Small Dimensionless Parameter to Characterize Multiple-Pomeranchon Phenomena*

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We identify a small dimensionless parameter η_P associated with the triple Pomeranchon vertex, which governs both the rate of high-mass diffractive dissociation processes and the fine structure in the J-plane spectrum near $J=1$. Theoretical arguments are given that $\eta_P \lesssim 1-\alpha_P(0)$, and a possible experiment to measure η_P is discussed. A formula for η_P, based on a multiperipheral model, shows that in such models this parameter does not vanish, and a connection of η_P with a perturbation formalism for the Pomeranchon propagator is suggested.

The apparent conflict between two descriptions of high-energy production processes, namely "diffractive dissociation" and "multiperipheralism," has been widely noted.[1] Chew, Rogers, and Snider[2] have shown that these concepts could be unified by splitting the kernel of a multiperipheral integral equation into a low-energy or resonance part and a weak Pomeranchuk component. In this paper we sharpen the issues involved and reduce their model dependence by defining an experimentally measurable parameter η_P, which determines both (1) the probability of diffractive dissociation into large masses and (2) the strength of the logarithmic branch point in the "Pomeranchon propagator."

We consider a collision of the type depicted in Fig. 1(a), where particle B dissociates into large mass $(s')^{1/2}$ with the same quantum numbers, while particle A merely recoils with momentum transfer $(-t)^{1/2}$. In the limit when the total energy $s^{1/2}$ is large, as well as s' and s/s', many authors[3] have argued that the differential cross section for this process should show the characteristic energy dependence $(s/s')^{2\alpha_P(t)}(s')^{\alpha_P(0)}$, and DeTar *et al.*,[4] following the reasoning of Mueller,[5] identified the coefficient of the energy dependence with a triple-Pomeranchon vertex.[6] The cross section obtained by summing over all the undetected hadrons of total mass $(s')^{1/2}$ in the high-energy limit is described symbolically by Fig. 1(b) where the link $\alpha_P(0)$ results from the above summation. We set the scale of our triple-Pomeranchon vertex by writing the spin-averaged differential cross section in question in the asymptotic limit as

$$s^2\frac{d\sigma_{AB}}{ds'dt} \underset{s/s'\to\infty}{\underset{s'\to\infty}{\simeq}} \frac{1}{16\pi}|\beta_{AP}(t)|^2\tilde{\beta}_{BP}(0)g_P(t)\left(\frac{s}{s'}\right)^{2\alpha_P(t)}(s')^{\alpha_P(0)}. \tag{1}$$

The normalizations of the Regge residues β are chosen so that

$$s^2\frac{d\sigma_{AB}{}^{\mathrm{el}}}{dt} \underset{s\to\infty}{\simeq} \frac{1}{16\pi}|\beta_{AP}(t)|^2|\beta_{BP}(t)|^2 s^{2\alpha_P(t)}, \tag{2}$$

and

$$s\sigma_{AB}{}^{\mathrm{tot}} \underset{s\to\infty}{\simeq} \tilde{\beta}_{AP}(0)\tilde{\beta}_{BP}(0)s^{\alpha_P(0)}. \tag{3}$$

With this choice,

$$\tilde{\beta}_{AP}(0)\tilde{\beta}_{BP}(0)=\mathrm{Im}[\beta_{AP}(0)\beta_{BP}(0)]. \tag{4}$$

The triple-Pomeranchon vertex, $g_P(t)$, has two legs at mass $(-t)^{1/2}$ and the other at mass zero; see Fig. 1(b).

The same quantity (essentially[7]) enters the cross section for double diffractive dissociation in which

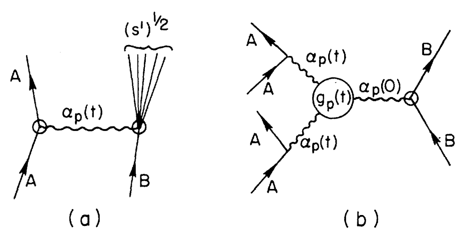

FIG. 1. (a) Single diffractive dissociation of A and B to produce a large mass $(s')^{1/2}$. (b) Emergence of the triple Pomeranchon vertex, $g_P(t)$, as $s'\to\infty$, $s/s'\to\infty$.

both A and B dissociate into large masses $(s'')^{1/2}$ and $(s')^{1/2}$, respectively, as shown in Figs. 2(a) and 2(b). With our normalization we find

$$s^2 \frac{d\sigma_{AB}}{ds'ds''dt} \underset{s/s's''\to\infty}{\widetilde{s'\to\infty,\, s''\to\infty}} \frac{1}{16\pi}\tilde{\beta}_{AP}(0)\tilde{\beta}_{BP}(0)g_P{}^2(t)\left(\frac{s}{s's''}\right)^{2\alpha_P(t)}(s's'')^{\alpha_P(0)}. \quad (5)$$

Note that if different ways of grouping final particles into two clusters satisfy the requirements on s' and s'', a single event should be counted more than once.

Although $g_P(t)$ is in principle determined by either of the above two multiple-production processes, the less experimentally accessible double diffractive dissociation described by Fig. 2 and Eq. (5) is more directly related to the important dimensionless parameter η_P which we now introduce. The vertex functions β and $g_P(t)$ are not dimensionless; however at $t=0$, if $\alpha_P(0)=1$, the quantity

$$\eta_P \equiv \frac{1}{16\pi}\frac{1}{2\alpha_P'(0)}g_P{}^2(0) \quad (6)$$

is dimensionless. Furthermore, if we divide Eq. (5) by the total cross section, Eq. (3), and integrate over the diffraction peak in t, we find that the *probability* of double diffractive dissociation is given by

$$\frac{1}{\sigma_{AB}{}^{\text{tot}}}\frac{d\sigma_{AB}}{d\ln s'd\ln s''} \sim \eta_P \frac{[s/s's'']^{\alpha_P(0)-1}}{\ln(s/s's'')}. \quad (7)$$

Formula (7) may be used to obtain the probability distribution in the central (pionization) region for large gaps between adjacent longitudinal rapidities of produced particles. Identifying the large rapidity gap as $\Delta \sim \ln(s/s's'')$, and with attention to the previously mentioned multiple counting, we find

$$\frac{dP(\Delta)}{d\Delta} \underset{\Delta \text{ large}}{\sim} \eta_P \frac{\langle\Delta\rangle}{\Delta}e^{-a_P\Delta}, \quad (8)$$

where $a_P \equiv 1-\alpha_P(0)$ and $\langle\Delta\rangle$ is a "mean gap." More precisely, if the mean number of produced particles (or of gaps) increases as $\langle n\rangle \sim k\ln s$, as predicted by multiperipheral theory, then $\langle\Delta\rangle \equiv k^{-1}$.

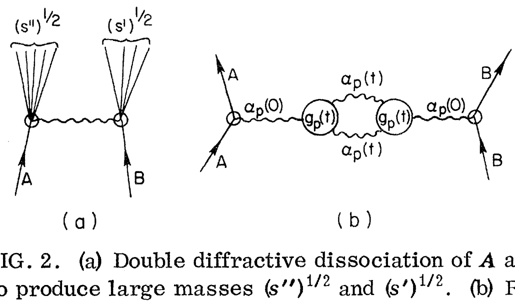

FIG. 2. (a) Double diffractive dissociation of A and B to produce large masses $(s'')^{1/2}$ and $(s')^{1/2}$. (b) Regge approximation to double diffractive dissociation as $s'\to\infty$, $s''\to\infty$, $s/s's''\to\infty$. Note the appearance of the one-loop correction to the Pomeranchon propagator.

With $\alpha_P(0)=1$ ($a_p=0$) and $\eta_P\neq 0$, formula (8) leads to a contradiction because the upper limit on Δ is of order $\ln s$. The integrated double-diffractive probability then increases $\alpha\ln(\ln s)$ and eventually exceeds unity. A corresponding disaster occurs for the single diffractive cross section of Eq. (1) which also grows $\propto\ln\ln s$. This is the well-known Finkelstein-Kajantie disease[8]; its cure is to assume either $\alpha_P(0)<1$ or $\eta_P=0$.

Accepting the experimental indication that $k\approx 1$,[9] so that the mean value in the Δ distribution is ≈ 1, it follows that the integrated probability in the large-gap tail of the distribution not only must be finite but much less than 1. Conservatively defining the "tail" to be $\Delta \gtrsim 3$, we thus require

$$\int_3^\infty (dP/d\Delta)d\Delta \ll 1,$$

which with insertion of formula (8) leads to $\eta_P\langle\Delta\rangle\ln(3a_P)^{-1}\ll 1$; since $a_P<0.03$, one may be confident that $\eta_P\ll 1$. A sharper limit will be obtained below.

The diagram depicting the double diffractive dissociation process, Fig. 2, strongly suggests the association of η_P with a single loop modification of a Pomeranchon Regge propagator. Such propagator concepts have been extensively discussed by Gribov and Migdal[10] (however without the physical association with multiparticle production processes emphasized here). Within the framework of multiperipheral models where one deals with "partial-wave" integral equations, the Fredholm denominator $D(J)$ is a natural candidate for identification with a Regge propagator. It is probable that this is a much more general S-matrix concept. In any event, we have found for a large class of multiperpheral models that

$$D(J) \approx J-\alpha_R-\eta_P\ln(J-\alpha_c)^{-1} \quad (9)$$

when J, α_R, and α_c are all near unity. Here

$$\alpha_c = 2\alpha_P(0)-1 \quad (10)$$

is the position of the Amati-Fubini-Stanghellini branch point, and α_R is the position of the leading

J-plane singularity when the Pomeranchon contribution to the kernel is turned off; that is, $\eta_P = 0$ or no diffractive dissociation. Omitted from Eq. (9) are nonsingular terms of order η_P and terms of order $(J-\alpha_R)^2$ or smaller.

If the leading zero of $D(J)$ occurs at $J = 1-a_P$, so $\alpha_c = 1-2a_P$, it is clear from Eq. (9) that $a_P = 0$ only when $\eta_P = 0$. Furthermore since phenomenological analysis of high-energy data[11] tells us that the strength of the Pomeranchon residue, proportional to

$$[D'(\alpha_P(0))]^{-1} = a_P/(\eta_P + a_P), \tag{11}$$

is not small compared to the size of lower vacuum singularities, we conclude that a_P is not a great deal smaller than η_P. By measuring η_P we will thus be able to predict the asymptotic rate of decrease of total cross sections:

$$\sigma^{\mathrm{tot}}(s) \propto s^{-a_P}, \quad a_P \gtrsim \eta_P. \tag{12}$$

Of course, it may transpire that $\eta_P = 0$. What arguments can we adduce toward the vanishing or nonvanishing of η_P? Again we turn to multiperipheral models for a hint and are able to report that within the context of an Amati-Bertocci-Fubini-Stanghellini-Tonin (ABFST) structure this parameter is nonzero. If the exchanged mesons along the multiperipheral chain have masses m_i and the kernel is deduced from elastic unitarity contributions to (off-shell) m_i, m_i scattering, we find

$$g_P(t) = \frac{\Gamma(\alpha_P(0)+1)}{16\pi^3} \sum_i \int_{-\infty}^{0} du [2(ut)^{1/2} \sinh q_i]^{2\alpha_P(t)} \left(\frac{u}{t}\right)^{[\alpha_P(0)+1]/2} \times P_{2\alpha_P(t)}{}^{-\alpha_P(0)-1}(\coth q_i) \frac{|\beta_{Pii}(t,u,m_i^2)|^2 \tilde{\beta}_{Pii}(0,u,u)}{(m_i^2-u)^2}, \tag{13}$$

with

$$\cosh q_i = (m_i^2 - t - u)/2(ut)^{1/2}. \tag{14}$$

Each term in the sum contributes a positive amount at $t=0$, so no cancelation is possible. An estimate based on a solution of the ABFST model of the off-shell vertex functions appearing in Eq. (13) leads to a value of $g_P(0)$ of the order of 1 GeV^{-1} and thus an η_P of about 0.02. These calculations will be described in detail elsewhere.[13]

A calculation of $g_P(t)$ in the context of the dual resonance model has been made recently.[14] In that model $g_P(0)$ vanishes; however, it also happens that at $t=0$ the Pomeranchon decouples from all integer spins. Given the prevailing theoretical uncertainty concerning η_P, there appears to be no substitute for a clean experimental measurement.

As already noted, the process of Fig. 1 and formula (1) is far more accessible to experiment than that of Fig. 2. In addition to less demanding requirements on initial energy and on final-state measurements, single diffractive dissociation into large masses is proportional to the square root of η_P and so may be of respectable magnitude even when η_P is extremely small. For example, if we assume that the t dependence of AB elastic scattering is roughly the same as that of large-mass diffractive dissociation, we may use the quotient of formulas (1) and (2), each integrated over t, to obtain the estimate

$$\frac{1}{\sigma_{AB}{}^{\mathrm{el}}} \frac{d\sigma_{AB}}{d\ln s'} \approx \frac{g_P(0)}{\sqrt{\sigma_{BB}{}^{\mathrm{tot}}}} = \left[16\pi\eta_P \frac{2\alpha_P'(0)}{\sigma_{BB}{}^{\mathrm{tot}}}\right]^{1/2}. \tag{15}$$

Here we have taken $\alpha_P(0) \approx 1$, so that $|\beta_{BP}(0)| \approx \tilde{\beta}_{BP}(0) \approx \sqrt{\sigma_{BB}{}^{\mathrm{tot}}}$. Thus, if the dissociating particle has a high-energy total cross section (on itself) of the order 20 mb $\approx 50\ \mathrm{GeV}^{-2}$, and $\alpha_P'(0) \approx 0.5\ \mathrm{GeV}^{-2}$, the ratio in question is of the order $\sqrt{\eta_P}$, easily measurable even if η_P is of the order 0.01.

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[7]We have been cavalier in our treatment of helicity indices labeling the triple-Pomeranchon vertex. Even though we consider a spin average for each of our cross sections, the quantity $g_P(t)$ appearing in Eq. (1) is really $\sum_m |\beta_{aP}{}^m|^2 g_P{}^m(t)$, where m is the helicity flip undergone by particle a. In Eq. (5) we encounter really $\sum_m [g_P{}^m(t)]^2$, where m is the helicity of the Pomeranchon of mass $\sqrt{-t}$. At $t=0$ only $m=0$ will survive since the Pomeranchon is a zero-mass object. Therefore, $g_P(0)$ is the same parameter in both the single and double diffractive dissociations.

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