ is seen to be in good agreement for Barker's shell-model eigenfunctions, but a discrepancy exists when the eigenfunctions of Norton and Goldhammer are used. The difficulty lies in the fact that the $J=1, T=0$ basis state appears to be poorly determined by the shell-model fits. The overlap between the Barker and Norton-Goldhammer eigenfunctions in this case is only 53%. In addition, the isospin mixing ratio for the pair of $1^+$ states is found to be 94/6 and consequently the empirical value of $W_1$ is most sensitive to any small change in this ratio.

The observed isospin mixing is found to be well accounted for by the simple Coulomb interaction alone, when correlated wave functions derived from realistic potentials are employed in the calculation. The Reid soft-core interaction yields matrix elements which are modestly greater than those obtained with hard-core potentials.

Two approximations have been made which could have significant influence on the final results. The first is the suppression of cross terms between the Coulomb force and the noncentral components of the nuclear force, as indicated in Eq. (3). Second, we have selected an oscillator-strength parameter ($\hbar\omega$) for $\varphi$ consistent with electron scattering data on neighboring $1p$-shell nuclei,[10] and have made no attempt to adjust this value to fit the data tested here. We estimate that each of these effects could produce a variation of no more than 10% in the calculated values of $W_J$.

All numerical work was done with the Honeywell 635 computer at the University of Kansas Computation Center.

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# Low-Energy Physics from a High-Energy Standpoint*

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Interaction of 10.1- and 15.8-GeV/$c$ positive and negative muons with nuclear emulsion nuclei is studied for events with one gray or black prong in addition to the outgoing muon track. The events involving very low energy exchange are consistent with "giant dipole" resonance. The line shape, its width, and the cross section of this resonance is discussed, as well as the angular distribution of the low-energy protons.

In the past, low-energy photons, electrons, nucleons, deuterons, $\alpha$ particles, etc. have been the common probes in the study of nuclear structure. Recently, the particles $\pi^-, K^-, \bar{p}, \Sigma^-$, etc., that traditionally are the concern of "elementary particle" physics, have also been used in the study of nuclear structure. High-energy photons and electrons have also been widely used for studying the details of nuclear structure. But these probes have some inherent drawbacks. On the other hand, the high-energy muon beam now available has a formidable advantage over the electron beam. The muon's heavier mass makes energy loss by radiation a negligible process $[\sim 1/(4\times 10^4)]$. Experiments studying nuclear radiation require resolution on the order of 1 MeV. In order to avoid the background radiation, we used a high-energy muon beam and for the target we used nuclear emulsion which is not only a target but also acts as an ideal detector. Here

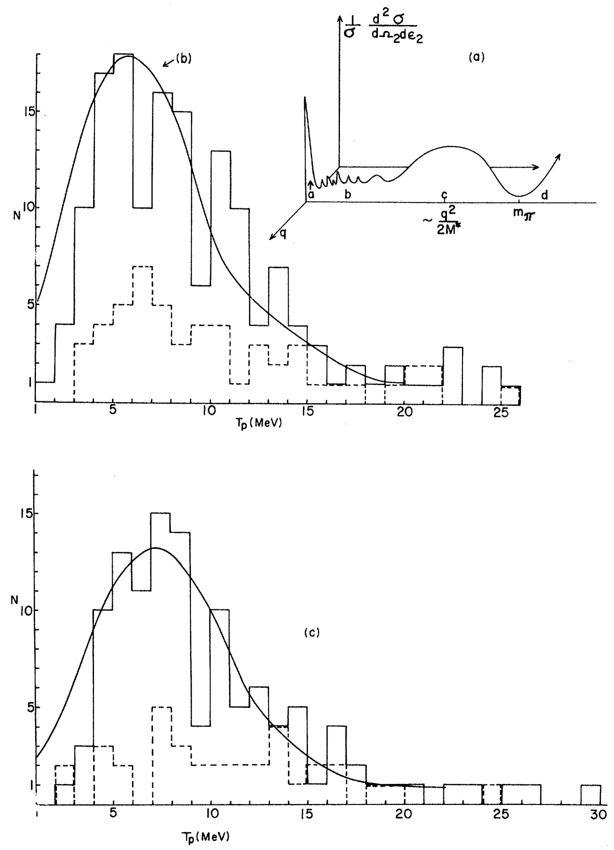


FIG. 1 (a) A typical double-differential electron-scattering cross section. (b) Energy distribution of one-prong (proton) events in 10.1-GeV/$c$ muons. Histograms with solid and dashed lines are due to the interaction with heavy and light nuclei, respectively. The solid curve is a Breit-Wigner form with $E_{max} = 5.5$ MeV and $\Gamma = 8$ MeV. (c) As in (b) but with $E_{max} = 7.5$ MeV.

we shall demonstrate the beginnings of positive achievement in the use of a high-energy muon beam in seeking information about nuclear structure. But first we look at a typical lepton-scattering cross section with a nuclear target as shown[1] in Fig. 1(a). First we have elastic scattering (region $a$) where the target is left in its ground state. Then we see spikes (region $b$) corresponding to excitation to discrete nuclear levels, including the giant-resonance region. The quasielastic peak (region $c$) is due to a collection of noninteracting nucleons at rest in a nucleus and is given at an energy transfer $\epsilon = q^2/2M_N$. Because the nucleons are not at rest inside the nucleus, the peak is spread out because of the Fermi motion of the nucleons in the nucleus. Finally, after an energy loss $\epsilon = m_\pi$, there is the region $d$ of meson production. During the last few years, high-energy muon beams have been used very effectively in revealing the structure of nucleons through their elastic[2,3] (region $a$), quasielastic[2,3] (region $c$), and inelastic[2,4-8] (region $d$) interactions with nucleons. A systematic study of the production of secondary particles[9] and the existence of "Roper" resonance,[10] which were not studied with electron beams, were studied through muon-nucleon interactions. High-energy muon beams were also instrumental in clearing up long-standing questions in muon-electron interactions.[11] The region which so far has not been studied systematically with a high-energy muon beam is the region of giant dipole resonance. Here we are interested in the study of giant dipole resonance in emulsion nuclei through the interaction of one (virtual) photon exchanged from the high-energy muon to the emulsion nucleus. In emulsion there are light elements H, C, N, and O (18 %) and medium-heavy elements Ag (48 %) and Br (34 %). Medium-heavy elements ($A \simeq 90$) are by far in abundance. So in practice we are studying the giant dipole resonance in Ag and Br nuclei.

Details of the exposure at 10.1 and 15.8 GeV/$c$ of positive and negative muons, respectively, were given earlier.[7,9-11] For the detection of the giant dipole resonance in emulsion nuclei, we are mainly concerned with one-prong events [i.e., (1+1) type[13]] which were selected under very stringent selection criteria. All one-prong events were checked for coplanarity and the elastic events were removed from all the calculations. From these we selected only those events which involve small momentum transfers and we thus limited ourselves to events with small scattering (<3°) of the out-going muon and with a large angle of the secondary black or gray track attached to each event. We selected 250 and 225 one-prong events in the 10.1- and 15.8-GeV/$c$ muon beams, respectivley, out of a total of 2500 muon interactions from both the beams. There were 50 and 36 one-prong events in the 10.1- and 15.8-GeV/$c$ muon beams, respectively, which had rather clear vertices and belonged to light elements. The remaining one-prong events with a heavy blob or a stem due to the "recoil" (dense, black, thick track[12] shorter than 5 $\mu$m) of heavy nuclei belonged to Ag and Br targets. Those events which had momentum transfer greater than pion threshold were rejected. The thickness and the stopping behavior of tracks in emulsions were extensively used in their identities. The projected and the dip angles were measured for both the continuing and the secondary (black or gray) track,[12] and thus their space angles were calculated. A Koristka scattering microscope to which was attached a filar micrometer that could be read to an accuracy of 0.02 $\mu m^{-1}$ was used for all the measurements for the secondary black and gray prongs. Whenever there was any doubt about the identity of the secondary particle through its track-following or -stopping behavior, the constant sagitta method[13] (with variable cell size) was used for its identification. The ranges of all the different tracks were corrected for their straggling effects and density variation[14] in nuclear emulsion. In the present work, the error in the range measurement was observed to be between 2 and 5 $\mu$m corresponding to an error in energy of about 0.2 to 0.35 MeV. This brings out the importance of the present technique used in such an experiment.

In one-prong events the mass determination of the secondary gray or black track gave us about 79% protons, 13% deuterons, 7% tritons, and 1% $\alpha$ particles in the 10.1-GeV/$c$ muon beam, and 74% protons, 13% deuterons, 5% tritons, and 1% $\alpha$ particles in the 15.8-GeV/$c$ muon beam. In Figs. 1(b) and 1(c) are shown the energy distributions of secondary protons from the 10.1- and 15.8-GeV/$c$ beams, respectively, for both the light (H, C, N, and O; dotted-line histogram) and heavy (Ag and Br) elements. We can see that there is a peak due to a giant resonance for heavy elements around 6 and 7 MeV for the 10.1- and 15.8-GeV/$c$ beams, respectively; for light elements it is at little higher values but the data are relatively small. The proton-emission threshold for Ag and Br is considered to be about 8 MeV. Experimental data are fitted by a theoretical curve of Breit-Wigner line shape, with the energy at the maximum intensity $E_{max} = 5.5$ and 7.5 MeV, and $\Gamma \sim 8$ MeV (in both cases) in 10.1- and 15.8-GeV/$c$ beams, respectively. Detailed calculations of the width of a giant resonance are rather difficult to perform; however, Wuldermuth and Wittern[15] have found $\Gamma = \hbar/\tau \simeq 5.55(200/A)^{2/3}$ MeV. The predicted values for $\Gamma$ are ~8.5 MeV. The 10.1-GeV/$c$ beam shows structure at 5, 8, and 10.5 MeV and the 15.8-GeV/$c$ beam at 5.5, 8, and 10.5 MeV. We know that the deformed nuclei do predict splitting in the giant resonance. As far as we know, there is no systematic study on $(\gamma, p)$ with either Ag or Br nuclei, so it is difficult to compare the present spectra with other experiments. By using an expression of Steinwedel and Jensen[16] one obtains $E_{max} = 208h/R[(8/M)(NZ/A^2)]^{1/2}$; for Ag and Br, one finds that $E_{max} \sim 5.85$ and 5.88 MeV, respectively. These assume a binding energy of 7.98 MeV. On the other hand, the hydrodynamical collective model predicts dipole peaks at energies of 6.7 and 8.3 MeV for Ag and Br, respectively, from an approximate expression[16] $E \simeq 70A^{-1/3}$.

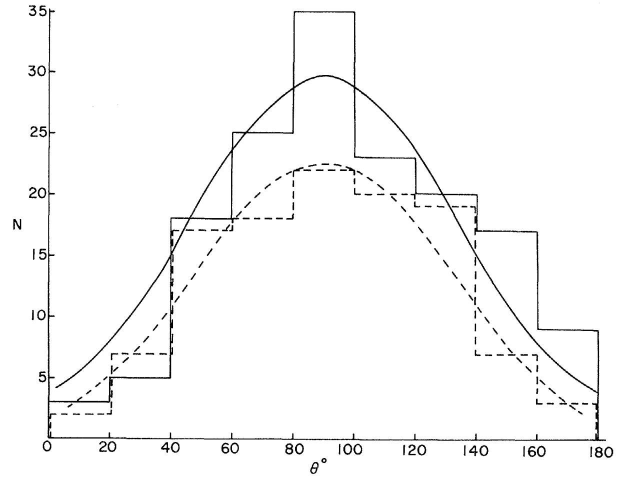


FIG. 2. Angular distribution of protons produced in heavy nuclei. The theoretical curve is given by $f = a + b\sin^2\theta$, where $a = 5.0$ and $b = 2.4$ for 10.1 GeV/$c$ (solid lines), and $a = 2.70$ and $b = 2.02$ for 15.8 GeV/$c$ (dashed lines).

In Fig. 2 is shown the angular distribution of the protons produced in heavy nuclei in both the beams. Most of the secondaries are preferably emitted at right angles to the muon beam. Both angular distributions are fitted by a function of the type $f = a + b\sin^2\theta$ which is evidence for the dipole nature of $\gamma$-ray absorption in the process we investigated. The ratio $b/a$ is then a measure of the degree of anisotropy. These parameters are determined by the method of least squares. The ratios are $b/a = 2.4/5.0 = 0.48$ and $2.02/2.70 = 0.75$ for 10.1- and 15.8-GeV/$c$ beams, respectively. Thus the degree of anisotropy of the 15.8 distribution is greater, which one should expect with the increase in primary energy.

In order to find the cross section for giant dipole resonance, the "meson-production" stars were separated from "giant-resonance" events. We also considered events with zero prongs (neutron emission). For this the photonuclear cross sections of emulsion nuclei were obtained. The scanning efficiency was also taken into consideration. With all of these corrections, the $\sigma_{dip} = 12.5 \times 10^{-30}$ and $14.1 \times 10^{-30}$ cm$^2$ per nucleon in the 10.1- and 15.8-GeV/$c$ beams, respectively. Very little work has been done on the giant-resonance cross section for emulsion nuclei and whatever is done is very crude[9] since no attempt (prior to the present experiment) was made to *identify* the single prong in one-prong events. So we believe our result is a start in the right direction. Further data at different primary energies are highly desirable.

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