Foundation.

[1]C. W. Wong, to be published.

[2]F. Iwamoto and M. Yamada, Progr. Theor. Phys. 17, 543 (1957).

[3]S. A. Moszkowski and B. L. Scott, Ann. Phys. (New York) 11, 65 (1960); H. A. Bethe, B. H. Brandow, and A. G. Petschek, Phys. Rev. 129, 225 (1963).

[4]A. de-Shalit and V. F. Weisskopf, Ann. Phys. (New York) 5, 282 (1958).

[5]B. H. Brandow, Rev. Mod. Phys. 39, 771 (1967).

[6]J. W. Clark and P. Westhaus, Phys. Rev. 141, 833 (1966).

π-Carbon Inelastic Scattering Near the 33 Resonance*

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Inelastic π^--C scattering from 120 to 280 MeV is calculated in the distorted-wave Born approximation using the Kisslinger optical potential and treating the 2^+ (4.44 MeV and 3^- (9.64 MeV) levels as rotational states. Agreement with experimental data is good, especially in the forward hemisphere. Fairly consistent values for the deformation parameters are obtained.

Elastic π^--C scattering in the energy region of the 33 π-nucleon resonance has been calculated recently by Sternheim and Auerbach.[1] Using the Kisslinger optical potential[2] with parameters derived from π-nucleon phase shifts, these authors obtain remarkably good fits to the elastic scattering data of Binon *et al.*[3] They improve the fits somewhat by varying the parameters, thus arriving at a phenomenological optical potential.

The Binon experiment also resolved inelastic scattering to several of the widely separated low-lying states of ^{12}C. We have treated the strongly excited 2^+ (4.44 MeV) and 3^- (9.64 MeV) levels as rotational excitations and calculated inelastic scattering to these states using the distorted-wave Born approximation (DWBA). The resulting inelastic fits are at least as good as the elastic fits, apparently justifying the use of the rotational model and the direct reaction formalism for the (π, π') reaction in this energy region.

The Kisslinger optical potential in coordinate space is given by[4]

$$V(r) = (\hbar^2c^2/2E)\{-Ab_0k^2\rho(r) + Ab_1\nabla\cdot[\rho(r)\nabla]\}, \tag{1}$$

where A is the mass of the target, k and E are respectively the lab momentum and total lab energy of the incident pion, and $\rho(r)$ is the nucleon density mormalized to unity. The complex Kisslinger parameters b_0 and b_1 are directly related to the π-nucleon phase shifts.[5] For ^{12}C, electron scattering data suggest a modified Gaussian nucleon density,[6]

$$\rho(r) = \rho_0[1 + (Z-2)r^2/3a^2]\exp(-r^2/a^2), \quad a = 1.5 \text{ F}. \tag{2}$$

Equation (2) is actually the charge distribution for two $1s$ protons and $(Z-2)$ $1p$ protons in a central harmonic potential; for even-even nuclei like ^{12}C the neutron density is assumed to be the same.

Differential cross sections for elastic scattering from the optical potential are obtained by numerically solving an approximate Klein-Gordon equation,

$$\hbar^2c^2(-\nabla^2 + \mu^2)\psi = [E - V_C(r) - V(r)]^2\psi \approx [(E-V_C)^2 - 2EV]\psi. \tag{3}$$

Here $\mu = m_\pi c/\hbar$, V_C is the Coulomb potential, V is the optical potential in Eq. (1), and E is the total incident-pion energy; the position of the target nucleus is assumed to be fixed. For a complete discussion of the elastic scattering calculations, the reader is referred to the papers of Auerbach and Sternheim.[1,4]

We now consider a deformed nucleon density $\rho(r, a(\theta'))$ obtained from (2) by making the parameter a functionally dependent upon the body-fixed polar angle θ'. We choose

$$a(\theta') = a_0[1 + \beta_L Y_0^L(\theta')], \quad a_0 = 1.5 \text{ F}, \tag{4}$$

where Y_0^L is the spherical harmonic of order L and β_L is a deformation parameter. To first order in the deformation we have

$$\rho(r, a) \approx \rho(r, a_0) + (a-a_0)[\partial\rho/\partial a]_{a_0} = \rho(r, a_0) + \beta_L F(r) Y_0^L(\theta'), \tag{5}$$

where $F(r) = a_0[\partial\rho/\partial a]_{a_0}$. The deformed density gives rise to a deformed Kisslinger potential

$$V(r) = V^{(0)}(r) + V^{(1)}(r, \theta'),$$

which we have written as the sum of a spherical term $V^{(0)}$, and a deformed term

$$V^{(1)} = \beta_L(\hbar^2c^2/2E)\{-Ab_0k^2F(r)Y_0^L(\theta') + Ab_1\nabla\cdot[F(r)Y_0^L(\theta')\nabla]\}. \tag{6}$$

While the spherical term $V^{(0)}$ can give rise only to elastic scattering, excited rotational states of the deformed nucleus are strongly coupled to the ground state by the deformed term $V^{(1)}$. In particular, we treat the 2^+ (4.44 MeV) and 3^- (9.64 MeV) states of ^{12}C as pure rotational states; they are coupled to the 0^+ ground state by quadrupole ($L = 2$) and octupole ($L = 3$) deformations, respectively.

In the distorted-wave Born approximation, the transition amplitude for excitation of the 2^L-pole rotational state is

$$T_{fi}(M) = \int d^3r\, \chi_f^{(-)*}(\vec{k}_f, \vec{r})\langle LM| V^{(1)}|00\rangle \chi_i^{(+)}(\vec{k}_i, \vec{r}), \tag{7}$$

where χ_i and χ_f are the distorted waves representing the elastic scattering of a pion with incident momentum $\vec{k}_i$ and final momentum $\vec{k}_f$, respectively. They are solutions of the Klein-Gordon equation using the spherical optical potential $V^{(0)}$. The matrix element of the deformed potential, $\langle LM| V^{(1)}|00\rangle$, is taken between the 0^+ nuclear ground state and the excited state of angular momentum L and projection M.

Integration over internal nuclear coordinates gives

$$\langle LM| V^{(1)}|00\rangle \propto \beta_L\{-b_0k^2F(r)Y_M^{L*}(\hat{r}) + b_1\nabla\cdot[F(r)Y_M^{L*}(\hat{r})\nabla]\}, \tag{8}$$

where $\hat{r}$ denotes the orientation of the pion position $\vec{r}$ relative to the lab-fixed z axis. The distorted waves χ_i and χ_f are each expanded in a partial-wave series, so that

$$T_{fi} \propto \sum\int d^3r[r^{-1}\chi_{l'}^{(-)}(r)Y_{m'}^{l'*}(\hat{r})]\,[-b_0k^2FY_M^{L*} + b_1\nabla\cdot(FY_M^{L*}\nabla)][r^{-1}\chi_l^{(+)}(r)Y_m^l(\hat{r})].$$

Writing $\nabla = \hat{r}\partial/\partial r - (i/r)\hat{r}\times\vec{L}$ and using standard techniques of angular momentum algebra, we find that

$$T_{fi} \propto \beta_L\sum[-b_0k^2G_{\rm I} - b_1G_{\rm II} + b_1G_{\rm III}\Delta_{l'l}^L]\int d\hat{r}\,Y_{m'}^{l'*}Y_M^{L*}Y_m^l. \tag{9}$$

The integral over spherical harmonics is easily evaluated. The radial integrals are

$$G_{\rm I} = \int dr\,\chi_{l'}^{(-)}F\chi_l^{(+)},$$
$$G_{\rm II} = \int dr\left(\frac{d\chi_{l'}^{(-)}}{dr} - \frac{\chi_{l'}^{(-)}}{r}\right)F\left(\frac{d\chi_l^{(+)}}{dr} - \frac{\chi_l^{(+)}}{r}\right), \tag{10}$$
$$G_{\rm III} = \int dr\frac{\chi_{l'}^{(-)}}{r}F\frac{\chi_l^{(+)}}{r},$$

and the quantity $\Delta_{l'l}^L$ is

$$\Delta_{l'l}^L = \tfrac{1}{2}[L(L+1) - l'(l'+1) - l(l+1)]. \tag{11}$$

$G_{\rm I}$ is the usual radial integral encountered in distorted-wave calculations using a local optical potential. $G_{\rm II}$ and $G_{\rm III}$ arise from the Kisslinger modification and result from the radial and azimuthal components of ∇, respectively. The differential cross section is proportional to $|T_{fi}|^2$ summed over the nuclear orientation M.

The distorted-wave program DWUCK[7] was extensively modified to carry out the calculation. We used the Kisslinger parameters tabulated in Ref. 1, and the reader is referred to the same paper for plots of the elastic cross sections. We calculated inelastic differential cross sections at 120-, 150-, 180-, 200-, 230-, 260-, and 280-MeV incident-pion energy. The results are shown in Figs. 1 and 2.

The solid-line cross sections ("Fermi-averaged parameters") use Kisslinger parameters derived directly from free π-nucleon phase shifts and corrected for the Fermi notion of the nucleons. These curves are of the greatest interest since, except for the deformation parameter β_L, they represent a parameterless model. The long-dash curves ("best-fit parameters") use Kisslinger parameters which have been varied to improve the elastic fits; in some cases they also improve the inelastic fits. Finally, the short-dash curves

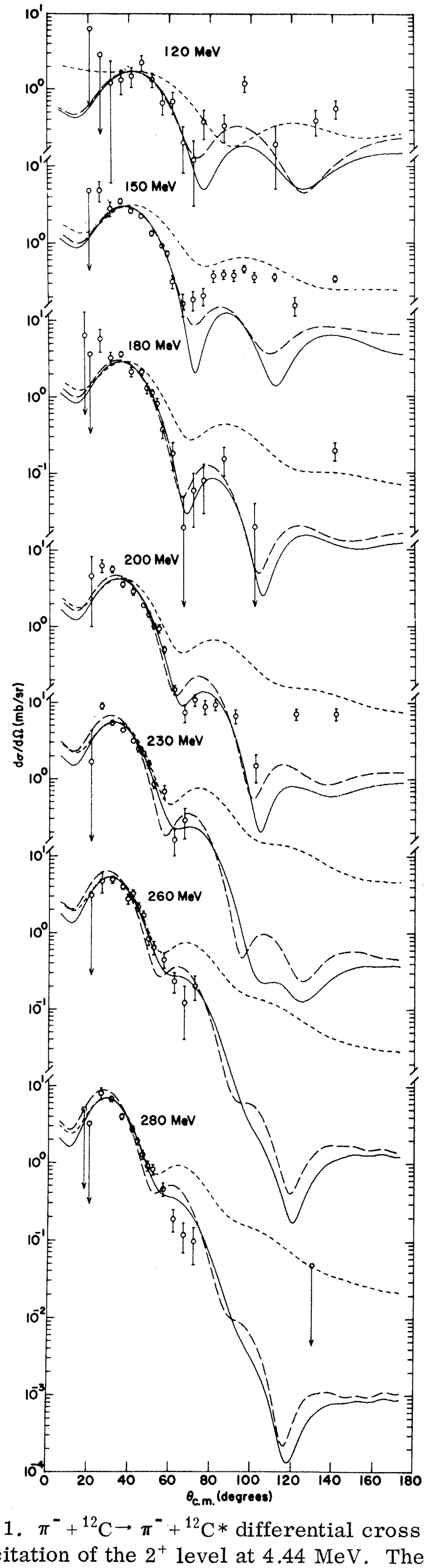

FIG. 1. $\pi^- + {}^{12}C \rightarrow \pi^- + {}^{12}C^*$ differential cross sections for excitation of the 2^+ level at 4.44 MeV. The Kisslinger model using Fermi-averaged parameters (solid lines) and best-fit parameters (long-dash line) is compared to the simple optical model (short-dash line).

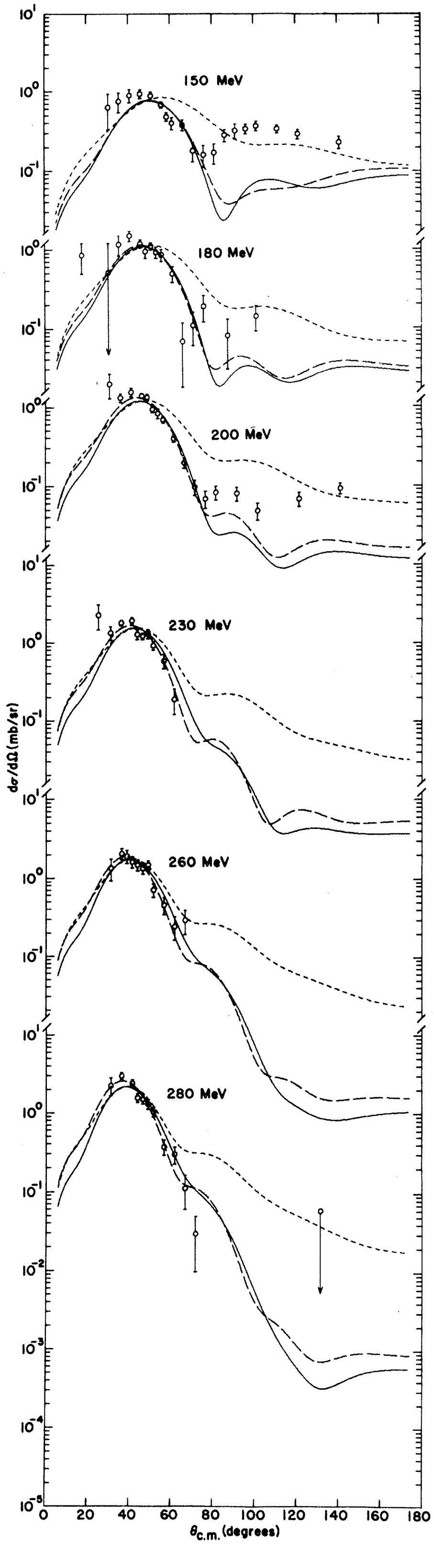

FIG. 2. Differential cross sections for excitation of the 3^- level at 9.64 MeV.

Table I. Deformation parameters for the 2^+ level (β_2) and the 3^- level (β_3), using both Fermi-averaged and best-fit Kisslinger parameters.

Energy (MeV)	β_2 Fermi averaged	Best fit	β_3 Fermi averaged	Best fit
120	0.70	0.68	...	...
150	0.80	0.66	0.70	0.55
180	0.70	0.65	0.72	0.66
200	0.81	0.76	0.71	0.66
230	0.87	0.85	0.74	0.68
260	0.82	0.80	0.74	0.68
280	0.93	0.93	0.80	0.77

("simple optical model") are included to illustrate the importance of the p-wave term in the Kisslinger potential. The simple optical model follows from a forward-angle approximation in (1) in which $b_1 \to 0$ and $b_0 \to b_0 + b_1$; it is inadequate beyond the first diffraction peak.

In all cases the Kisslinger model successfully predicts the position and shape of the first diffraction peak. This feature permits a fairly accurate determination of the deformation parameters. Because both the deformation and the transition amplitude are calculated only to first order in β_L, the differential cross section scales with $|\beta_L|^2$. The magnitude of the deformation (but not the sign) was thus determined in each case by displacing the cross section vertically to obtain a visually best fit to the data.

The position of the first diffraction minimum is fairly well predicted, although the minimum is too deep at the lower energies. The position and shape of the second peak are only qualitatively correct at the lower energies. The wide-angle data (where they exist) sometimes exceed the predicted cross sections by as much as an order of magnitude. This quantitative failure of the model at large angles, which is seen in the elastic fits as well, is probably due to ignoring nucleon recoil in the derivation of the optical potential and to the omission of higher-than-p-wave terms in the π-nucleon amplitude.[8]

Deformation parameters for the 2^+ and 3^- levels are listed in Table I. If the deformation is to be interpreted as a property of the nucleus independent of the scattering particle, we expect deformation parameters which are independent of pion energy and comparable in magnitude to those obtained from nuclear inelastic scattering. In fact the π-nucleus deformations tend to increase slowly with pion energy and are somewhat higher than those obtained using nuclear projectiles, which are typically on the order of 0.5.[9] It is also apparent that deformations this large should not be treated in first order; improved calculations are under way.

We conclude that fairly accurate distorted-wave calculations can be carried out in the 33 resonance region using the Kisslinger optical potential. Both the collective model and the direct reaction formalism appear to be applicable to π-nucleus inelastic scattering.[10]

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[1]M. M. Sternheim and E. H. Auerbach, Phys. Rev. Lett. 25, 1500 (1970).

[2]L. S. Kisslinger, Phys. Rev. 98, 761 (1955).

[3]F. Binon, P. Duteil, J. P. Garron, J. Gorres, L. Hugon, J. P. Peigneux, C. Schmit, M. Spighel, and J. P. Stroot, Nucl. Phys. B17, 168 (1970).

[4]E. H. Auerbach, D. M. Fleming, and M. M. Sternheim, Phys. Rev. 162, 1683 (1967).

[5]See Ref. 4, Eqs. (25) and (26).

[6]D. G. Ravenhall, Rev. Mod. Phys. 30, 430 (1958).

[7]P. D. Kunz, private communication.

[8]M. M. Sternheim, Phys. Rev. 135, B912 (1964).

[9]See, for example, B. G. Harvey, J. R. Meriwether, J. Mahoney, A. Bussière de Nercy, and D. J. Horen, Phys. Rev. 146, 712 (1966). The $^{12}C(\alpha,\alpha')$ DWBA fits used by these authors are poor in comparison with our $^{12}C(\pi,\pi')$ DWBA fits.

[10]After completing this manuscript we received a preprint by H. K. Lee and H. McManus, who investigated elastic and inelastic π-^{12}C scattering using a microscopic formulation. Their inelastic-scattering calculations employ particle-hole nuclear wave functions for ^{12}C with a WKB-Glauber approach for the reaction mechanism. The fits to the data are comparable to the macroscopic DWBA fits which we have obtained. The macroscopic approach, while less fundamental, is probably more easily applied to the analysis of inelastic pion scattering from nuclei heavier than ^{12}C.