Thermally Driven Superfluid-Helium Film Flow*

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Measurements of dissipation versus velocity for the flowing helium film are reported. A new probe technique allows measurement of the dissipation as a temperature difference in a small region of the film. The data are interpreted in terms of the Langer-Fisher fluctuation model as modified for driven flow where we find $\Delta T \propto \exp[\beta\rho_s/k\rho T v_s]$.

The flow of superfluid helium through fine pores[1,2] has been interpreted in terms of the thermal fluctuation theory developed by Langer and Fisher[3] (LF). Recent measurements by Notarys[2] indicate that pressure-driven flow through fine pores obeys the functional relation of a modified LF theory at temperatures well below T_λ. The energy of excitation of the phase-destroying fluctuation has the same functional dependence ($1/v_s$) on the superfluid velocity as the energy of a usual vortex ring, but a significantly smaller magnitude. We report measurements of the dissipation as a function of velocity in a thermally driven, saturated helium film. An aluminum film probe is used to measure small temperature differences along the film. These data are found to be related to the extension by Notarys of the Langer and Fisher fluctuation theory.

When a constant driving force is applied to a superfluid film a steady-state flow is established and phase destruction proceeds at a rate proportional to the superfluid velocity. The equation for driven motion is[4]

$$\rho_s(dv_s/dt)=\rho_s(F/m)-\rho_s(dv_s/dt)_{\text{fluct}},$$

where ρ_s is the superfluid density, F/m is the acceleration due to a force acting on the system in the direction of v_s, $dv_s/dt=0$ in steady state, and $(dv_s/dt)_{\text{fluct}}$ is defined below. Set $F/m=\nabla\mu$, where μ is the chemical potential and

$$\nabla\mu=-\frac{1}{\rho}\nabla P-S\nabla T-\frac{\rho_n}{2\rho}\nabla(v_n-v_s)^2,$$

where ρ_n and ρ are, respectively, the normal component and total fluid densities, P is the pressure, S is the bulk fluid entropy per unit mass, and v_n is the normal fluid velocity; $v_n=0$ is assumed in a thin helium film. For the present experiment the dominant term is $S\nabla T$. The fundamental equation of the LF theory[3] is

$$(dv_s/dt)_{\text{fluct}}=-A_f f_0(h/m)\exp(-E_0/kT),$$

where A_f is the film cross-sectional area, f_0 is the fluctuation attempt frequency, and $E_0=\rho_s K^3/16\pi v_s$ is the approximate energy of a critical fluctuation; $K\equiv h/m$ is the quantum of circulation. This energy is obtained from a dimensional analysis using a hydrodynamic model for a vortex ring.[3] Finally a relation between ΔT and v_s is obtained when $\nabla T=\Delta T/l$, and l is the length of the dissipation region along the film. A pressure $\rho S\Delta T$ is written for convenient comparison between the present data and that for fine pores; combining the above equations one finds that

$$\rho S\Delta T=\rho(h/m)V_f f_0\exp(-\beta\rho_s/k\rho T v_s), \tag{1}$$

where $\beta=K^3/16\pi$ in the above approximation for E_0.[4]

The experimental apparatus shown in Fig. 1 consists of a glass post extending vertically from the bath in an enclosed chamber. This post has a diameter of 2 cm at the bottom and is necked down smoothly to the top section of 1 cm diam. The outer surface of the post provides a substrate for the helium film under discussion as well as a support for the heater and a thin-film thermometer probe[5] which is an evaporated aluminum film used near its superconducting transition temperature. This probe extends about 1 cm

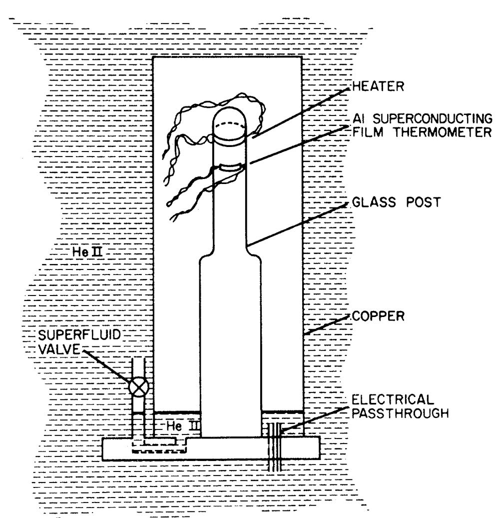

FIG. 1. Schematic diagram of the experimental apparatus.

around the cylinder and has a width of 0.02 cm, which is small compared to the height of the film probe above the bath, usually 5 to 10 cm. Calibration of the film probe is described in Ref. 5. The small mass of the film gives a rapid response to small temperature fluctuations since the heat capacity of the film is much less than the helium film covering the probe. The sensitivity of the probe is of the order $10^4\ \Omega\ \mathrm{K}^{-1}$ at a power dissipation of $\leq 10^{-9}$ W. A four-wire potentiometric method is used to determine temperature differences to with 10 μK. Because the power dissipation is low, no appreciable thermal boundary (Kapitza) resistance occurs. In addition the point of temperature measurement is external to the heater. This technique offers an improvement over previous experiments[6] when the Kapitza resistance caused temperature differences which would mask the small differences associated with the thermohydrodynamic dissipation measured in this experiment.

The heater is of Nichrome and a four-wire potentiometer system gives the resistance directly and as a function of current. A precision standard resistor in the heater circuit is used for accurate determination of the heater power, substantially all of which is transferred to the helium film.

The velocity of the helium film in response to a heat input $\dot{Q}$, is given from the energy balance

$$\dot{Q} = \sigma(H_v + TS),$$

where H_v is the heat of vaporization and TS is the energy added to the superfluid component ρ_s. The mass flow rate σ is given as

$$\sigma = \rho_s d v_s P$$

for a film of thickness d over a surface of limiting perimeter P. The film thickness is calculated directly from the relation $d = kH^{-1/3}$ for a given height H above the bulk liquid surface.[7] As shown by Keller[8] there is no change in thickness between a static film and one moving with a subcritical velocity.

The experimental procedure was as follows. First a liquid level was established in the chamber from the external He II bath through the superfluid-tight valve. A temperature within the region of the aluminum probe sensitivity was selected and controlled to ± 3 μK with an electronic controller. A saturated equilibrium superfluid film formed on the glass post. Superfluid flow was initiated through the film toward the heater as the heat input $\dot{Q}$ to the heater was increased and evaporation removed the energy to the bath by recondensation. The flow remained subcritical on the 2-cm-diam section of the post. Increasing $\dot{Q}$ caused a measurable ΔT at the temperature probe. A temperature difference ΔT was measured only at constant $\dot{Q}$ was found to be constant over long periods of time (30 min). When $\dot{Q}$ was suddenly turned on, the constant value of ΔT was reached in a time limited by the recording potentiometer to the order of 1 sec.

The data are used to derive values of $\log(\rho S \Delta T)$ and $\rho_s/\rho T v_s$, which are shown plotted in Fig. 2. The values ρ, ρ_s, and S at a temperature T are obtained from the recent literature. Measurements are taken over the range of sensitivity of the aluminum temperature probes from 1.7 to 2.02 K. The measured values of ΔT extend over the range 10 μK to 1 mk.[9] Typical values of v_s were 24-60 cm/sec over the range of temperature and driving force in these experiments. These values are comparable to those obtained from gravitational-flow experiments.[10] In the temperature region studied the temperature dependence of $v_s \propto \rho_s/T$ is well satisfied as is seen in Fig. 2. An estimate of the experimental errors is shown by the error bars. The measured ΔT

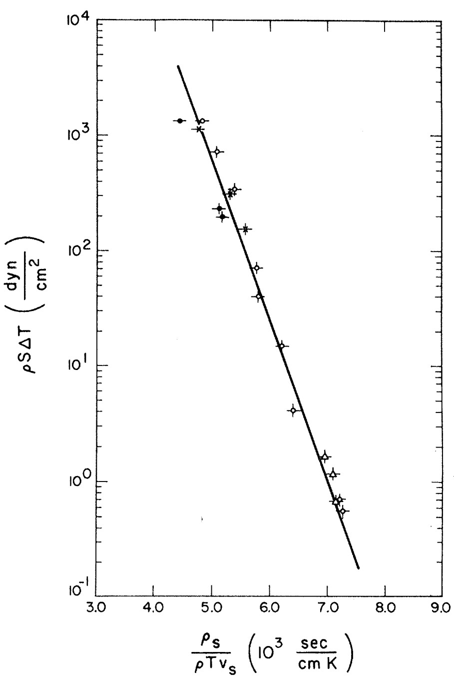

FIG. 2. The pressure $\rho S \Delta T$ versus the quantity $\rho_s/\rho T v_s$ for thermally driven flow of a saturated helium film. Open circles, 1.976 K; closed circles, 1.997 K; crosses 2.025 K; and triangles, 1.745 K. The solid line fit to the data is $\rho S \Delta T = 3.10 \times 10^9 \exp[(-3.2\times 10^3)\rho_s/\rho T v_s]$.

depends on dR/dT determined for the aluminum probe[5] and the assumption that the function is smooth is checked by making small temperature changes in the bath with the electronic controller. The errors in $\rho_s/\rho T v_s$ arise primarily from determination of the heater current and resistance. These measurements are in excellent agreement with the functional dependence between ΔT and v_s given by Eq. (1).

The data were obtained from several runs spanning a year in which a different temperature controller and Dewar-pumping system were used. We used different glass substrates of both drawn and polished tubing and aluminum films, both metallic and granular.[11] The heater element in one run was constructed from wire wound and epoxied to the glass post and in the other runs was evaporated completely around the glass post. No detectable systematic variations of the data were found as a result of these changes.

The constants A and β are determined from a fit of Eq. (1) to the data as shown by the solid line in Fig. 2, where

$$\rho S \Delta T = A \exp(-\beta \rho_s / k\rho T v_s).$$

The constant A includes the cross-sectional area of the film, the length l of the film over which dissipation occurs, the bulk fluid density, and the attempt frequency. From the measured perimeter of the post, the calculated film thickness, and the length l (taken to be the distance along the 1-cm-diam section of the post to the temperature probe), a value $f_0 = 2.19\times10^{17}$ is determined.

The value of β determined by a fit of Eq. (1) is $\beta = 4.4\times10^{-13}$. This value is significantly less than the calculated value $\beta = 50\times10^{-12}$. From the pressure driven flow in 800-Å-diam pores, Notarys[2] obtains $\beta = 13\times10^{-13}$, also small compared to the calculated value. The film thickness calculated for the present experiment was near 150 Å. However, the further decrease in β compared to the measurements of Notarys is less than a scaling with the helium thickness would predict.

No complete theory describing vortex formation in the film exists presently. According to Clark[12] the surface tension at the free surface should be strong enough to eliminate surface effects due to a vortex. The energy required to form a vortex is decreased in a constricted geometry and this is in qualitative agreement with the present observations.

An alternate interpretation of the measured temperature differences is possible according to Huggins.[13] On the basis of a hydrodynamic model for subcritical flow where no change in film thickness occurs a temperature gradient along the film was predicted. The magnitude of the temperature difference expected is a factor of 10 greater than that observed and does not have the expected functional dependence on v_s, that for small temperature differences gives $\Delta T \propto v_s^2$. That this interpretation is not important for thermally driven film flow is in agreement with isothermal flow measurements of Keller.[14]

The results indicate that in helium-film flow the magnitude of dissipation is functionally related to the superfluid velocity by the Langer-Fisher fluctuation theory, in the particular case of a vortex ring model. Close agreement with the temperature dependence inherent in this model is found as in Fig. 2. The quantitative identification of the fluctuation energy with the energy required to form a vortex ring is not good. We feel that in the case of helium-film flow a more careful description of vortex formation in a helium film may improve the quantitative agreement.

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films were not studied.

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[9]The experimental technique has been extended to higher power and consequently larger temperature differences. The data for $\Delta T \geq 5$ mK deviate from the rest of the data presented here. Several effects must be considered at the higher power levels, including the stability of bath control to larger power inputs, a modification of the film-flow process related to evaporation along the film below the heater, and participation of the normal fluid component in the dissipation process. Further experiments are in progress.

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Propagation of Electron Cyclotron Waves Along a Magnetic Beach

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The wave dispersion characteristic of whistler waves in a plasma is directly recorded as a function of the ratio of electron cyclotron frequency to wave frequency along their propagation path in a magnetic mirror. In the limit of small wavelengths the dispersion characteristic approaches that of plane waves in an unbounded plasma.

The dispersion relation of right-hand circularly polarized waves in plasma (whistler waves),

$$n^2 = \frac{k^2c^2}{\omega^2} = 1 + \frac{(\omega_p/\omega)^2}{\omega_b/\omega - 1}, \quad (1)$$

suggests an experimental study of these waves along a weakening magnetic field (magnetic beach). Here, c is the speed of light, k is the wave number, ω is the signal frequency, ω_p is the electron plasma frequency, and ω_b is the electron cyclotron frequency. Near electron cyclotron resonance, in the region $\omega_b/\omega \geq 1$, this dispersion relation, which is valid for an unbounded plasma, should also hold for free-space wavelengths several times larger than the transverse dimension of a laboratory plasma. In Eq. (1) the momentum-transfer collision frequency has been neglected and the angle between the direction of propagation and the static magnetic field is assumed to be zero.

The importance of experimental verification of Eq. (1) along a magnetic beach lies in the possibility of performing tests of validity for the microscopic progagation theory of the whistler mode with allowance for Landau damping, cyclotron damping, and plasma acceleration.[1-3] Until now it has been very difficult to study whistler-mode propagation in a laboratory plasma either because of large coupling losses of wave-exciting probes[2] or because of the complexity of separate excitation of circularly polarized waves.[4,5] Hitherto, measurements of $n(\omega_b/\omega)$ in a uniform magnetic field have had the disadvantages that in changing ω_b one is confronted with variations of the plasma parameters; or if ω is changed, the amplitude of the test wave varies depending on the coupling losses of the launching device used.[2,4,5]

In this Letter the validity of Eq. (1) in the limit of small wavelengths is confirmed by experimental data taken along a propagation path in a magnetic beach, in the vicinity of the resonance region $\omega_b/\omega \geq 1$. The wave-dispersion characteristic is directly recorded as a function of ω_b/ω by sampling the whistler wave along its propagation path in the magnetic-mirror plasma. This