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Ground-State Energy of a Charged-Boson Gas

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The ground-state energy of a charged-boson gas at high density is re-examined and it is found that Brueckner's exact result for the second-order energy is reproduced precisely by Feenberg's variation-perturbation method.

In recent years the charged-boson gas has been an interesting subject in the theoretical study of many-body boson problems. The ground-state energy of the system at high density was first evaluated exactly to second order by Brueckner,[1] who carried out complete summations of one- and two-ring diagrams using the Bogoliubov theory; his exact result is[2]

$$E/N = -0.8031/r_s^{3/4} + 0.0280 + O(r_s^{3/4}),$$ (1)

which agrees *numerically* with the variational result obtained by Lee and Feenberg[3] in the Bijl-Dingle-Jastrow (BDJ) wave-function space. Because of this numerical agreement considerable energy has been expended in attempts to prove that two results agree analytically. The purpose of this report is to clarify this point by showing that the variational energy is higher in second order than the exact energy by an amount exactly equal to the energy shift generated by the three-phonon vertex.[4]

In order to evaluate the difference between the exact and variational energies in second order, here we introduce

$$t = \frac{1}{\sqrt{2}} \left[\frac{\hbar^2}{4\pi m e^2 \rho} \right]^{1/4} k, \quad \bar{t}'' = \bar{t} - \bar{t}', \quad \omega = [1 + t^4]^{1/2}, \quad S = t^2/\omega, \quad \{f\} = f + f' + f'' = f(t) + f(t') + f(t'').$$ (2)

Then, Brueckner's formula for the exact second-order energy term can be expressed as

$$\mathcal{E}^{(2)}[\text{exact}] = \mathcal{E}^{(2)}(1 \text{ ring}) + \mathcal{E}_X^{(1)} + \mathcal{E}_{Y\lambda}^{(1)}, \tag{3}$$

where

$$\mathcal{E}^{(2)}(1 \text{ ring}) = \frac{1}{4\pi^4}\int \frac{d\vec{t}\,d\vec{t}'}{SS'}(\omega + t^2)(1-S)^2(1-S')^2, \tag{4}$$

$$\mathcal{E}_X^{(1)} = \frac{1}{8\pi^4}\int \frac{d\vec{t}\,d\vec{t}'}{SS'(t'')^2}(1-S)(1-S')(1+SS'), \tag{5}$$

$$\mathcal{E}_{Y\lambda}^{(1)} = -\frac{1}{24\pi^4}\int \frac{d\vec{t}\,d\vec{t}'}{SS'S''\{\omega\}}\left[\left\{\frac{1}{\omega}\right\} - SS'S''\left\{\frac{1}{t^2}\right\}\right]^2. \tag{6}$$

Also, Lee and Feenberg's expression for the variational second-order energy is, from Eq. (57) of Ref. 3,

$$\mathcal{E}^{(2)}[\text{variation}] = (2\pi^4)^{-1}\int d\vec{t}\,d\vec{t}'(1-S)(1-S')(1-S'')t^2. \tag{7}$$

Re-expressing the integrals of Eqs. (4)-(7) so that integrands are linear in the function ω and symmetric in the variables t, t', and t'', we find after some lengthy algebra that the energy difference in second order becomes

$$\mathcal{E}^{(2)}[\text{exact}] - \mathcal{E}^{(2)}[\text{variation}] = \Delta_1 + \Delta_2, \tag{8}$$

with

$$\Delta_1 = -\frac{1}{24\pi^4}\int \frac{d\vec{t}\,d\vec{t}'}{SS'S''\{\omega\}}[SS'S''(2\{\omega\} - \{S\omega\} - \{S\}\{S\omega\} + 2\{S^2\omega\}]^2$$

$$= -\frac{1}{24\pi^4}\int \frac{d\vec{t}\,d\vec{t}'}{SS'S''\{\omega\}}(1-S)^2(1-S')^2(1-S'')^2\left[2\left\{\frac{S^2\omega}{1-S}\right\} - \left\{\frac{S}{1-S}\right\}\{S\omega\}\right]^2, \tag{9}$$

$$\Delta_2 = \frac{1}{12\pi^4}\int d\vec{t}\,d\vec{t}'\left[\left(4 - \left\{\frac{1}{S}\right\} + 2\{S\} + SS'S''\right)\{\omega\} + \left(\{S\}\left\{\frac{1}{S}\right\} - 3\right)\{S\omega\} + \left\{\frac{\omega}{S}\right\} + \{S^3\omega\}\right.$$

$$\left. + \left(4 + \frac{1}{SS'S''} - 2\left\{\frac{1}{S}\right\} - \{S\}\right)\{S^2\omega\}\right]. \tag{10}$$

It is not difficult to show that the expression for Δ_1 is exactly the same as the second-order perturbation correction [Eq. (6) of Ref. 4] derived for liquid He[4] by Davison and Feenberg,[4] who realized that three-phonon components, which are absent in the BDJ-type wave function, contribute to the energy in second order. On the other hand, the integral of Eq. (10) can be reduced to

$$\Delta_2 = \frac{1}{2\pi^4}\int d\vec{t}\,d\vec{t}'(\vec{t}\cdot\vec{t}')\frac{(1-S)^2(1-S')^2}{SS'} = 0. \tag{11}$$

Thus, we arrive at the significant conclusion that Brueckner's exact result[1] for the ground-state energy is reproduced precisely by Feenberg's variation-perturbation method.[3,4] The details of our analysis are extremely complicated, and will be reported elsewhere together with the numerical value of the perturbation correction Δ_1.

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