ble I.

In the table, τ_s^* is the skin time without the viscosity correction.

It thus appears that the anomalous skin effect, as observed in Tokamaks, may be traceable to this effect. An experimental check should be relatively easy, especially if the time of current penetration is measured after the plasma is heated to a relatively high temperature. Oscillation over the ion cyclotron frequency should also be measurable.

Somewhat paradoxical results are indicated in Eq. (22), because it may appear that increasing the drift velocity increases τ_s. However, in reality $\langle(n-n_0)^2/n_0^2\rangle_{av}$ should be a function of v_{d0}, and presumably an increasing function. This probably compensates for the inverse-square dependence on v_{d0}.

Aside from the application to the Tokamak, this mechanism may also be responsible for the anomalously thick skin of collisionless shocks perpendicular to the magnetic field, as well as the anomalous viscosity hypothesized by Morse and Stovall.[1]

Furthermore, we have introduced here a new concept for the mechanism of transport of various plasma parameters. This technique may be useful for calculating heat conductivity and diffusion coefficients.

In the absence of both this viscosity effect and ion Landau damping, the electron and wave momenta would reach equilibrium values such as predicted by Drummond and Pines[7,8] and the resistivity due to the phonon-electron interaction would disappear. Thus either ion Landau damping (which also leads to anomalous resistivity) or anomalous viscosity is essential in explaining the anomalous skin effect.

Table I. Skin times calculated for different electron temperatures.

kT_e (eV)	τ_s (msec)	τ_s^* (msec)
100	2.1	7.2
400	1.45	57.6
1000	0.95	220
2000	0.65	640

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Comment on the Proposed Magnetic Cooling of He^3

W. P. Wolf, M. F. Thorpe, and R. Alben
Becton Center, Yale University, New Haven, Connecticut 06520
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We examine a recent proposal by Goldstein for producing very low temperatures by freezing He^3 in the presence of magnetic fields. We conclude that the highly isotropic nature of solid He^3 will completely suppress the particular cooling mechanism envisaged, but a related magnetothermal effect should be observable. At readily accessible temperatures this effect will be small, but below about 1 mK a potentially useful amount of magnetic cooling is predicted. The field required for the maximum cooling is about 70 kOe.

In a recent Letter,[1] Goldstein proposed a new method for obtaining extremely low temperatures by freezing He^3 in the presence of a magnetic field. It was implied that the method depends on the peculiar nature of the anisotropy of solid He^3, but the precise role of the anisotropy was not discussed explicitly.[2] In this note we wish to point out that (1) the particular mean-field calcu-

lation carried out by Goldstein is appropriate only when the anisotropy is large, and such an anisotropy can be ruled out on the basis of published NMR results; (2) the mean-field calculation appropriate to an isotropic solid, which Goldstein intended to treat,[2] indicates no magnetic cooling whatsoever; and (3) a linear spin-wave calculation which should be much better than mean-field theory at low temperatures does in fact indicate a potentially interesting cooling effect, but with a behavior significantly different from that described by Goldstein.

For example, if solid He^3 were produced at the melting pressure in zero field with an entropy of $0.05R$, its temperature would be about 1.2 mK. The mean-field calculation of Goldstein predicted that the application of a field of about 27 kOe would reduce the temperature for this case to about one eighth of the zero-field value, a striking and potentially useful effect. On the other hand the mean-field calculation appropriate to an isotropic system gives no cooling upon application of any field, and the more accurate spin-wave treatment shows that the temperature would in fact be reduced by only about one-half, but that this would require a field of about 70 kOe. We conclude, therefore, that while Goldstein's interesting idea of cooling solid He^3 by using magnetic fields is physically correct, the detailed mechanism will in fact make the effect far less dramatic than that proposed and less accessible experimentally.

We first note that the process described in Ref. 1 can be carried out in two steps. First one could freeze liquid He^3 by adiabatic compression in the usual way[3,4] and then one could apply a field to magnetize the He^3. The first step will result in solid He^3 at some temperature T_1 with a corresponding entropy $S(T_1, H=0)$. The value of T_1 will of course depend on the initial conditions of the liquid, but this part of the process is irrelevant for the present discussion. The vital question is whether a different temperature T_2 significantly lower than T_1 can be achieved by the further application of a magnetic field H. To obtain the largest possible cooling it is clearly most advantageous to consider the field applied adiabatically and reversibly, and in this case $S(T_2, H)=S(T_1, H=0)$. Since the entropy at constant field is always a monotonically increasing function of T, a sufficient condition for T_2 to be less than T_1 is $(\partial S/\partial H)_T>0$, as pointed out by Goldstein.[1] The problem then boils down to finding a situation where this condition holds, and where, moreover, $(\partial S/\partial H)_T$ is large over an appreciable range of fields.

For most magnetic systems, $(\partial S/\partial H)_T$ is in fact negative, corresponding to ordering imposed by the magnetic field, and cooling can only be obtained by adiabatic demagnetization. Exceptions to this rule may occur in multisublattice systems, such as antiferromagnets, since the gain in entropy of part of the system may then exceed the reduction in the rest.

A good example of this is the case discussed by Goldstein. Following Garrett[5] he considered a two-sublattice antiferromagnet in a field applied parallel to one of the two sublattices, and he calculated the change of entropy with field assuming that the two sublattices remain in their initial collinear configuration. This assumption implicitly demands an amount of anisotropy sufficient to stabilize this arrangement, and in the specific calculation used by Goldstein,[1,2] this anisotropy was assured by considering only the Ising-model part of the complete interaction Hamiltonian and neglecting all transverse terms. This approximation is in fact not stated explicitly either by Goldstein or in the earlier paper by Garrett,[5] and indeed both seem to imply that the model is valid for a two-sublattice, isotropic (Heisenberg) antiferromagnet. This is not the case, however, since in the limit of a completely isotropic system there is a very different type of solution which in fact always has a lower energy. This solution corresponds to the well-known[6] spin-flop phase of usual antiferromagnets and it implies that no matter in which direction the field is applied, the sublattices will turn so that they are initially perpendicular to H. The variation of entropy with field in this configuration is of course quite different from that in the parallel case.

It is therefore of paramount importance to determine the nature of the appropriate interaction Hamiltonian before attempting to calculate the thermodynamic properties using one of the statistical approximations. In the past it has always been assumed[7,8,2] that the dominant term for He^3 is an isotropic coupling between the nearest-neighbor nuclear spins,

$$\mathcal{H}_e = \sum_{i>j} J\vec{I}_i\cdot\vec{I}_j, \tag{1}$$

and that the only anisotropic terms arise from the much weaker magnetic-dipole coupling

$$\mathcal{H}_d = \sum_{i>j} A_{ij}[\vec{I}_i\cdot\vec{I}_j - 3(\vec{I}_i\cdot\vec{r}_{ij})(\vec{I}_j\cdot\vec{r}_{ij})/r_{ij}^2], \tag{2}$$

with $A_{ij} = g_n^2 \mu_n^2 / r_{ij}^3$, where $\vec{r}_{ij}$ is the distance between spins i and j and g_n is the nuclear g factor. Fitting J to various empirical data such as the susceptibility,[9] specific heat, and nuclear-spin-relaxation times,[8] it has proved possible to obtain a generally consistent explanation of the available data; and the adequacy of a Hamiltonian of the form $\mathcal{H} = \mathcal{H}_e + \mathcal{H}_d$ has never really been called into doubt. However, inasmuch as Goldstein's proposed cooling method would demand an anisotropy many times larger than that provided by $\mathcal{H}_d$, it now becomes important to examine critically whether additional terms in $\mathcal{H}$ can really be excluded by the available data.

Support for a Hamiltonian of the form $\mathcal{H}_e$ plus anisotropic terms with the same form as Eq. (2) but with unspecified magnitude is provided by the observed frequency dependence of the transverse NMR relaxation time T_2, which obeys quite accurately the "$\frac{10}{3}$ effect" increase as the measuring frequency is varied.[8] The observed frequency at which the effect takes place provides a measure of the isotropic exchange J, and this is in good agreement with other estimates.[7-9] Using the measured values of T_2 and J, one can then estimate the order of magnitude of the anisotropic terms; and one finds general agreement with the magnitudes given by $\mathcal{H}_d$, about three orders of magnitude smaller than $\mathcal{H}_e$ in the region of interest.[10]

If we now consider the macroscopic consequences of such a Hamiltonian applied to He^3 in its bcc phase, we find a further reduction in the effective anisotropy as a result of the overall cubic symmetry. This has the effect of reducing the first-order contribution of any dipolar anisotropy identically to *zero* so that only higher order contributions will be effective. The precise size of these is quite difficult to estimate, especially if we allow for the possibility that the sublattices may not remain collinear, but we can readily estimate the order of magnitude[11] as $\sim A^2/J$. We thus see that the effective anisotropy is some *six orders of magnitude smaller* than the isotropic part of the energy, so that it is quite clear that the Ising model must be an extremely poor approximation.

Conversely, we can conclude that the completely isotropic Heisenberg Hamiltonian should be a rather good approximation and it is now of interest to ask if this will show any significant magnetothermal effects.

Using the same mean-field approximation as that used by Goldstein and Garrett, but including the effect of the transverse terms, we can readily calculate the phase boundary and isentropes shown in Fig. 1. It can be seen that the lines within the antiferromagnetic phase $[H < H_c(T)]$[12] are straight and vertical, corresponding to the condition $(\partial S/\partial H)_T = (\partial M/\partial T)_H = 0$. This corresponds to a well-known result due to Néel[13] that the perpendicular susceptibility of an antiferromagnet is independent of field and temperature in the mean-field approximation. We see, therefore, that far from obtaining a dramatic cooling effect, *the mean-field theory predicts no cooling of any kind.*

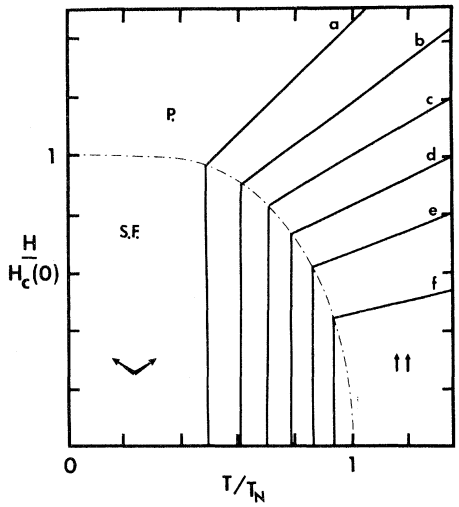

FIG. 1. Mean-field calculation of the phase boundary and isentropes for a spin $I = \frac{1}{2}$ two-sublattice Heisenberg antiferromagnet with intersublattice exchange only. $T_N = zJI(I+1)/3k_B$ is the Néel temperature, where z = number of neighbors (8 for the bcc lattice), and $H_c(0) = 4k_B T_N/g_n\mu_n$. The broken line divides the paramagnetic phase denoted by P from the "spin-flop" phase denoted by S.F. In a reversible adiabatic magnetization experiment, the system moves along a line of constant entropy. The lines labeled a, b, c, d, e, and f correspond to $S/Nk_B = \ln 2 - (0.6, 0.5, 0.4, 0.3, 0.2, \text{and } 0.1$, respectively), where N is the total number of spins.

Of course for the real system the isentropes will not be exactly vertical. One factor contributing to nonzero magnetothermal effects will be the small but finite anisotropy and in principle this could be included in the mean-field calculation, provided the anisotropic terms are included correctly to second order. However, it seems quite clear that an effective anisotropy of one part in 10^6 will not affect the thermodynamic properties significantly, and we have therefore not carried through the rather complex calculation.

Far more important is the inadequacy of the

mean-field approximation itself. Since there are no theories which accurately describe a large interacting spin system at all temperatures, we must consider various limiting cases to obtain an improved description of the entropy. Following Johnson *et al.*,[14] we have used linear spin-wave theory for the entropy at low temperatures, with a graphical interpolation to meet the entropy curve calculated from a high-temperature series[14-16] at T_N. The results for zero field are shown as the solid line in Fig. 2. To obtain an estimate of the maximum entropy difference in a field, we consider the system close to the antiferromagnetic-paramagnetic phase boundary, $H = H_c(T)$, where the field is just strong enough to overcome the antiferromagnetic exchange and align the spins.[17] In this state the spin-wave spectrum becomes similar to that in a ferromagnet with a dispersion law $\hbar\omega \propto k^2$, in contrast to the zero-field antiferromagnet for which $\hbar\omega \propto k$. Correspondingly, the entropy for this state varies as $T^{3/2}$, compared with the zero-field variation proportional to T^3, and this results in the variation with temperature shown by the dashed line in Fig. 2. Here we have again extrapolated the exact low-temperature variation to meet the calculated zero-field entropy at $T = T_N$. We see from Fig. 2 that the isentropic application of a magnetic field at the very lowest temperature should result in a significant amount of cooling, but that the effect should become quite small as T approaches T_N, as predicted by the corresponding mean-field calculation. Similar behavior has been found in exact solutions of linear Heisenberg chains.[18]

It is clear that our estimates of the entropy could be refined in different ways and indeed it would seem worth making a detailed study of the magnetothermal properties of solid He^3 to throw further light on both the nature of the interactions and the statistical approximation used to calculate thermodynamic properties. However, we can conclude even now that the cooling which can be produced by the application of a magnetic field on He^3 will be very much harder to observe than the effect predicted in Ref. 1.

We are grateful to Dr. L. Goldstein for clarifying some of the ideas contained in Ref. 1 and for sending us a preprint of his full paper[2] on the subject. We would also like to thank Dr. J. C. Bonner for a number of helpful discussions.

Note added in proof.—Similar conclusions concerning the approximations used in Ref. 1 and the predictions of linear spin-wave theory have recently been reached by Walstedt, Walker, and Varma.[19]

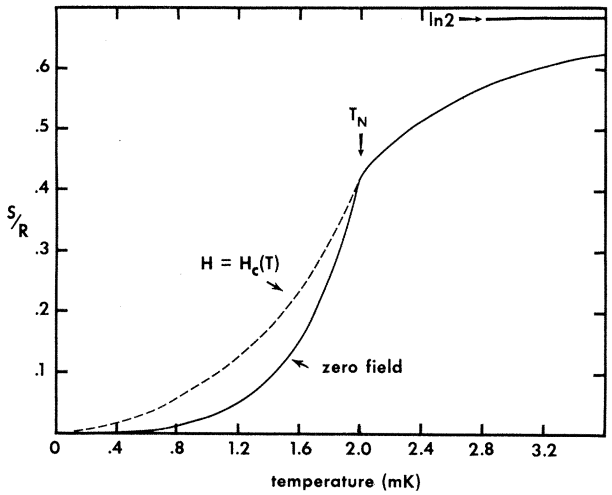

FIG. 2. Estimated variation of entropy with temperature for solid He^3 assuming isotropic nearest-neighbor exchange with $J/k_B \sim 1.44$ mK (Ref. 14). The solid curve corresponds to zero field, and was derived following Ref. 14, using an exact spin-wave result ($S/R \propto T^3$) for the low-temperature part, and a nine-term series expansion for the high-temperature part (down to $T_N \approx 2$ mK). The intermediate range is estimated using an approximate extrapolation of the spin-wave result adjusted to join the two curves at T_N. The broken curve is a similar estimate for fields along the antiferromagnetic-paramagnetic phase boundary. In the low-temperature spin-wave region where $H_c(T) \approx H_c(0) \approx 74$ kOe, the entropy $S/R \propto T^{3/2}$; as $T \to T_N$, $H_c(T) \to 0$, and the curve converges with the zero-field estimate.

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As we shall see, anisotropic nuclear interactions do indeed appear to be negligible, but this by no means supports the *omission* of "spin-flop phenomena." Quite to the contrary, in the case of negligible anisotropy the spin-flop solution is always the one with lowest energy.

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Thermoreflectance of Graphite

M. Anderegg, B. Feuerbacher, and B. Fitton
European Space Research Organisation, Noordwijk, Holland
(Received 11 February 1971)

The thermoreflectance of graphite has been measured in the energy range 4.4 to 5.5 eV at 80°K. Three peaks at 4.52, 4.76, and 4.82 eV have been identified as M_1-type singularities and assigned to transitions taking place at the points Q and L in the Brillouin zone.

A new spectrum of the thermoreflectance of graphite near the 4.8-eV optical transition, which differs markedly from another measurement,[1] is presented. A splitting of this transition, due to multilayer interaction, is observed for the first time. The observed transition energies of 4.76 and 4.82 eV are compared with other optical measurements that localize the transition at 4.6,[2,3] 4.8,[4] and 5.1 eV.[1] From a line-shape analysis of the thermoreflectance peaks the transitions are identified as M_1-type singularities. The thermal modulation effect appears to be mainly due to gap-energy shift, while broadening modulation gives a minor contribution. The measurements provide evidence for an inversion of the parity of the π-electron states at point Q as proposed by several band-structure calculations,[2,5-7] but in disagreement with other calculations[8,9] which predict two peaks in the thermoreflectance spectrum separated by 1.6 eV.

The measurements have been performed using a rectangular crystal of stress-annealed pyrolytic graphite[10] glued to a copper heat sink, which was cooled to liquid-nitrogen temperature. The sample was cleaved to a thickness of about 0.2 mm prior to its installation into the vacuum system, where it was kept at a pressure of 10^{-9} Torr. Temperature modulation was achieved by passing a square-wave current at 3.6 Hz with an amplitude of 2 A rms through the sample, corresponding to a power dissipation of 0.1 W/cm^2. The mean temperature of the sample was 80°K, the temperature modulation amplitude being 1.4°K peak to peak. The sample was illuminated under 45° angle of incidence with light from a high-pressure mercury lamp after passing through a monochromator with a spectral resolution of 0.06 eV.

The results of the thermoreflectance measurements between 4.4 and 5.5 eV are shown in Fig. 1 as a solid line. The effect of thermal modulation may be considered as a superposition of broadening and stress modulations. The stress may be regarded as uniaxial parallel to the c axis, be-