els in ${}^{50}_{22}Ti_{28}$ at 3.88 MeV (0^+) and 4.32 MeV (2^+) which, besides the ground state, are the most strongly excited states in the (t,p) reaction[5] are not observed in the (${}^{16}O$, ${}^{14}C$) two-proton-transfer reaction. A similar situation is found in ${}^{52}_{24}Cr_{28}$. However, the first 0^+ excited state at 2.66 MeV which has an important $(pf)_{J=0}{}^2$ neutron component is strongly populated by both (t,p) and (${}^{16}O$, ${}^{14}C$) reactions. A thorough analysis of these cross sections will involve a nuclear-structure form factor, taking into account the $(nlj)_{J,\,T=1}{}^2$ pair-correlation and shell-model configuration mixing, as in the (t,p) reaction formalism.[6] It would be worthwhile in the future to compare the two-proton transfer with the two-neutron transfers also induced by heavy ions such as (${}^{18}O$, ${}^{16}O$).

In the four isotopes investigated, the two-proton-transfer cross-section values display a certain correlation with the known electromagnetic transition probabilities[3]: A decrease of $B(E2)$ values is observed in ${}^{46}Ti$ and ${}^{50}Ti$ with respect to ${}^{52}Cr$ and ${}^{54}Cr$. This emphasizes the role of pairing forces in the interpretation of the so-called vibrational levels in spherical nuclei. Furthermore, the fact that α transfers do not populate these levels[1] shows the dominance of pairing over quartetting[7] in the structure of low-lying Ti and Cr states.

It is a pleasure to thank Dr. G. Bruge and A. Chaumeaux for their assistance during the experiment.

[1]H. Faraggi, M.-C. Lemaire, J.-M. Loiseaux, M. C. Mermaz, and A. Papineau, to be published.

[2]H. Faraggi, A. Jaffrin, M.-C. Lemaire, M. C. Mermaz, J.-C. Faivre, J. Gastebois, B. G. Harvey, J.-M. Loiseaux, and A. Papineau, Ann. Phys. (New York) (to be published, de-Shalit memorial volumes).

[3]G. Bruge, J.-C. Faivre, H. Faraggi, and A. Bussière, Nucl. Phys. A146, 597 (1970); A. M. Bernstein, in *Advances in Nuclear Physics*, edited by M. Baranger and E. Vogt (Plenum, New York, 1969), Vol. 3, p. 325.

[4]L. R. Dodd and K. R. Greider, Phys. Rev. Lett. 14, 959 (1965).

[5]S. Hinds and R. Middleton, Nucl. Phys. A92, 422 (1967); R. Chapman, S. Hinds, and A. E. MacGregor, Nucl. Phys. A119, 305 (1968).

[6]N. K. Glendenning, Phys. Rev. 137B, 102 (1965).

[7]V. Gillet, in *Proceedings of the International Conference on Properties of Nuclear States, Montréal, Canada, 1969,* edited by M. Harvey *et al* (Presses de l'Université de Montréal, Montréal, Canada, 1969), p. 483.

Intermediate Structure and the Giant-Dipole Resonance in O^{16}†

C. M. Shakin
Department of Physics, Case Western Reserve University, Cleveland, Ohio 44106

and

W. L. Wang
Department of Nuclear Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 16 February 1971)

We develop a projection-operator theory of photodisintegration. It is shown that the coupling of 3p-3h (three-particle, three-hole) modes to the 1p-1h modes can explain the intermediate structure of the giant resonance in oxygen when the 1p-1h states are treated as "doorways" for γ-ray absorption and for particle emission. The structure problem is treated using the interacting-boson approximation of Iachello and Feshbach.

One of the more interesting problems of nuclear-structure theory centers about the understanding of the giant-dipole resonance. The general features of the phenomenon can be accounted for by either the collective model[1] or the particle-hole models.[2] These models reproduce only the gross structure and do not account for intermediate structure, such as that observed in oxygen[3] and other light nuclei. Naturally this structure is believed to be due to more complicated configurations coupled to the simple 1p-1h modes.

In the photonuclear cross sections for O^{16}, such intermediate structure is especially pronounced in the 20- to 26-MeV region.[3] Experiments with energy resolutions of several hundred keV reveal resonances at about 21.0, 22.3, 23.0, 24.3, and 25.0 MeV (see Fig. 1). The main features at 22.3 and 24.3 MeV have been reproduced by continuum shell-model calculations.[5] However, the cross sections in these models are too large by factors of 2 or 3. The peaks at 21, 23, and 25 MeV cannot be explained

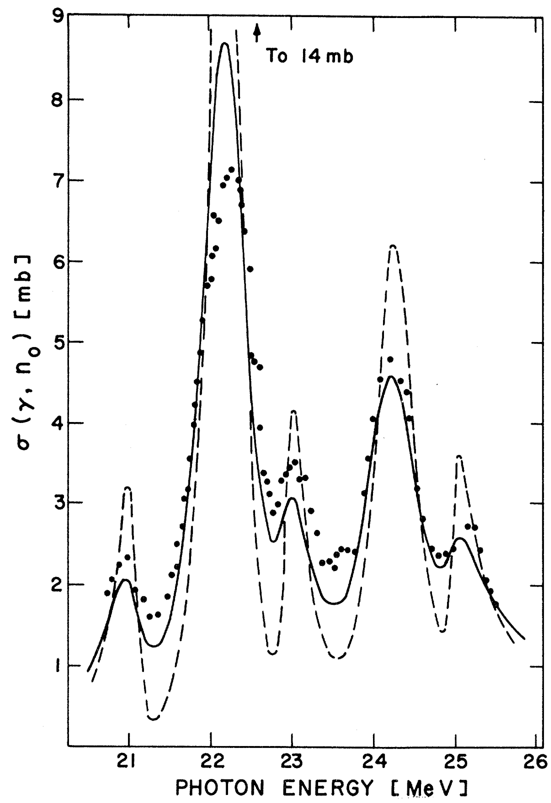

FIG. 1. The total cross section, $O^{16}(\gamma, n_0)O^{15}$ to the O^{15} ground state. The points are the experimental data of Ref. 4. The dashed curve is obtained from the T matrix of Eq. (7). The solid curve is obtained from the dashed curve by averaging with a function describing the experimental energy resolution ($\Delta E = 250$ keV). Increasing the assumed energy resolution to $\Delta E = 350$ keV makes for very good agreement with the main peak at 22 MeV but makes the fit at the other peaks somewhat worse.

in the 1p-1h models. Gillet, Melkanoff, and Raynal[6] have made an attempt to explain this structure by using 2p-2h states.

In this work, it will be shown that the intermediate structure in this region can be explained by a proper extension of the 1p-1h model to include 3p-3h states.

We begin by formulating a T-matrix expression for photonuclear reactions using a projection operator technique.[7] We define a P space to include some limited number of open channels, a doorway d, and a q space which includes those parts of the Hilbert space not contained in either P or d. Treating the electromagnetic coupling in perturbation theory and including only a single doorway state for simplicity in writing, we have that

$$T(E) = \langle \Phi_c^{(-)} | \mathcal{H}^\gamma | 0 \rangle + \frac{\langle \Phi_c^{(-)} | \mathcal{H}_{Pd} | \Phi_d \rangle \langle \Phi_d | \mathcal{H}^\gamma + \mathcal{H}_{dP}(E^{(+)} - \mathcal{H}_{PP})^{-1} \mathcal{H}^\gamma | 0 \rangle}{E - \langle \Phi_d | \mathcal{H}_{dd} + \mathcal{H}_{dP}(E^{(+)} - \mathcal{H}_{PP})^{-1} \mathcal{H}_{Pd} | \Phi_d \rangle}, \tag{1}$$

where

$$(E - \mathcal{H}_{PP}) | \Phi_c^{(-)} \rangle = 0, \tag{2}$$

$$(E - \mathcal{H}_{dd}) | \Phi_d \rangle = 0, \tag{3}$$

$$\mathcal{H} \equiv H + Hq(E - H_{qq})^{-1} H, \tag{4}$$

and

$$\mathcal{H}^\gamma \equiv H^\gamma + Hq(E - H_{qq})^{-1} H^\gamma, \tag{5}$$

with $\mathcal{H}_{Pd} = P\mathcal{H}d$, etc. Here H^γ is the electromagnetic coupling term, $|0\rangle$ denotes the target ground state, $|\Phi_d\rangle$ is a doorway state (of particle-hole structure), and $|\Phi_c^{(-)}\rangle$ is a continuum state of the P space.

The structure of the T matrix is complicated but may be greatly simplified by making some assumptions. First we assume that $\langle q | H^\gamma | 0 \rangle = 0$ so that $\mathcal{H}^\gamma \to H^\gamma$, i.e., the states $|q\rangle$ are assumed to have no significant electromagnetic coupling to the ground state. Second, we assume that the states $|q\rangle$ are not coupled to the few P-space states, $|\Phi_c^{(-)}\rangle$, so that $\mathcal{H}_{Pd} \to H_{Pd}$.

The treatment of $\mathcal{H}_{dd}$ is somewhat more complicated. To understand the structure of this term we make a further division of the q space: $q = q' + q''$. The q' space is taken to contain certain 3p-3h states to be specified below; these states are considered as doorways for the coupling of the 1p-1h dipole states to even more complex modes. The q'' space contains more complicated modes *as well as* the many *open channels* not included in the P space. Application of these ideas to the specification of $\mathcal{H}_{dd}$ leads to the following approximation:

$$\langle \Phi_d | \mathcal{H}_{dd} | \Phi_d \rangle \simeq E_d + \sum_{q'} \frac{|\langle \Phi_d | H | q' \rangle|^2}{(E - E_{q'} + \frac{1}{2} i \Gamma_{q'})} + \sum_{q''} [\Delta_{q''}(E) - \tfrac{1}{2} i \Gamma_{q''}(E)], \tag{6}$$

where $\Gamma_{q'}$ describes the width of the q' states due to their coupling to the q'' space. The term $\Delta_{q''}(E) - \frac{1}{2}i\Gamma_{q''}(E)$ describes the contribution to the T-matrix denominator due to the coupling of the $|\Phi_d\rangle$ *directly* to the open channels in the q'' space. Putting these ideas together and neglecting $\Delta_{q''}$ and a small term in the numerator, $\mathcal{H}_{dP}(E-\mathcal{H}_{PP})^{-1}\mathcal{H}^{\gamma}$, we have

$$T(E) \simeq \langle \Phi_c^{(-)}|H^{\gamma}|0\rangle + \frac{\langle \varphi_c^{(-)}|H_{Pd}|\varphi_d\rangle\langle\varphi_d|H^{\gamma}|0\rangle}{E-E_d-\sum_{q'}V_{q'}^2/(E-E_{q'}+\frac{1}{2}i\Gamma_{q'})+i\sum_{q''}\frac{1}{2}\Gamma_{q''}-\langle\Phi_d|H_{dP}G_P^{(+)}H_{Pd}|\Phi_d\rangle} \tag{7}$$

with $V_{q'} = \langle q'|H|\varphi_d\rangle$ and $G_P^{(+)} = (E-H_{PP})^{-1}$. The extension to several doorway states $|\varphi_d\rangle$ is easily made and, if desired, the energy average of T may be obtained by replacing E by $E+\frac{1}{2}iI$, where I is the width of the averaging interval.

For O^{16} we *define* the P space to include only the channels in which the residual nucleus (O^{15} or N^{15}) is in its ground state or first excited state. The single-particle $d_{3/2}$ resonance is removed from the P space to the d space by the methods of Wang and Shakin.[8] The states $|\varphi_d\rangle$ are then constructed by doing a particle-hole calculation using the $J=1^-$, $T=1$ states made from the orbits $1d_{5/2}$, $2s_{1/2}$, $1p_{1/2}^{-1}$, $1p_{3/2}^{-1}$ and the $1d_{3/2}$ "wave-packet."[8] The single-particle wave functions are obtained from a Woods-Saxon potential and the residual interaction is chosen to be

$$V_{ij} = -V_0(\Pi_t + 0.46\Pi_S)\delta(\vec{r}_i - \vec{r}_j), \tag{8}$$

where Π_t and Π_s are spin-triplet and -singlet projection operators. The strength $V_0 = 4\pi(1.76)^3 \times 8.5$ MeV was chosen to reproduce the energies of the dipole states at 22.3 and 24.3 MeV. These two states were then used as doorway states $|\varphi_d\rangle$.

In order to describe the intermediate structure we need to construct the q' states which we take as 3p-3h states, or three-"boson" states. The "multiboson states" may be constructed by taking direct products of low-lying "single-boson states." For reference we list the following low-lying states to be used as building blocks for the three-boson states: for $T=0$, $J=3^-$ (6.1 MeV) and $J=1^-$ (7.1 MeV); for $T=1$, $J=3^-$ (13.2 MeV), $J=2^-$ (12.7 MeV), $J=1^-$ (13.5 MeV), and $J=1^-$ (17.43 MeV). The energies given for the $T=0$ bosons are experimental values. Also, in our calculation, we take the lowest $J=1^-$, $T=0$ state obtained to be the spurious state and disregard it.

Using the listed states we construct a 3p-3h state: The states $|J_1, T_1\rangle$ are coupled to an intermediate 2p-2h state $|J_{12}, T_{12}\rangle$, which is then coupled to $|J_3, T_3\rangle$ to form a 3p-3h state. For our purposes we only construct states with total $J=1^-$, $T=1$. The unperturbed energies (the sums of the "boson" energies) are generally higher than that of the 1p-1h dipole states. These states are brought down in energy by considering the effects of the particle-particle and hole-hole interactions. For the treatment of the p-p and h-h correlations we find the methods of Iachello and Feshbach[9] very useful. We find that the p-p and h-h interactions treated as perturbations in the multiboson states have diagonal matrix elements which are much larger than the (neglected) off-diagonal ones, indicating the utility of the classification scheme. In Table I we present the calculated spectra of the three-boson states as well as their coupling-matrix elements to the two 1p-1h doorways. (Only a few states are strongly coupled.) The energies listed in Table I for the three-"boson" states represent the sum of the energies of the noninteracting bosons listed above and the diagonal matrix elements of the p-p and h-h interactions. These latter matrix elements are calculated in an approximation discussed in Ref. 9 where certain complicated recoupling terms, which are expected to be small, are neglected. We have not attempted to calculate the recoupling terms, but instead we allowed ourselves the liberty of making some relatively small adjustments of the three-boson spectrum to obtain the best fit to the experiment. The adjusted energies are given in the table. Most of the adjustments made are small compared to the matrix elements of the p-p and h-h interactions; in the most extreme case the lowest state (at 20.2 MeV) is moved up 1 MeV which is still less than 20% of the p-p and h-h matrix element of -5.5 MeV.

In order to compare with the experimental data, it is necessary to choose $\Gamma_{q'}$ and $\sum_{q''}\Gamma_{q''}$. Since little is known concerning these quantities, we choose them to be equal and independent of energy. Their value is determined by requiring that the integrated cross section for $O^{16}(\gamma, n_0)O^{15}$ be equal to the experimental value.[4] In this way we determined $\Gamma_{q'} = \sum_{q''}\Gamma_{q''} = 0.25$ MeV. The calculated (γ, n) cross section (dashed line) to the O^{15} ground state is shown in Fig. 1. In order to compare with the experimental data we have av-

Table I. The energy spectrum, configurations, and coupling-matrix elements to the dipole states, for the three-boson states.

Energy (MeV)	Adjusted Energy (MeV)	Configuration $(J_1T_1)(J_2T_2)J_{12}(J_3T_3)$	Coupling Matrix Element to Dipole State at 22.3 MeV	24.3 MeV
26.7	26.6	(10) (10) 2 (11)	0.014 MeV	0.0069 MeV
26.4	26.6	(10) (10) 2 (31)	0.0033	0.0016
25.9	25.7	(10) (10) 2 (21)	0.042	0.013
25.8	25.7	(30) (10) 4 (31)	-0.061	-0.030
25.6	25.7	(30) (10) 3 (31)	0.19	0.092
25.1	25.0	(30) (10) 3 (21)	-0.14	-0.07
25.0	25.0	(30) (10) 2 (11)	-0.047	-0.02
24.7	24.85	(30) (10) 2 (31)	0.23	0.11
24.2	24.85	(30) (30) 0 (11)[a]	0.40	0.20
24.1	24.85	(30) (10) 2 (21)	-0.22	-0.11
23.3	22.8	(30) (30) 4 (31)	0.0033	0.0017
23.3	22.8	(30) (30) 2 (11)	0.098	0.049
23.1	22.8	(10) (10) 0 (11)	0.29	0.14
23.0	22.8	(30) (30) 2 (31)	0.090	0.045
22.5	22.8	(30) (30) 2 (21)	0.13	0.066
20.2	21.25	(30) (30) 0 (11)	0.48	0.24

[a]This (11) is the 1^- state at 17.43 MeV. The other (11) states are the 13.5-MeV, $J=1^-$ states.

eraged the theoretical curve with an experimental energy resolution assumed to be $\Delta E = 250$ keV, yielding the solid curve. The result is in excellent agreement with the data of Caldwell *et al.*[4] Other experiments show essentially the same structure.[4]

Angular distributions have been measured by Jury, Hewitt, and McNeill,[10] Thompson and Baglin,[10] Stewart,[10] and Verbinski and Courtney.[10] Our calculation provides strong support for the assumption that the major components of the intermediate structure are due to $J=1^-$ states. (A small admixture of positive-parity states in this region is, of course, possible.) It is interesting to note that in a study of $C^{13}(He^3, \gamma)O^{16}$ to the O^{16} ground state, resonances were observed at 24.05 and 25.1 MeV.[11] This observation is in keeping with our assumption of important 3p-3h components in the giant-resonance structure.

Finally, we wish to point out that part of the success of our model is due to the use of the experimental energies for the $T=0$, $J=3^-$ and $J=1^-$ states, as our 1-particle, 1-hole calculation puts these states at somewhat higher energies. We assume that a more extensive calculation in which we were to consider the effects of 3p-3h states in depressing the energies of the low-lying 1p-1h states would improve this aspect of our calculation. Also, it would be interesting to obtain more information concerning the parameters $\Gamma_{q'}$ and $\sum_{q''}\Gamma_{q''}$, as well as the possible modification of our results through considering the effects of ground-state correlations.

†Work supported in part by the U. S. Atomic Energy Commission.

[1]M. Goldhaber and E. Teller, Phys. Rev. 74, 1046 (1948).

[2]J. P. Elliot and B. H. Flowers, Proc. Roy. Soc., Ser. A 242, 57 (1957); G. E. Brown and M. Bolsterli, Phys. Rev. Lett. 3, 472 (1959).

[3]For example, see F. W. K. Firk, Nucl. Phys. 52, 437 (1964); B. C. Cook, J. E. E. Baglin, J. N. Bradford, and J. E. Griffin, Phys. Rev. 143, 712 (1966).

[4]J. T. Caldwell, R. L. Bramblett, B. L. Berman, R. R. Harvey, and S. C. Fultz, Phys. Rev. Lett. 15, 976 (1965); P. F. Yergin, R. H. Augustson, N. N. Kanshal, H. A. Medicus, W. R. Moyer, and E. J. Winhold, Phys. Rev. Lett. 12, 733 (1964).

[5]B. Buck and A. D. Hill, Nucl. Phys. A95, 271 (1967); J. D. Perez and W. M. MacDonald, Phys. Rev. 182, 1066 (1969); W. P. Beres and W. M. MacDonald, Nucl. Phys. A91, 529 (1967); J. Raynal, M. A. Milkanoff, and T. Sawada, Nucl. Phys. A101, 369 (1967); A. M. Saruis and M. Marangoni, Nucl. Phys. A132, 433

(1969).
[6]V. Gillet, M. A. Melkanoff, and J. Raynal, Nucl. Phys. A97, 631 (1967).
[7]H. Feshbach, Ann. Phys. (New York) 19, 287 (1962).
[8]W. L. Wang and C. M. Shakin, Phys. Lett. 32B, 421 (1970).
[9]F. Iachello, thesis, Massachusetts Institute of Technology, 1969 (to be published).
[10]J. W. Jury, J. S. Hewitt, and K. G. McNeill, Can. J. Phys. 48, 1638 (1970); M. N. Thompson and J. E. E. Baglin, Phys. Lett. 25B, 256 (1967); J. E. E. Baglin and M. N. Thompson, Nucl. Phys. A138, 73 (1969); R. J. J. Stewart, Aust. J. Phys. 21, 107 (1968); D. E. Frederick, R. J. J. Stewart, and R. C. Morrison, Phys. Rev. 186, 992 (1969); V. V. Verbinsky and J. C. Courtney, Nucl. Phys. 73, 398 (1965).
[11]N. G. Puttaswamy and D. Kohler, Phys. Lett. 20, 288 (1966).

Importance of the D-State Probability of the Deuteron in Threshold $p + p \to \pi + d$ †

A. W. Thomas and I. R. Afnan
School of Physical Sciences, The Flinders University of South Australia, Bedford Park, South Australia 5042
(Received 12 February 1971)

Using a number of realistic, local and nonlocal, separable two-nucleon interactions, it is shown that threshold s-wave pion production is sensitive to the D-state probability of the deuteron and the nonlocality of the interaction. This enables one to use the reaction $p+p \to \pi^+ +d$ to study the relative strength of the central to tensor potential and the nonlocality of the nucleon-nucleon interaction at short distances.

Recent calculations using two-nucleon potentials with varying short-range behavior,[1] and relative strengths of the central and tensor components,[2] have shown large variation in the binding energy and saturation density of nuclear matter. These variations are a direct result of uncertainties in our knowledge of the nucleon-nucleon (N-N) interaction.

At present it is known that the long range ($r > 1.6$ F) part of the N-N interaction is due to one-pion exchange (OPE). However, the intermediate and short-range part of the potential is not uniquely determined. Consequently, this region has been represented by different phenomenological models, some of which are local while others are nonlocal or velocity dependent. It is the lack of real theoretical understanding, together with uncertainties in the experimental information, which leads to the large variations in the calculated properties of nuclear matter.

One such experimental uncertainty is the D-state probability of the deuteron, P_D. It has been shown[2] that variation of P_D from 1 to 9%, while keeping all other properties of the deuteron constant, leads to large variation in the binding energies of both nuclear matter and finite nuclei. The reason is that the quadrupole moment of the deuteron Q, and the coupling parameter ρ_1, which at low energy is related[3] to Q, do not uniquely determine the range and strength of the tensor part of the N-N interaction.

In an attempt to constrain the value of P_D to better than the present range of 3-8%,[4] and possibly gain information about the N-N potential at short range, we have studied the sensitivity of low-energy s-wave pion production in the reaction

$$p+p \to \pi^+ +d \tag{1}$$

to variations in these properties. One expects Reaction (1) to give information about the N-N interaction at momentum transfer greater than the pion mass.

The transition amplitude, to second order in the pion-nucleon interaction V, is given by[5]

$$T_{d,p} = \langle \chi_d | V | \chi_p^{(+)} \rangle + \langle \chi_d | V(E-H)^{-1} V | \chi_p^{(+)} \rangle, \tag{2}$$

where χ_d is the wave function for a deuteron plus a free pion, and $\chi_p^{(+)}$ is the two-proton scattering wave function. Here H is the full two-nucleon Hamiltonian plus a pion kinetic-energy operator. The pion-nucleon interaction density is taken to be of the same form as that used by Woodruff[6] and Koltun and Reitan,[7] namely

$$V = H_0 + H_1 + H_2, \tag{3}$$

where

$$H_0 = (4\pi)^{1/2}(f/u) i\vec{\sigma} \cdot \{\nabla_\pi[\tau \cdot \varphi(x)] + [\vec{p}\tau \cdot \pi(x) + \tau \cdot \pi(x)\vec{p}]/2M\}, \tag{3a}$$

$$H_1 = 4\pi\lambda_1 \mu^{-1} \varphi^2(x), \tag{3b}$$

$$H_2 = 4\pi\lambda_2 \mu^{-2} \tau \cdot \varphi(x) \times \pi(x). \tag{3c}$$

As usual, $\vec{\sigma}$ and τ are the nucleon-spin and -isospin operators, respectively, and $\vec{p}$ is the nucle-