Light-Cone Expansions for Exclusive Processes*

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(Received 28 December 1970)

It is shown that light-cone dominance ideas apply to exclusive processes as well. These processes must include a photon of large virtual mass, as for example deep pion electroproduction. The method provides for a new general parametrization for amplitudes in the Bjorken limit. If correct, it will lend strong support to the ideas of light-cone expansions of operator products and provide a way to study the structure, at short distances, of the commutators of electromagnetic currents and strong sources.

The expansion of operator products when their space-time distance approaches the light cone, recently suggested by one of us[1,2] and others,[3] seems to be a useful tool for analyzing processes involving high virtual-mass photons. The region of interest is the one for which both the energy and the virtual mass of the photon are large with a fixed ratio between the two. This is the limit for which Bjorken's suggestion[4] of the scaling behavior in deep inelastic scattering of electrons on protons seems to be well verified experimentally.[5] We shall call this the Bjorken limit.

A possible important use of the light-cone expansions is that it provides for a parametrization of the amplitudes in the Bjorken limit, in the same way that Regge behavior controls the parametrization of the high-energy limit of hadronic processes when all external masses are kept finite. There, the energy dependence is predicted, while the residue function is sensitive to the particular model chosen. We shall see that here an analogous result is obtained.

Applications of the above-mentioned expansions have been proposed for inclusive processes and, except for the well-known case of deep inelastic electron-proton scattering, these tests of the light-cone expansions involve extra assumptions. Typically one must assume that the same terms that are relevant in the Regge limit are also dominant here or, if not, one must assume that limiting procedures can be manipulated safely as discussed in Ref. 2.

Our purpose here is to show that further tests of the idea can be made for exclusive processes as well, with one or two electromagnetic currents, using certain assumptions about limiting procedures to be specified below. By exclusive processes we mean, following Feynman, processes leading to a given final state. As shown below, the general structure of these processes will provide for a stringent test of the validity of the operator expansion. The predictions can be tested in electroproduction of pions or ρ's in the deep inelastic region off hydrogen. Such experiments are, in fact, being performed at Cornell.[6] These tests are relevant for checking the operator expansions of one electromagnetic current with a source of a strongly interacting particle. The more difficult experiment in which one produces real photons instead of pions would test the structure of two electromagnetic currents.

Our result is that for the processes just mentioned (see Fig. 1) the invariant amplitudes A_i can be expressed in the Bjorken limit as

$$A_i(\nu, \sigma^2, t) \sim \nu^{d_i} f_i(\omega, t), \qquad (1)$$

where t is the momentum transfer to the target, σ^2 is the absolute value of the square of the four-momentum of the virtual photon (spacelike momenta have a negative square in our metric), ν is the energy of the photon in the laboratory system, and $\omega = 2M\nu/\sigma^2$, where M is the target mass. The fact that the d_i are independent of ω and t is a test of the conjecture that states that the singularities of the operator expansions near the light cone are c numbers.[7]

The power of our detailed predictions may be increased if further assumptions are made. If we conjecture that naive dimensionality holds for the physical operator products,[8] as happens to be the case in deep inelastic electron scattering, we can predict the values of d_i. These may hold for real photon production since we are dealing with

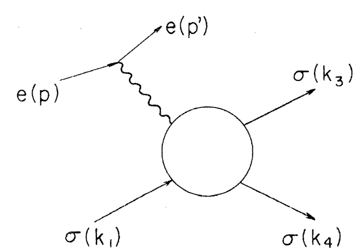

FIG. 1. Kinematics of two-particle final states in electroproduction.

two electromagnetic currents. For the case of pion or ρ electroproduction the commutator includes a purely hadronic source. Since it is known from models that the compositeness of the hadrons affects the commutator structure,[9] it is by no means obvious that naive dimensionality should hold in this case as well. Nevertheless we have computed for these processes the naive d_i so that comparison with the data may be easily done for this model.

We first illustrate the method by studying the scattering of scalar photons on scalar isospinless particles, since the operator structure relevant to our analysis remains essentially unchanged by spin and isospin complications. Straightforward application of the reduction formalism for the reaction (see Fig. 1)

$$\sigma(k_1)+\gamma(q)\rightarrow\sigma(k_3)+\sigma(k_4)$$

allows us to write the amplitude as

$$T\sim\int dx\, e^{iq\cdot x}\langle\sigma(k_1)|[j(x),j_\sigma(0)]|\sigma(k_4)\rangle\theta(-x_0), \tag{2}$$

where $j(x)$ is the electromagnetic current and $j_\sigma(x)$ is the source of the scalar field. Factors irrelevant to our discussion have been omitted. In the rest frame of the target, choosing the photon momentum in the Z direction, we have

$$q=(q_0,0,0,q_z);\quad k_1=(M,0,0,0);\quad k_i=(k_{i0},0,|\vec{k}_i|\sin\theta_i,|\vec{k}_i|\cos\theta_i)\quad(i=3,4).$$

Let us define

$$q^2=-\sigma^2,\quad \sigma^2>0;\quad \omega=2M\nu/\sigma^2;\quad \nu=k_1\cdot q/M,\quad t=-\tau=(k_1-k_4)^2,\quad s=(q+k_1)^2=M^2-\sigma^2+2\nu M;\quad q_0=\nu.$$

Thus in the limit $\nu\rightarrow\infty$, ω and t fixed, we have

$$q_z\approx\nu+M/\omega$$

and

$$2M|\vec{k}_4|=[\tau(\tau+4M^2)]^{1/2},\quad \cos\theta_4\approx(2M^2+\omega\tau)/\omega[\tau(\tau+4M^2)]^{1/2}, \tag{3}$$

and ω restricted by

$$2M^2/\omega\leqslant[\tau^2+4\tau M^2]^{1/2}-\tau.$$

The next step is to discuss the matrix element defined in expression (2). On invariance grounds it follows that the matrix element can depend on x^2, $x\cdot k_1$, $x\cdot k_4$, or $k_1\cdot k_4$. Because of our choice of fixed τ and ω, all these invariants are independent of ν. Hence, all the ν dependence has been isolated in the exponential.

The standard argument can now be applied. The integral picks up contributions from $|x_0-z|\sim\nu^{-1}$ and, barring pathological behavior, from $|x_0+z|\lesssim\omega/M$. Hence $|x_0{}^2-z^2|\lesssim\omega/M\nu$ and because of causality [see Eq. (2)] we have, finally, $x^2\leqslant\omega/M\nu\rightarrow0$ and hence light-cone dominance.

We can now apply the techniques of operator products expansions near the light cone.[1-3] We get

$$T\approx\int dx\, e^{iqx}c(x)F(x\cdot k_1,x\cdot k_4,\tau), \tag{4}$$

where

$$F(x\cdot k_1,x\cdot k_4,\tau)=\int\int d\alpha d\beta g(\alpha,\beta,\tau)\exp[i(\alpha k_1+\beta k_4)\cdot x], \tag{5}$$

$$c(x)=\theta(-x_0)[(-x^2+i\epsilon x_0)^d-(-x^2-i\epsilon x_0)^d]. \tag{6}$$

We can now use the identity

$$\int dx\, e^{ikx}\theta(-x_0)[(-x^2+i\epsilon x_0)^d-(-x^2-i\epsilon x_0)^d]=c(d)h(k^2,k_0), \tag{7}$$

where

$$c(d)=\pi2^{2d+3}\Gamma(d+1)\Gamma(d+2),\quad h(a,b)=e^{-i\pi\epsilon(b)}[(-a+i\epsilon)^{-d-2}-(-a-i\epsilon)^{-d-2}]+[(a+i\epsilon)^{-d-2}-(a-i\epsilon)^{-d-2}],$$

and we arrive at

$$T\approx c(d)\int\int d\alpha d\beta g(\alpha,\beta,\tau)h(\xi^2,\xi_0), \tag{8}$$

where

$$\xi_0 = (q + \alpha k_1 + \beta k_4)_0 \approx \nu, \quad \xi^2 = (q + \alpha k_1 + \beta k_4)^2 \approx 2M\nu[\alpha + \beta(1-\omega^{-1}) - \omega^{-1}]. \tag{9}$$

In (9) we have assumed that only finite α and β contribute to (8).[10] Finally, we get

$$T \sim \nu^{-d-2} f(\omega, \tau), \tag{10}$$

where

$$f(\omega, \tau) = (2M)^{-d-2} c(d) \int\int d\alpha d\beta g(\alpha, \beta, \tau) h(\alpha + \beta(1-\omega^{-1}) - \omega^{-1}, 1). \tag{11}$$

A similar analysis can be performed for the production process

$$A_1 + \gamma \to A_3 + A_4 + \cdots + A_N$$

shown in Fig. 2. Setting

$$t_i = (k_1 - k_i)^2 \quad (i = 4, \cdots, N),$$

we get

$$\lim A(\nu, \sigma^2, t_3, t_4, \cdots, t_N) = \nu^{-d-2} f(\omega, t_3, t_4, \cdots, t_4), \tag{12}$$

with the limit taken as $\nu \to \infty$ and $\sigma^2 \to \infty$ while $t_i = \text{const}$, $\omega = 2M\nu/\sigma^2 = \text{const}$.

We now apply our formalism to the more interesting case of electroproduction of pions from hydrogen. We specialize to this final state because it is currently being measured at Cornell,[6] but we can analyze other systems as well.

Without deriving the results of the kinematical analysis in detail, we quote from Salin[11] the relevant formulas. To second order in electromagnetism, the amplitude can be expressed as

$$T_{fi} = \epsilon_\mu \langle k_3, k_4 | J_\mu | k_1 \rangle; \quad \epsilon_\mu = \bar{u}(p') \gamma_\mu u(p). \tag{13}$$

(See again Fig. 1. In this case k_1 and k_4 are the proton momenta and k_3 the pion momentum.) The spin-averaged differential cross section has the following expression[11]:

$$d\sigma = \frac{e^4 g^2}{2(2\pi)^5} \frac{m_e^2 M^2}{[(pk_1)^2 - m_e^2 M^2]^{1/2}} \frac{\delta^4(p + k_1 - p' - k_3 - k_4)}{q^4} \frac{d^3 p'}{p_0'} \frac{d^3 k_3}{k_{3,0}} \frac{d^3 k_4}{k_{4,0}} |T_{fi}|^2, \tag{14}$$

where M and m_e are the proton and electron masses and

$$|T_{fi}|^2 = \sum_{i=1}^{4} L_i(q^2, \nu, \cos\varphi, \Phi) T_i(q^2, \nu, \tau). \tag{15}$$

φ is the electron scattering angle in the laboratory frame, and Φ is the angle between the plane of the electrons and the final πN plane. The expressions for L_i may be found in Ref. 11. It can be checked that T_i are dimensionless (as far as asymptotic dimensions are concerned) so that assuming naive dimensionality, our predictions for the Bjorken limit are[12]

$$T_i(q^2, \nu, \tau) \approx T_i(\omega, \tau) \quad (i = 1, \cdots, 4). \tag{16}$$

The same analysis can be done, for example, for ρ electroproduction. We conclude that in the Bjorken limit, the amplitudes for one-current exclusive processes have (disregarding logarithmic terms) a simple form given by (1). This suggests a natural way to parametrize the data obtained from the deep electroproduction experiments now under way at Cornell.

The precise forms needed to fit the data will provide information on the dimensionality of the operators involved in the operator expansions.

We thank M. Kugler for discussions.

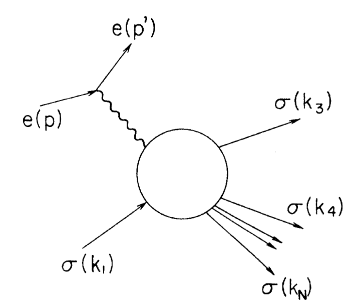

FIG. 2. Kinematics of many-particle final states in electroproduction.

*Work performed under the sponsorship of the U. S. National Bureau of Standards.

[1]Y. Frishman, Phys. Rev. Lett. 25, 966 (1970).

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[3]R. A. Brandt and G. Preparata, CERN Report No. TH-1208, 1970 (to be published).

[4]J. D. Bjorken, Phys. Rev. 179, 1547 (1969).

[5]E. D. Bloom *et al.*, Stanford Linear Accelerator Center Report No. SLAC-PUB-796, 1970 (to be published).

[6]Some preliminary results at rather low values of the virtual-photon mass are contained in D. E. Andrews *et al.*, "Preliminary Results on Hadron Production in Deep Inelastic Electron Scattering," to be published.

[7]This is, of course, a main feature of the operator expansions suggested in Refs. 1, 2, and 3 and is a generalization to the light cone of Wilson's short-distance expansion. For the latter, see K. Wilson, Phys. Rev. 179, 1499 (1969).

[8]By naive dimensionality we mean those values of d_i given from considering a free-field scale-invariant theory. See Refs. 2 and 3 for examples.

[9]H. R. Rubinstein, G. Veneziano, and M. Virasoro, Phys. Rev. 167, 1441 (1968).

[10]The integration over the variables α and β cannot be restricted in this case, using spectral conditions, to a finite region. In fact we find $\alpha < 1$, $\beta > -1$.

[11]Ph. Salin, Nuovo Cimento 32, 521 (1964).

[12]Note that had we considered, instead of Eq. (2), the expression

$$\int dx\, e^{-ik_3\cdot x} \langle\sigma(k_1)|[j(0), j\sigma(x)]|\sigma(k_4)\rangle\, \theta(x_0)$$

which is certainly equal to the one in Eq. (2), we would not have succeeded in establishing light-cone dominance by looking at the exponential $e^{-ik_3\cdot x}$ only. To establish it, we have to assume that the part which gives the dominant contribution when $q^2 \to -\infty$ involves an extra exponential factor which brings us back, essentially, to Eq. (2). Similiar assumptions have to be made also in the application of light-cone expansions to form factors. For the latter application see R. Brandt and G. Preparata, Phys. Rev. Lett. 25, 1530 (1970).

Speculative Determination of the Intermediate-Boson Mass*

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(Received 8 February 1971)

It is shown that the algebra generated by the space integrals of the fourth components of the electromagnetic current and the weak-interaction current is that of an SU(2)⊗U(1) group. Let $\vec{K}$ be the generator of this SU(2) group. By requiring the $|\Delta\vec{K}| = 1$ part of these current operators, including their respective coupling constants, to form a $\vec{K} = 1$ triplet, one finds that the semiweak coupling constant g is related to the fine-structure constant α by $(4\pi)^{-1}g^2 = \frac{1}{8}\alpha$; therefore, in the absence of radiative corrections, the mass of the (hypothetical) spin-1, charged intermediate boson is 37.29 GeV.

This note is motivated by the view that, perhaps, the electromagnetic interaction and the weak interaction are of the same origin, and that the semiweak coupling constant g may, in fact, be of the same order of magnitude as the electric charge e. In order to give such an idea a quantitative basis, let us first examine the algebra generated by the space integrals of the fourth components of the electromagnetic current $\mathcal{J}_\mu{}^\gamma$ and the weak interaction current $\mathcal{J}_\mu{}^{\rm wk}$, in the *absence* of any strong interaction currents. As usual, one may make the decompositions

$$\mathcal{J}_\mu{}^\gamma = j_\mu{}^\gamma + J_\mu{}^\gamma$$

and

$$\mathcal{J}_\mu{}^{\rm wk} = j_\mu{}^{\rm wk} + J_\mu{}^{\rm wk}, \qquad (1)$$

where $j_\mu{}^\alpha$ and $J_\mu{}^\alpha$ ($\alpha = \gamma$ or wk) denote, respectively, the lepton current and the hadron current. The lepton currents are known explicitly in terms of the charged-lepton field ψ_l and the neutrino field ψ_{ν_l} ($l = e$ and μ):

$$j_\lambda{}^\gamma = i\textstyle\sum_l \psi_l{}^\dagger \gamma_4\gamma_\lambda\psi_l$$

and

$$j_\lambda{}^{\rm wk} = i\textstyle\sum_l \psi_l{}^\dagger \gamma_4\gamma_\lambda(1+\gamma_5)\psi_{\nu_l}, \qquad (2)$$

where the symbol † denotes Hermitian conjugation.

For the purpose of this note, one may just as well consider a hypothetical world in which there is no strong interaction; the only physical observables in this hypothetical world are assumed to be simply the electromagnetic and the weak-