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1*s*-3*d* Excitation of Atomic Hydrogen by Electron and Proton Impact

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The Glauber approximation has been applied to calculate the total and the differential cross sections of 1*s*-3*d* excitation of the hydrogen atom by electron and proton impact. The results obtained have been compared with other existing theoretical findings.

The problem of the scattering of a charged particle by a hydrogen atom has been the subject of wide theoretical investigation because of its simplicity and the availability of an exact wave function. In this Letter we have considered the excitation of the hydrogen atom to the 3*d* level from its ground state by electron or proton impact using the Glauber approximation,[1] a high-energy approximation which is expected to hold good for small-angle scattering. This approximation differs from almost all high-energy approximations, such as the Born, Vainshtein,[2] and impulse[3] approximations, in that for inelastic scattering it explicitly takes account of the interaction of the incident particle with the proton, whereas in the other approximations, the contribution of this interaction vanishes or has been neglected.

Apart from its application to other fields,[4] the Glauber model has recently been used successfully to calculate the scattering cross sections for the elastic and inelastic 1*s*-2*s*, 1*s*-2*p*, 1*s*-3*s*, and 1*s*-3*p* transitions in electron- (proton-) hydrogen collision problems.[5] In view of the surprisingly good agreement of the calculated results with the available experimental findgs even at the intermediate energy region, it seems worthwhile to investigate the 1*s*-3*d* transition in the same approximation. Vainshten[6,7] has applied the distorted-wave method and the first Born approximation (FBA) to calculate the 1*s*-3*d* excitation of the hydrogen atom by electron impact. The 1*s*-3*d* excitation of the hydrogen atom by proton impact has been investigated by Bates and Griffing[8] using the FBA.

Initially the hydrogen atom is in its ground state represented by the wave function Φ_i. Because of a collision with a particle of charge Ze the atom undergoes a transition to a final state f with wave function Φ_f. The scattering amplitude $F_{fi}(\vec{q})$ for this process is given by (notation is the same as that used by Ghosh and Sil[5])

$$F_{fi}(\vec{q}) = (ik_i/2\pi)\int \Phi_f^*(\vec{r})\Gamma(\vec{b},\vec{r})\Phi_i(\vec{r})\exp(i\vec{q}\cdot\vec{b})d^2b\,d\vec{r}, \tag{1}$$

with

$$\Gamma(\vec{b},\vec{r}) = 1-\exp[-2i\eta Z\ln(|\vec{b}-\vec{s}|/b)], \text{ and } \eta = e^2/\hbar v_i,$$

where $\vec{q}=\vec{k}_i-\vec{k}_f$, $\hbar\vec{k}_{i,f}=M\vec{v}_{i,f}$, M being the reduced mass of the system and v_i and v_f the velocities of the incident and scattered particle. Here the possible final states are

$$\Phi_{3d_0} = (2\times 3^9\pi a_0^7)^{-1/2} r^2\exp(-r/3a_0)(3\cos^2\theta-1),$$

$$\Phi_{3d_{\pm1}} = (3^8\pi a_0^7)^{-1/2} r^2\exp(-r/3a_0)\sin\theta\cos\theta\exp(\pm i\varphi),$$

$$\Phi_{3d_{\pm2}} = (2^2\times 3^8\pi a_0^7)^{-1/2} r^2\exp(-r/3a_0)\sin^2\theta\exp(\pm i2\varphi). \tag{2}$$

The term $r\cos\theta$ in the wave function $\Phi_{3d_{\pm1}}$ makes the corresponding scattering amplitude vanish. The scattering amplitudes for $\Phi_{3d_{+2}}$ and $\Phi_{3d_{-2}}$ come out to be the same. Therefore we need to calculate only two scattering amplitudes, viz. $3d_0$ and $3d_{+2}$. Substituting the initial and final wave functions in Eq. (1) we can express the scattering amplitude for $1s$-$3d_0$ excitation in e-H scattering as

$$F_{fi}(\vec{q}) = -\frac{3^4}{2^6\sqrt{6}}\, ik_i a_0^2 \int_0^{\pi/2} \sin^5\theta\cos\theta\left[1-\left(\frac{|\cos\theta|}{\cos\theta}\right)^{2i\eta}|\cos\theta| F\left(\tfrac{1}{2}+i\frac{\eta}{2},\, 1+i\frac{\eta}{2},\, 1,\, \sin^2 2\theta\right)\right] \times\frac{\sin^4\theta-10(\frac{3}{4}a_0q)^2\sin^2\theta\cos^2\theta+9(\frac{3}{4}a_0q)^4\cos^4\theta}{(\sin^2\theta+(\frac{3}{4}a_0q)^2\cos^2\theta)^6}\,d\theta. \tag{3}$$

Similarly for the $3d_{+2}$ state we may write

$$F_{fi}(\vec{q}) = \frac{3^7}{2^{12}}(1-i\eta)nk_i a_0^4 q^2 \int_0^{\pi/2} \sin^5\theta(\cos\theta)^{3-2i\eta}\sin^2 2\theta\, F\left(1-i\frac{\eta}{2},\, \frac{3}{2}-i\frac{\eta}{2},\, 3,\, \sin^2 2\theta\right) \times\frac{\sin^2\theta-\frac{2}{3}(\frac{3}{4}a_0q)^2\cos^2\theta}{[\sin^2\theta+(\frac{3}{4}a_0q)^2\cos^2\theta]^6}\,d\theta. \tag{4}$$

The scattering amplitude for the H^+-H collision is nothing but the complex conjugate of the above expressions. The differential and total cross sections are calculated using the standard relations.

The differential and total cross sections are obtained numerically using the Gaussian quadrature method. In Fig. 1 we have compared our results for the differential cross sections in e-H scattering at 100, 200, and 400 eV incident electron energies with the corresponding results of FBA. In Fig. 2, we have plotted our curves for the total cross sections for e-H scattering along with the curves of the FBA and the distorted-wave approximation. Figure 3 shows our results for the total cross sections for proton-hydrogen scattering along with those of the FBA.

The Glauber and FBA curves for the differential cross section of $1s$-$3d$ excitation of the hydrogen atom by electron impact fall monotonically with increasing scattering angle, and the higher the energy the steeper the fall. However, the Glauber curves are more sharply peaked in the forward direction. Nearly the same type of behavior of the differential cross section has also been noticed for the elastic and the other inelastic cases ($2s$, $2p$, $3s$, and $3p$ excitations) of e-H scattering as shown by Ghosh and Sil[5] and by Tai, Teubner, and Bassel.[5] At all three incident energies, Glauber's values are always greater than the corresponding values of FBA at an angle $\theta \geq 30$.

It is apparent from Fig. 2 that all the methods give essentially the same results for the total cross sections above 200 eV and that significant differences between the results do not set in until the incident energy is decreased below 100 eV. Further, we note that the Glauber predictions

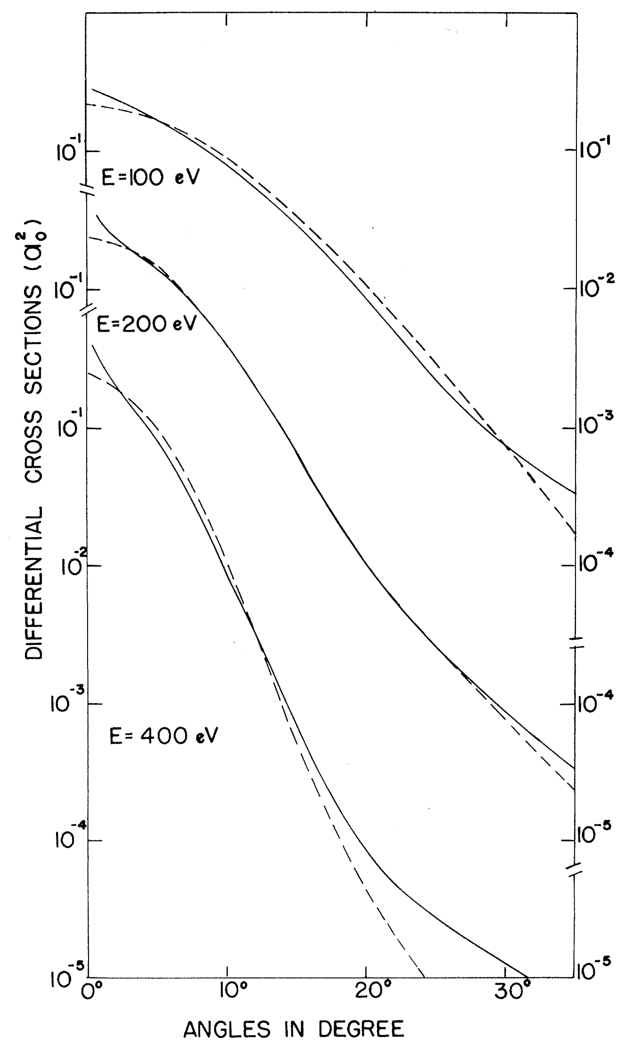

FIG. 1. Differential cross sections for $1s$-$3d$ excitation in electron-hydrogen collisions at 100, 200, and 400 eV versus the scattering angle. Solid line, Glauber approximation; dashed line, FBA.

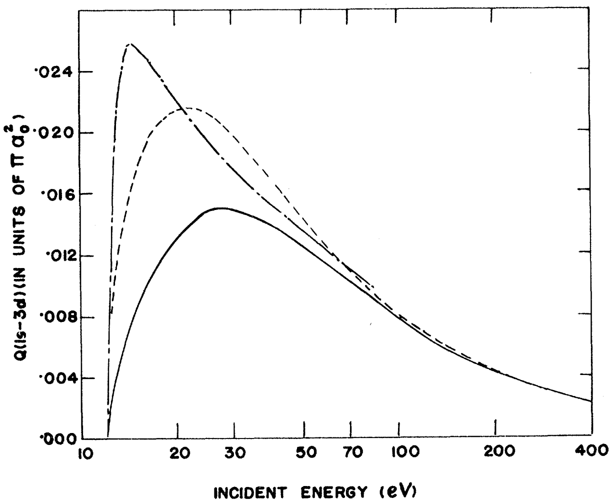

FIG. 2. Total cross section (Q) versus incident electron energy. Solid line, Glauber approximation; dashed line, first Born approximation; dash-dotted line, distorted-wave approximation.

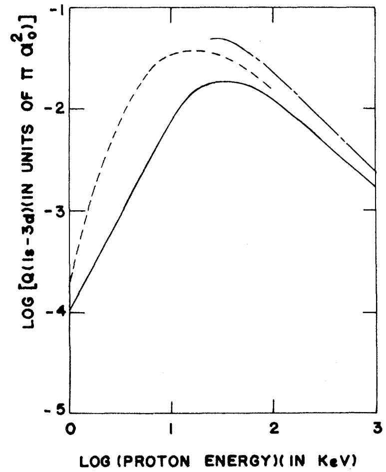

FIG. 3. Total cross section (Q) for proton impact versus incident proton energy in keV. Solid line, Glauber approximation; dashed line, FBA; dashed-dotted line, distortion approximation.

for the total cross section of inelastic e-H scattering[5] always lie below the other theoretical results. This behavior of the Glauber approximation is in contrast to the behavior in the elastic case.

From Fig. 3 it is evident that at high incident energies the Glauber total-cross-section curve for $1s$-$3d$ excitation of the hydrogen atom by proton impact approaches the corresponding FBA curve. In our curve we have obtained a peak value around 35 keV whereas the FBA peak value is around 20 keV, somewhat displaced towards lower energy. The curve obtained by Sen, Bhattacharya, and Sil[9] by applying a distortion approximation also yields a peak value around 30 keV. Our curve always passes below the other two curves. It may be noted that also in the inelastic cases of $1s$-$2p$ excitation in H^+-H collisions,[10] the Glauber values for the total cross section are always lower than the corresponding FBA values.

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