tion: The difference of nuclear electric quadrupole coupling constants for the two states is $\Delta eQq = -940 \pm 30$ MHz, and the magnetic coupling in the excited state is[12] $C = 53 \pm 5$ KHz.

The difference in quadrupole coupling constants, ΔeQq, is nearly the same for both the presently studied $P(117)$ line of the 21-1 band and the formerly observed $R(127)$ line of the 11-5 band,[6] despite widely differing vibrational states. An estimate of the ground-state quadrupole coupling, extrapolated from heteronuclear molecules,[1] indicates that the hyperfine splitting in the electronically excited state is about four times smaller than in the lower state.

The experimentally determined spectrum agrees quite well with the prediction of the model. The present measurements, with their complete coverage, well-defined zero level, and good signal-to-noise ratio, reveal no structures other than the predicted 21 components. This confirms that the transitions where the nuclear spin orientations change do not occur and proves, moreover, that there is negligible cross relaxation between different molecular hyperfine levels.[4] The saturation techniques utilized would also reveal a possible collisional cross relaxation in velocity space, as was demonstrated in a recent experiment investigating gas-laser plasma.[8]

The strict selection rules simplify the spectrum, but they also make it impossible to determine the hyperfine splittings of the levels separately: Only the difference is observed in experiments of this kind. However, the individual level spacings could be determined by applying a radio-frequency field to mix hyperfine levels.

The strong, narrow, iodine-vapor resonances reported in this paper may be used for long-term frequency stabilization of the krypton ion laser. Such a light source, with its excellent frequency stability combined with significant output power, will certainly prove a very useful tool for a variety of applications.

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[1]M. Kroll and K. K. Innes, J. Mol. Spectrosc. 36, 295 (1970).

[2]P. H. Lee and M. L. Skolnick, Appl. Phys. Lett. 10, 303 (1967).

[3]R. L. Barger and J. L. Hall, Phys. Rev. Lett. 22, 4 (1969).

[4]R. G. Brewer, M. J. Kelley, and A. Javan, Phys. Rev. Lett. 23, 559 (1969).

[5]G. R. Hanes and C. E. Dahlstrom, Appl. Phys. Lett. 14, 362 (1969).

[6]M. Kroll, Phys. Rev. Lett. 23, 631 (1969).

[7]The same linewidth γ_{ab} is observed in Lamb-dip experiments although an erroneous factor of 2 is occasionally found in the literature (see, for example, Refs. 3 and 5).

[8]P. W. Smith and T. W. Hänsch, Phys. Rev. Lett. 26, 740 (1971).

[9]C. Bordé, in Proceedings of the Sixth International Quantum Electronics Conference, Kyoto, Japan, September 1970 (unpublished).

[10]G. H. Wannier, *Statistical Physics* (Wiley, New York, 1966).

[11]G. W. Robinson and C. D. Cornwell, J. Chem. Phys. 21, 1436 (1953).

[12]The notation ΔeQq is the same as in Ref. 6. The interpretation of C depends on the assumed coupling scheme of the angular momenta. For a comparison with Ref. 6 one may set $C = K/[J(J+I)]$, which yields $K = 730 \pm 70$ MHz.

Anomalously Rapid Skin-Current Penetration and Heating in Pulsed-Plasma Experiments*

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Electrostatic instabilities in a plasma having electrons flowing along a magnetic field are generated by a shear in the electron drift velocity. This can cause an effective resistivity much higher than the Buneman value and consequently increase the rates of penetration of a skin current and of heating.

To explain the anomalous resistivity observed in several turbulent heating experiments,[1] the growth rate of the electron-ion two-stream instability[2,3] has been interpreted as an effective electron collision frequency. To get agreement between the observed conductivity[1] and the theory,[2,3] the effective ion-electron collision time has been assumed equal to the e-folding growth time for the instability.[1] Thus the conductivity is given by

$$\sigma \simeq (M/m)^{1/3}\omega_{pe},$$

where M is the ion mass, m is the electron mass, and ω_{pe} is the electron plasma frequency. However, the conductivity computed by the above theoreticians[2,3] is a factor of 20 greater than that obtained from the above assumption. Furthermore, we suspect that the condition assumed of $u \gg \beta_e$ (where u is the electron drift velocity and β_e the electron-thermal velocity) is difficult to obtain in practice. Thus, the explanation of the observed conductivity should be sought in terms of more realistic conditions such as finite plasma size, presence of magnetic field, etc., which we now do.

Since in usual turbulent heating experiments plasmas are generally prepared before application of strong electric fields, the electric field cannot penetrate into the plasma medium instantaneously. During the time of field penetration, it is expected that the axial electron drift velocity has a radial dependence such that

$$du(r)/dr > 0,$$

which can be an origin of electrostatic instabilities.[4,5] Since the instability due to the velocity gradient tends to reduce this gradient, we suspect that the observed electric field penetration may be achieved with a time scale on the order of a growth time which is much shorter than the "classical" penetration time

$$\tau = (4\pi\sigma/c^2)r^2,$$

with σ the Buneman conductivity, r the plasma radius, and c the speed of light.

Let us assume density and drift-velocity gradients perpendicular to an external magnetic field. If the characteristic scale lengths of these gradients are much longer than cyclotron radii and the perpendicular wavelength, a linear dispersion relation of electrostatic waves can be easily found in terms of a geometric optics approximation which we have derived and which will be discussed in detail in a longer publication:

$$k^2 + \sum_j k_{\mathrm{D}j}^2(1+\Omega_j L_j) + \sum_j \omega_{pj}^2\left[\frac{k_\perp K_j}{\omega_{cj}} - \frac{(\partial u_j/\partial y)k_\parallel k_\perp}{\omega_{cj}}\frac{\partial}{\partial\Omega_j}\right]L_j = 0, \tag{1}$$

where $k_{\mathrm{D}j}$ is the Debye wave number; ω_{pj} is the plasma frequency; ω_{cj} is the cyclotron frequency; K_j is the density-gradient constant, given by

$$K_j = n_{0j}^{-1}dn_{0j}/dy; \quad \Omega_j = \omega - k_\parallel u_j;$$

and

$$L_j \equiv (k_\parallel\beta_j)^{-1}e^{-\lambda_j}\textstyle\sum_n I_n(\lambda_j)Z(\zeta_{nj}),$$

where β_j is the thermal velocity, $I_n(\lambda_j)$ is the nth-order modified Bessel function with argument $\lambda_j = k_\perp^2\beta_j^2/2\omega_{cj}^2$, and $Z(\zeta)$ is the plasma dispersion function with argument $\zeta_{nj} = (\omega - k_\parallel u_j + n\omega_{cj})/k_\parallel\beta_j$. There are several limiting cases of interest in Eq. (1), depending on frequency and thermal effects.

(A) *Cold plasma.*—If we assume $\beta_e, \beta_i \to 0$ in Eq. (1), we obtain

$$k^2 = k_\parallel^2\frac{\omega_{pe}}{\Omega_e^2} + k_\perp^2\frac{\omega_{pe}^2}{\Omega_e^2-\omega_{ce}^2} - \frac{\omega_{pe}^2 du_e/dy}{\omega_{ce}\Omega_e^2}k_\parallel k_\perp + \frac{\omega_{pe}^2 k_\perp K}{\omega_{ce}\Omega_e} + k^2\frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pi}^2 k_\perp K}{\omega_{ci}\omega}, \tag{2}$$

where we have assumed that $k_e = K_i \equiv K$, $u_i = du_i/dy = 0$, and $|\omega| \gg \omega_{ci}$. The case in which $|\omega|^2 \gg \omega_{pi}^2$ has been discussed by Mikhaivlovskii and Rukhadze[5] who found the solution for ω given by

$$\omega \simeq k_\parallel u + i\min(\omega_{pe}, \omega_{ce}).$$

Since, $k_\parallel u \ll \min(\omega_{pe}, \omega_{ce})$ in general, the density-gradient term has little influence on the instability. This can be seen from

$$\Omega_e^{-1} - \omega^{-1} \simeq 0$$

in Eq. (2). Thus, the statement made in Ref. (5) on the stabilizing effect of the density gradient is not correct if the plasma is quasineutral ($n_i \simeq n_e$). The growth rate is very large and the instability is almost nonoscillatory. Nonlinear effects of the instability can reduce the velocity gradient within a time ω_{pe}^{-1} or ω_{ce}^{-1}, whichever is the larger, giving rise to rapid electric field penetration into the plasma core. At the same time, electrons are rapidly heated and the cold plasma model may become inapplicable. However, instability still persists if the condition

$$\frac{du}{dy} > \frac{\omega_{pe}^2 \omega_{ce}^2 \cos^2\theta + k_\parallel^2 \beta_e^2 (\omega_{pe}^2 \sin^2\theta + \omega_{ce}^2)}{\omega_{pe}^2 \omega_{ce} \cos\theta \sin\theta},$$

where θ is the angle between $\vec{k}$ and $\vec{B}_0$, is satisfied.

In the frequency range $\omega_{ci} \ll |\omega| \ll \omega_{ce}$, Eq. (2) gives another solution for ω:

$$\omega \simeq \frac{1+i\sqrt{3}}{2\sqrt[3]{2}} \frac{\{(1+\tan^2\theta)[1-\tan\theta(du/dy)\omega_{ce}^{-1}]^{1/2}\}^{1/3}}{[1+\tan^2\theta(1+\omega_{pe}^2/\omega_{ce}^2)]^{1/2}} (\omega_{pi}^2 \omega_{pe})^{1/3}$$

which is very similar to the solution obtained by Buneman.[2] In the absence of the velocity gradient, the growth rate is maximum at $\tan\theta = 0$, so that turbulence wave vector is directed along the magnetic field and the problem is essentially one dimensional. Only electrons are preferentially heated even though the effective electron-ion collision frequency is much higher than the classical value. If the velocity gradient is present, the growth rate does not vanish at $\theta = \frac{1}{2}\pi$, and we may expect that effective ion heating takes place in this case as a result of resonance at the lower hybrid frequency; this will be discussed later in a longer note. Rapid ion heating observed in turbulent-heating experiments[1,6] could be due to the instability caused by velocity gradients.

(B) *Hot electron plasma.* – Let us assume that

$$\Omega_e / k_\parallel \beta_e \ll 1, \quad \lambda_e \ll 1,$$

$\beta_i = 0$ (cold ions), and $|\omega| \gg \omega_{ci}$ in Eq. (1). Diamagnetic effects all vanish under these assumptions and we obtain a simple dispersion relation,

$$\omega^2 = \frac{T_e}{M} k^2 \left(1 - \tan\theta \frac{du/dy}{\omega_{ce}} + \frac{k^2}{k_D^2}\right)^{-1}. \tag{3}$$

If the velocity gradient is sufficiently large, Eq. (3) yields a nonoscillatory solution

$$\omega = ikC_s[\omega_{ce}/(du/dy)\tan\theta]^{1/2},$$

where $C_s = (T_e/M)^{1/2}$ is the ion acoustic velocity. For $k \simeq k_D$, the growth rate is on the order of ω_{pi} or higher. The physical mechanism of this instability is somewhat similar to current convective instability[7] although the growth rate is much higher than ω_{ci} and the wavelength is correspondingly short. The direction of k is almost perpendicular to the magnetic field which is favorable for ion heating. The plasma diffusion associated with the instability is expected to be very small because of the short-wavelength nature. However, the velocity-gradient profile should be disrupted very rapidly within ω_{pi}^{-1} or less.

(C) *Collisional plasma instabilities.* – If a finite, electron–momentum-transfer collision frequency ν is introduced, Eq. (3) can be modified as

$$(\omega^2 - \omega_s^2)(\omega - k_\parallel u) = \tan\theta \frac{du}{dy} \frac{\omega^2(\omega - k_\parallel u + i\nu)}{\omega_{ce}}, \tag{4}$$

where $\omega_s \equiv C_s k$ ($k < K_D$) is the ion acoustic frequency. The above dispersion relation indicates a coupling between the drifting-electron mode $\omega = k_\parallel u$, which is a negative-energy wave, and the ion acoustic mode $\omega = kC_s \simeq k_\perp C_s$, in the presence of the collisions as well as the velocity gradient. The maximum growth rate is given by

$$\gamma = \frac{1}{2\sqrt{2}} \left(\tan\theta \frac{du}{dy} \omega_{ce}^{-1} \omega_s \nu\right)^{1/2} \quad (\lesssim \omega_s),$$

which can be on the order of ω_{pi}, and somewhat smaller than the growth rates previously obtained. It can easily be shown that in a plasma carrying a homogeneous current (no velocity gradient), perpendicular ion acoustic waves ($\omega \simeq k_\perp C_s$) can be excited only if $u > \omega/k_\parallel$, which is practically impossible under usual experimental conditions. However, as we have seen, the inhomogeneity in the electron drift velocity greatly reduces the critical velocity for the onset of perpendicular ion acoustic waves. The electron collision rate ν is not necessarily due to binary collisions with either ions or neutrals but could also be due to some turbulence present in the plasma.

We have shown that an electron drift along an external magnetic field having a gradient across the field may give rise to various high-frequency ($|\omega| \gg \omega_{ci}$) instabilities with $k_\perp \gg k_\parallel$. The growth rate γ is in the range $\omega_{pi} \lesssim \gamma \lesssim \min(\omega_{pe}, \omega_{ce})$. It is intuitively expected that the fast-growing instabilities rapidly weaken the velocity gradient (or help an electric field penetrate into the plasma core) and should be accompanied by rapid ion heating as well as electron heating due to unfrozen ion motions ($|\omega| \gg \omega_{ci}$) associated with some instabilities [(B) and (C)]. Detailed calculations including quasilinear effects will be reported as a separate report.

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[1]B. A. Demidov *et al.*, Zh. Eksp. Teor. Fiz. 48, 454 (1965) [Sov. Phys. JETP 21, 302 (1965)]; S. M. Hamberger and M. Friedman, Phys. Rev. Lett. 21, 674 (1968); T. Kawabe *et al.*, Phys. Rev. Lett. 25, 642 (1970); S. Q. Mah *et al.*, Phys. Rev. Lett. 25, 1409 (1970).

[2]O. Buneman, Phys. Rev. 115, 503 (1959).

[3]V. D. Shapiro, Zh. Tekh. Fiz. 31, 522 (1961) [Sov. Phys. Tech. Phys. 6, 376 (1961)].

[4]E. Harrison, Proc. Phys. Soc., London, Ser. B 82, 689 (1963).

[5]A. B. Mikhailovskii and A. A. Rukhadze, Zh. Tekh. Fiz. 35, 2143 (1965) [Sov. Phys. Tech. Phys. 10, 1644 (1966)].

[6]T. H. Jensen and F. R. Scott, Phys. Fluids 11, 1809 (1968); C. B. Wharton *et al.*, in *Proceedings of the Ninth International Conference on Phenomena in Ionized Gases, Bucharest, Rumania, 1969*, edited by G. Musa *et al.* (Institute of Physics, Bucharest, Rumania, 1969), p. 649.

[7]B. B. Kadomtzev and A. V. Nedospasov, J. Nucl. Energy, Part C 1, 230 (1960).

Self-Consistent Field Theory of Relativistic Electron Rings

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We investigate equilibria and compression of thin electron rings in cylindrically symmetric guide fields. It is found that particle energies are smaller and ring radii different from values obtained ignoring self-fields.

In recent years there has been considerable interest in the generation of relativistic electron rings.[1-7] Here the compression and final equilibrium of a thin ring is considered in a guide field of cylindrical symmetry describable with the vector potential $A_\theta = A$ (uniform field, mirror field, betatron field, etc.). The effect of the electric and magnetic self-fields of the ring can be significant in determining the final equilibrium particle energy, ring radius, and cross section. All ring electrons are assumed to have the same canonical angular momentum $P_\theta = P$, which is conserved during compression.

The Hamiltonian of an electron for the above conditions is

$$H = c[m^2c^2 + P_r^2 + P_z^2 + (P/r - qA)^2]^{1/2} + q\varphi, \quad (1)$$

where $A(r,z) = A_0 + A_r$. A_0 is the known guide field; A_r is the ring-field vector potential. For a thin ring one has that $P_r^2 + P_z^2 \ll m^2c^2 + (P/r - qA)^2 \simeq \gamma^2 m^2 c^2$, and one may expand Eq. (1) as

$$H = (2m\gamma)^{-1}(P_r^2 + P_z^2) + \psi, \quad (2)$$

where

$$\psi = mc^2\left[1 + \left(\frac{P/r - qA}{mc}\right)^2\right]^{1/2} + q\varphi \quad (3)$$

is an effective potential with a minimum at $r = R$, the ring radius. This may be determined from

$$0 = \left(\frac{\partial\psi}{\partial r}\right)_R = -\frac{1}{m}\left(\frac{P}{R} - qA\right)\left(\frac{P}{R^2} + q\frac{\partial A}{\partial r}\right)\left[1 + \left(\frac{P/R - qA}{mc}\right)^2\right]^{-1/2} + q\frac{\partial\varphi}{\partial r}, \quad (4)$$

where A, $\partial A/\partial r$, and $\partial\varphi/\partial r$ are to be evaluated at $r = R$. The ring current

$$I_r = qNv_\theta = \frac{qN}{\gamma m}\left(\frac{P}{R} - qA\right) \quad (5)$$

determines A_r with

$$A_r(R) = \lambda I = \frac{x}{\gamma}\left(\frac{P}{Rq} - A\right), \quad (6)$$

where N and λ are the electron number and ring inductance per unit ring length, respectively, and x is the dimensionless quantity $\lambda q^2 N/m$. From $A = A_0 + A_r$ and (6) we obtain

$$A_r(R) = \frac{x}{x+\gamma}\left(\frac{P}{Rq} - A_0\right). \quad (7)$$

For a thin ring, one finds to lowest order that

$$\lambda = \frac{\mu_0}{2\pi}\iint \sigma(\delta,\psi)\ln\left(\frac{8R}{\delta}\right)d\psi\,\delta\,d\delta, \quad (8)$$

where δ and ψ are polar coordinates in the ring cross section, and $\sigma(\delta,\psi)$ is the normalized current distribution.

One may further show that in the same approxi-