# Spin Dependence in Inclusive Collisions*

Henry D. I. Abarbanel† and David J. Gross†
*Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540*
(Received 7 January 1971)

Polarization phenomena in inclusive reactions are shown to exhibit remarkably simple features from the point of view of $J$-plane physics with factorizable singularities. In particular, straightforward tests of factorization are proposed in single- and multiple-particle inclusive experiments initiated with polarized photon beams– real and virtual – or polarized proton targets.

Multiple-production reactions in hadron physics have a greatly simplified structure when only a specified subset of produced particles is detected while the variables of the rest are summed over. These inclusive experiments, of the variety $a+b \to c+$anything, where $c$ represents the coordinates of the measured particles, have already yielded a number of precise predictions on the basis of various models of strong interactions. In particular, the ideas of limiting fragmentation and pionization[1] appear to be common features of a variety of models.[2-4]

The applicability of a Regge analysis to inclusive phenomena was realized a long time ago by Amati, Stanghellini, and Fubini.[3] Subsequent workers[4,5] have greatly modernized and generalized those results and have exhibited the role played by the vacuum trajectory in governing the fragmentation and pionization limits.

We shall discuss here a feature of inclusive experiments which depends on a presumed general characteristic of $J$-plane singularities; namely, factorization of the residues at Regge poles. The Regge analysis leads to quite striking predictions about the spin dependence of inclusive, differential cross sections for reactions initiated with a polarized beam or target and thus provides a rare opportunity to examine the validity of the factorizability of Regge residues in a single experiment.

Let us begin by discussing experiments with a single hadron detected. Consider $a(p_a,s_a)+b(p_b) \to 1(p_1)+X$; the undetected hadronic stuff is called $X$. All spin coordinates, except that of $a$, $s_a$, are summed over. The differential cross section for this process is given by

$$d\sigma(a(p_a,s_a)+b(p_b)\to 1(p_1)+X) = \frac{d^3p_1}{\pi m_1{}^2E_1}\frac{M(p_a,p_b,p_1;s_a)}{\Delta^{1/2}(s,m_a{}^2,m_b{}^2)}, \tag{1}$$

where $\Delta(x,y,z)=(x+y-z)^2-4xy$, $s=(p_a+p_b)^2$, and the dynamics are contained in

$$M(p_a,p_b,p_1;s_a)=\int\frac{d^4X\,e^{-ip_1\cdot X}}{32\pi^2}4E_aE_b\langle p_a,s_a,p_b\,\mathrm{in}|J_1(X)J_1(0)|p_a,s_a,p_b\,\mathrm{in}\rangle \tag{2}$$

with $J_1(X)$ the source density for particle 1. $M$ is clearly a piece of the absorptive part of the three-to-three *forward* scattering amplitude of $a$, $b$, and 1 as shown in Fig. 1. The basic assumption we will accept is that as the appropriate invariant subenergies, given by $p_a\cdot p_1$ or $p_b\cdot p_1$ here, become large, the usual $J$-plane singularities govern the asymptotic behavior of this piece of the three-to-three absorptive part.[4-6]

Now we ask what such a Regge analysis has to say about the spin dependence of $M$ on $s_a$. To this end we recall the following fact from two-body reactions: In the process

$$a(p_a,\lambda_a)+b(p_b,\lambda_b)\to a(p_a{}',\lambda_a{}')+b(p_b{}',\lambda_b{}'),$$

the $s$-channel barycentric helicity amplitudes for large $s$, forward scattering, conserve *separately* the helicities of $a$ and $b$, that is $\lambda_a=\lambda_a{}'$ and $\lambda_b=\lambda_b{}'$ up to terms of order $s^{\alpha_L-1}$, where $\alpha_L$ is the leading Regge-trajectory intercept, when the $t$-channel helicity amplitudes factorize at the Regge poles.

To establish this result[7] one needs first to remember that angular-momentum conservation

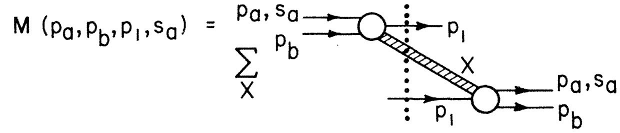


FIG. 1. The relation between $M$ and a piece of the three-to-three forward absorptive part. The dotted line cuts particles on the mass shell.

alone requires that total helicity is conserved: $\lambda_a - \lambda_b = \lambda_a' - \lambda_b'$. Factorization of the $t$-channel helicity amplitudes and the large-angle behavior of the rotation functions $d_{\lambda\mu}^J(Z)$, namely $d_{\lambda\mu}^J(Z) \to Z^J f_\lambda g_\mu + O(Z^{J-1})$, then leads via the helicty crossing matrix to factorization of the contribution of any given Regge pole to the $s$-channel helicity amplitudes in the $a$ and $b$ labels to order $s^{\alpha-1}$. The independence of the $a$ and $b$ helicities expressed in this factorization now requires separately that $\lambda_a = \lambda_a'$ and $\lambda_b = \lambda_b'$ for the leading, $s^\alpha$, contribution of each Regge pole. Thus, up to terms of order $s^{\alpha_L - 1}$, $s$-channel helicity is conserved for both $a$ and $b$ in the forward direction. Unfortunately tests of this effect in two-body processes are either trivial (if the target or the beam is unpolarized) or extremely difficult experimentally (polarized target and beam).

Since this derivation takes place at $t = (p_a - p_a')^2 = 0$, one might worry that so-called conspiracy relations could destroy the result. However, we note that because they contribute to total cross sections, the usual leading trajectories, $P$, $P'$, $\rho$, and $A_2$, are exempt from this possible criticism. The pion trajectory may be conspiring,[8] but $\alpha_\pi(0) \leq 0$ so that we are not concerned with it. The most likely candidate for a nonfactorizable asymptotic behavior is a Regge cut, and confirmation of this effect constitutes evidence against strong, nonfactorizable cuts in angular momentum.

The implications of this are direct for the inclusive production process under consideration. To see them, proceed to the Lorentz frame (FF) where $p_a + p_b + p_1$ has only a time component. This is the overall center of mass of the *forward* three-to-three absorptive amplitude and is rather funny from the point of view of the actual experiment. Note that the inclusive reaction has automatically taken us to $t = 0$. In FF we may treat the two particles $b$ and 1 as a blob of mass squared $= (p_1 + p_b)^2$ and consider what happens to $M(p_a, p_b, p_1; s_a)$ as this mass and the transverse momentum of $p_1$ are held fixed while the subenergy $(p_a + p_1)^2 = s_1$ becomes large. This is called the fragmentation region.[1-6] Now we are envisioning a quasi two-body forward-scattering process, so as soon as $s_1$ leaves the resonance region, say, $s_1 \geq 8$-$10\ \mathrm{GeV}^2$, and a finite sum of Regge poles will describe the behavior in $s_1$, we may expect the spin dependence of $M$ to be such that *no transitions between states with different helicities will occur.* Note that this transpires at a relatively low energy (the helicity-flip amplitudes are of order $1/s$ with respect to the nonflip amplitudes) and should be readily accessible at present accelerators.

There are some immediate and striking consequences of this helicity conservation in inclusive experiments: (1) Consider photon-induced reactions with a *polarized photon beam,* for example, $\gamma + p \to \pi + X$. If the incoming photon in FF has a mixture of helicities specified by $A|+\rangle + B|-\rangle$, $|A|^2 + |B|^2 = 1$, then transitions between $|+\rangle$ and $|-\rangle$ do not occur. Further, parity conservation requires the transitions $|\pm\rangle \to |\pm\rangle$ to be equal, so the inclusive production is *independent* of the state of polarization of the incoming photon. This is not only true in FF but also in any frame because of the photon which has only two allowed helicities. In particular, in the laboratory frame if we specify the photon polarization by the pseudovector $\vec{P}_\gamma$, then the only possible spin correlation in the process $\gamma + p_b \to p_1 + X$, namely $\vec{P}_\gamma \cdot (\vec{p}_b \times \vec{p}_1)$, must be absent for photon energies greater than, say, 4 or 5 GeV and slow detected hadrons, say, $E \lesssim 1$ GeV. One may search for this effect by examining events with fixed $\vec{P}_\gamma$, as in the Stanford Linear Accelerator Center polarized photon beam, and varying the azimuth of the outgoing detected hadron. On the basis of order-of-magnitude estimates alone, the size of such a spin correlation should be at worst ~(transverse momentum of $p_1$)/(mass of the proton) relative to the other allowed term with no spin dependence and thus not *a priori* negligible.

(2) Consider a *polarized proton target* in the laboratory and select those events in $b(p_b) + a(p_a, s_a) \to 1(p_1) + X$ for which $s_1 = (p_1 + p_a)^2$ is in the Regge region. The analysis above leads to the conclusion that $d\sigma(a + b \to 1 + X)$ should be independent of the state of polarization $\vec{P}$ of the proton for these events. Or again, spin correlations of the form $\vec{P} \cdot (\vec{p}_b \times \vec{p}_1)$ should be absent.

(3) A few moments thought shows that helicity *conservation* for particle $a$ in FF occurs even in the reaction $a + b \to 1 + 2 + \cdots + N + X$, whenever the smallest subenergy $s_j = (p_a + p_j)^2$, $j = 1, \cdots, N$, is in the Regge region. In general, the transitions $\pm\lambda \to \pm\lambda$ do not have to be equal when $N \geq 2$. However, since the helicity amplitudes in FF factorize in the Regge region, the dependence of the multiparticle amplitude on the helicity of $a$ is independent of $N$. Thus parity conservation guarantees that, up to terms of order $s^{\alpha_L - 1}$, the amplitude for $+\lambda \to +\lambda$ is equal to that for $-\lambda \to -\lambda$. If $a$ is either a photon or a spin-$\frac{1}{2}$ particle,

the result is again that $M$ is independent of the state of polarization of $a$. For $N=2$ this *rules out three independent spin correlations*, which, *a priori*, could be as large as the spin-independent term.

(4) In electroproduction experiments where $N$ final hadrons are detected,[9] $e+p_b \to e'+2+\cdots+N+X$, the virtual photon carrying four-momentum $q$ is in a superposition of helicity states. If the smallest of $s_j=(q+p_j)^2$, $j=1,\cdots,N$, is in the Regge region, helicity conservation and parity will as before reduce the number of transitions from six $(0\to\pm, \pm\to\pm, 0\to 0, +\to -)$ to two $(0\to 0, +\to + = -\to -)$. The usual description of electroproduction is in terms of hadronic structure functions. For inclusive experiments there are, for $N\geq 2$, six independent structure functions (for $N=1$ there are but four). The two that survive in the Regge region can easily be constructed by noting that helicity conservation in FF is equivalent to the statement that $M$ is invariant under rotation of all vectors save the spin of the photon about the direction of $\vec{q}$. This rotation is specified by requiring that $\Sigma=q+p_b+p_1+\cdots+p_n$, which only has a time component in FF, and $q$ are left invariant. Therefore, the only gauge-invariant tensors that will survive to the leading order in $s=(p_b+q)^2$ are those that can be constructed from $q$ and $\Sigma$, and for them we may write

$$W_{\mu\nu}(p_b,q,p_1,\cdots,p_n)=\sum\langle p_b|J_\mu(0)|p_1\cdots p_n X\rangle\langle X p_1\cdots p_n|J_\nu(0)|p_b\rangle\delta^4(q+p_b-p_1-\cdots-p_n-p_X)$$
$$=-W_1\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)+W_2\left(\Sigma_\mu-\frac{(\Sigma\cdot q)q_\mu}{q^2}\right)\left[\Sigma_\nu-\frac{(\Sigma\cdot q)q_\nu}{q^2}\right]. \qquad (3)$$

Similarly for inclusive neutrino-scattering experiments, in the appropriate Regge region, the number of structure functions will be reduced from six to three. These are rather strong statements and are amenable to direct verification. In addition the Regge analysis leads immediately to statements about the scaling behavior, for large $q^2$, of the structure functions, but we postpone that for a lengthier exposition.

We are grateful to G. Tiktopolous, S. B. Treiman, and C. De Tar for discussions.

---

*Research supported by the U. S. Atomic Energy Commission under Contract No. AT(30-1)-4159, and by the U. S. Air Force Office of Scientific Research under Contract No. AF49(638)-1545.

†Alfred P. Sloan Foundation Research Fellows.

[1]The experimental situation is reviewed by M. Koshiba and R. Panvini, in *High Energy Collisions—Third International Conference*, edited by C. N. Yang *et al.* (Gordon and Breach, New York, 1969), and is further analyzed by N. F. Bali, L. S. Brown, R. D. Peccei, and A. Pignotti, Phys. Rev. Lett. 25, 557 (1970), and L. L. Wang *et al.*, to be published.

[2]Droplet model: J. Benecke *et al.*, Phys. Rev. 188, 2159 (1969). Parton model: R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969); K. Wilson, Cornell University Report No. CLNS-131, 1970 (to be published).

[3]Multiperipheral model or multi-Regge analysis: D. Amati, A. Stanghellini, and S. Fubini, Nuovo Cimento 26, 896 (1962); L. Caneschi and A. Pignotti, Phys. Rev. Lett. 22, 1219 (1969); and others.

[4]A number of models have been surveyed in the generally multiperipheral work of C. De Tar, Lawrence Radiation Laboratory Report No. 19882, 1970 (to be published).

[5]A. Mueller, BNL Report No. BNL 15012, 1970 (to be published).

[6]A discussion of variables for $N$ particles detected and a defense of Regge asymptotics of $M$ is found in H. D. I. Abarbanel, Phys. Rev. D (to be published).

[7]G. C. Fox and E. Leader, Phys. Rev. Lett. 18, 628 (1967); R. F. Peierls and T. L. Trueman, Phys. Rev. 134, B1365 (1964).

[8]See the nice pedagogical article of F. Arbab and J. D. Jackson, Phys. Rev. 176, 1796 (1968), for a discussion of the issues.

[9]The only published discussion of such processes appears to be that of S. D. Drell and T. M. Yan, Phys. Rev. Lett. 24, 855 (1970), who consider $N=1$ averaged over the azimuth of the detected hadron. Further, they consider it in a different kinematic region from the one we emphasize here.