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Importance of the D-State Probability of the Deuteron in Threshold $p+p \to \pi + d$ †

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Using a number of realistic, local and nonlocal, separable two-nucleon interactions, it is shown that threshold s-wave pion production is sensitive to the D-state probability of the deuteron and the nonlocality of the interaction. This enables one to use the reaction $p+p \to \pi^+ + d$ to study the relative strength of the central to tensor potential and the nonlocality of the nucleon-nucleon interaction at short distances.

Recent calculations using two-nucleon potentials with varying short-range behavior,[1] and relative strengths of the central and tensor components,[2] have shown large variation in the binding energy and saturation density of nuclear matter. These variations are a direct result of uncertainties in our knowledge of the nucleon-nucleon (N-N) interaction.

At present it is known that the long range ($r > 1.6$ F) part of the N-N interaction is due to one-pion exchange (OPE). However, the intermediate and short-range part of the potential is not uniquely determined. Consequently, this region has been represented by different phenomenological models, some of which are local while others are nonlocal or velocity dependent. It is the lack of real theoretical understanding, together with uncertainties in the experimental information, which leads to the large variations in the calculated properties of nuclear matter.

One such experimental uncertainty is the D-state probability of the deuteron, P_D. It has been shown[2] that variation of P_D from 1 to 9%, while keeping all other properties of the deuteron constant, leads to large variation in the binding energies of both nuclear matter and finite nuclei. The reason is that the quadrupole moment of the deuteron Q, and the coupling parameter ρ_1, which at low energy is related[3] to Q, do not uniquely determine the range and strength of the tensor part of the N-N interaction.

In an attempt to constrain the value of P_D to better than the present range of 3-8 %,[4] and possibly gain information about the N-N potential at short range, we have studied the sensitivity of low-energy s-wave pion production in the reaction

$$p+p \to \pi^+ + d \tag{1}$$

to variations in these properties. One expects Reaction (1) to give information about the N-N interaction at momentum transfer greater than the pion mass.

The transition amplitude, to second order in the pion-nucleon interaction V, is given by[5]

$$T_{d,p} = \langle \chi_d | V | \chi_p^{(+)} \rangle + \langle \chi_d | V(E-H)^{-1} V | \chi_p^{(+)} \rangle, \tag{2}$$

where χ_d is the wave function for a deuteron plus a free pion, and $\chi_p^{(+)}$ is the two-proton scattering wave function. Here H is the full two-nucleon Hamiltonian plus a pion kinetic-energy operator. The pion-nucleon interaction density is taken to be of the same form as that used by Woodruff[6] and Koltun and Reitan,[7] namely

$$V = H_0 + H_1 + H_2, \tag{3}$$

where

$$H_0 = (4\pi)^{1/2}(f/u) i\vec{\sigma} \cdot \{\nabla_\pi [\tau \cdot \varphi(x)] + [\vec{p}\tau \cdot \pi(x) + \tau \cdot \pi(x) \vec{p}]/2M\}, \tag{3a}$$

$$H_1 = 4\pi\lambda_1 \mu^{-1} \varphi^2(x), \tag{3b}$$

$$H_2 = 4\pi\lambda_2 \mu^{-2} \tau \cdot \varphi(x) \times \pi(x). \tag{3c}$$

As usual, $\vec{\sigma}$ and τ are the nucleon-spin and -isospin operators, respectively, and $\vec{p}$ is the nucleon momentum. The pion mass is μ, while that of the nucleon is M. φ and π are the pion field operator and its conjugate, respectively. The coupling parameter $f^2=0.0822$ and the rescattering parameters λ_1 and λ_2 were taken to be 0.0058 and 0.0487, respectively.[8]

In evaluating the second term in Eq. (2)—the so-called "rescattering" contribution—we have assumed that the two-nucleon energy in the intermediate state is an average of the energies in the initial and final channels. This is equivalent to the approximation taken by Koltun and Reitan.[7]

The cross section for pion production in Reaction (1) at low energies is given by[9]

$$\sigma(pp\to\pi^+d)=\alpha\eta+\beta\eta^3, \tag{4}$$

where $\eta\equiv$ c.m. pion momentum in units of μc. Thus the restriction to s-wave pions means that one is essentially calculating the coefficient α. ($\alpha\eta$ is the s-wave and $\beta\eta^3$ the p-wave contribution to the total cross section.)

We have calculated α for several classes of N-N potentials. Firstly, to show explicitly the dependence on P_D, we used for the ^3S-^3D channel the one-term separable potentials of Afnan, Clement, and Serduke[2] (*PD*1, *PD*3, *PD*4, *PD*5.5, *PD*7, *PD*9—the names should be self-explanatory). These potentials all fit the deuteron properties and effective-range parameters, but have a D-state probability varying from 1 to 9%. Apart from P_D, they are equivalent.

Secondly, we used a class of realistic local potentials due to Hamada and Johnston[10] (H-J), Reid[11]—Reid hard-core (RHC), soft-core (RSC), and soft core alternative (RSCA) potentials—and also the one-boson exchange potential of Bryan and Scott[12] (B-S). These potentials have $P_D=7.03$, 6.5, 6.47, 6.22, and 5.38%, respectively, and fit all the scattering data and deuteron properties.

Table I. Cross section for threshold s-wave pion production, with the potentials of Afnan, Clement, and Serduke (Ref. 2) in the $\pi+d$ channel and RSC (Ref. 11) in the p-p channel.

Potential in $\pi+d$ channel ^3S-^3D	Deuteron D-state probability (%)	α (μb)
*PD*1.0	1.0	488
*PD*3.0	3.0	389
*PD*4.0	4.0	358
*PD*5.5	5.5	327
*PD*7.0	7.0	305
*PD*9.0	9.0	281

Thirdly, we have used the three-term separable potentials of Serduke,[13] potentials 1 and 2 (*PD*4.31, *PD*6.5), and Mongan (Case II)[14] for the ^3S-^3D channel. These have $P_D=4.31$, 6.5, and 1.1%, respectively, and fit all the n-p scattering data, as well as the deuteron properties.

In Tables I and II we give the results for these potentials. As we have restricted ourselves to s-wave pions only, the initial two-proton state is $^3P_1$. With the separable potentials in the deuteron channel we used the RSC potentials in the initial state. Otherwise we have remained consistent within each potential model.

Comparison of the results for the local potentials shows that the RHC gives by far the smallest cross section. This is because the RHC has a core radius over 0.1 F bigger in the $^3S_1$-$^3D_1$ than in the $^3P_1$ channel. Thus, an important part of the scattering wave function near the core does not contribute. This decrease in cross sec-

Table II. Cross section for threshold s-wave pion production for a number of local and nonlocal separable potentials with different deuteron D-state probabilities. All these potentials fit both the scattering and bound-state two-nucleon data.

Potential in p-p channel $^3P_1$	Potential in $\pi+d$ channel $^3S_1$-$^3D_1$	Deuteron D-state probability (%)	α (μb)
H-J	H-J	7.03	161
RHC	RHC	6.50	125
RSC	RSC	6.47	171
RSC	RSCA	6.22	180
B-S	B-S	5.38	196
RSC	*PD*6.5	6.5	229
RSC	*PD*4.31	4.31	285
RSC	Mongan Case II	1.1	421

tion is also present when the H-J rather than the RSC scattering wave function is used with the separable potentials. Thus, it is seen that taking different core radii in different channels can lead to observable effects. In view of the many uncertainties associated with the N-N interaction, we feel that in constructing a hard core N-N interaction one should have the same radius in all partial waves, unless the N-N data demand a change.

The variation in α with P_D is well illustrated by the results for the one-term separable potentials. The cross section decreases by 180 μb in going from 1 to 9%. Exactly the same trend is shown in the H-J, RSC, RSCA, and B-S results —a variation in P_D from 7 to 5.38% leads to an increase of 35 μb in α. The third class of potentials also shows this variation with P_D. Furthermore, the rate of increase in α with decrease in P_D is approximately the same for the local and separable nonlocal potentials.

We therefore conclude that the cross section for s-wave pion production is sensitive to the D-state probability of the deuteron, and that this sensitivity is independent of the model taken for the N-N interaction! In addition, comparison of the results for the local and nonlocal potentials indicates that the cross section is dependent on the form of the potential for a given P_D.

The above results can be compared with these of the experiment of Rose,[15] who found $\alpha = 240 \pm 20$ μb, after making Coulomb corrections. It is found that the local potentials give too small a cross section, while the separable nonlocal potentials bracket the experimental results. On the other hand, comparison with the results of Crawford and Stevenson[16] ($\alpha = 138 \pm 15$ μb) would lead one to conclude that the cross sections for all the potentials used here are too large.

This is obviously a poor situation, and it would be most desirable for the parameters α and β to be determined more accurately, preferably to within a few percent or better. The present discrepancy of a factor of 2 is intolerable if one wants to obtain quantitative information on the N-N interaction from pion production.

We must point out that, at present, there would be errors of about ±4% in the theoretical values of α even if the theory itself were exact. There errors arise from experimental uncertainties in the rescattering parameters λ_1 and λ_2. From this point of view, it would be most useful if the low-energy, s-wave, pion-nucleon phase shifts δ_1 and δ_3 could be found to within 1%. These two improvements in the experimental information could enable us to determine P_D to better than 1%, within a given N-N potential model and within the present description of the pion-production process.

Finally, we should point out that the experimental uncertainties do not in any way alter our conclusions that the pion-production cross section is quite sensitive to details of the N-N interaction, in particular, the relative strengths of the central and tensor components and the nonlocality of the short-range part of the N-N interaction.

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