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Behavior of Two-Point Correlation Functions at High Temperatures*

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The asymptotic decay of the general two-point correlation function $G_{AB}(\vec{R})$ at high temperatures is analyzed on the basis of the d-dimensional, spin-$\frac{1}{2}$ Ising model in general field H. For $H \neq 0$ the Ornstein-Zernike form $\approx D_A D_B e^{-\kappa R}/R^{(d-1)/2}$ is found for general $\hat{A}$ and $\hat{B}$. However for certain operators, including the energy, the amplitude of the Ornstein-Zernike term vanishes as H^2 and in zero field only the higher order decay $\sim e^{-2\kappa R}/R^d$ remains. The relation to approximate treatments and to critical-point phenomena is discussed briefly.

The asymptotic behavior as $R \to \infty$ of the correlation functions

$$G_{AB}(\vec{R}_1; \vec{R}) = \langle \hat{A}(\vec{R}_1)\hat{B}(\vec{R}_1+\vec{R})\rangle - \langle \hat{A}(\vec{R}_1)\rangle\langle \hat{B}(\vec{R}_1+\vec{R})\rangle \quad (1)$$

is of considerable interest in the general theory of condensed matter.[1] Thus in the scaling/homogeneity description of critical phenomena, the form of the decay is believed to specify the behavior of the basic scaling functions for large argument[1-3]; the asymptotic decay is also a touchstone of the validity of approximate theories. In a d-dimensional system with short-range forces, the Ornstein-Zernike (OZ) and phenomenological Landau-type theories predict[1-3]

$$G_{\Psi\Psi}(\vec{R}_1, \vec{R}) \approx D_{\Psi\Psi} e^{-\kappa R}/R^{(d-1)/2} \text{ as } \vec{R} \to \infty \quad (2)$$

provided $\vec{R}_1$ is far from the boundaries. Here $\hat{\Psi}(\vec{r})$ is the order parameter[4] with conjugate field ζ and $\kappa = \kappa(\zeta, T)$ is the inverse range of correlation. In a lattice system κ and $D_{\Psi\Psi}(\zeta, T)$ will also depend upon the orientation of $\vec{R}$. It can be argued heuristically[5] that the form (2) should remain valid with the same κ, for arbitrary operators $\hat{A}$ and $\hat{B}$. This generalized OZ hypothesis may be tested against the exact results for two-dimensional Ising models[6]: For field $H = 0$ and $T > T_c$ it is (i) confirmed for the spin-spin correlation function $G_{\Psi\Psi}$, but (ii) fails for the energy-energy correlations $G_{\mathcal{E}\mathcal{E}}$, where 2κ replaces κ and the exponent $\frac{1}{2}(d-1)=\frac{1}{2}$ becomes 2. (The hypothesis also fails below T_c, but we discuss that in a subsequent article.) It is important to understand the generality and form of this failure. Thus Jones[5] has suggested that (2) might fail only on special loci in the (ζ, T) plane on which the amplitude D_{AB} happened to vanish.[7] Higher-order terms must presumably then take over but no general suggestion as to their form has yet been made. Unfortunately, except at high fields (or low densities where the virial expansions are proven to converge), no exact information is available for $H \neq 0$, for $d>2$, or for other operators.

In this note we report calculations for classical spin systems[8] (where $\zeta = H$) which go beyond this general hypothesis to yield

$$G_{AB}(\vec{R}_1, \vec{R}) \approx D_A^{(1)}(H,T) D_B^{(1)}(H,T)(e^{-\kappa R}/R^{(d-1)/2})[1+O(R^{-1})] + D_A^{(2)}(H,T) D_B^{(2)}(H,T)(e^{-2\kappa R}/R^d)[1+O(R^{-1})] + O(e^{-3\kappa R}) \quad (3)$$

as $R \to \infty$, provided $\vec{R}_1$ is far from all boundaries. (In a subsequent note, we consider $\vec{R}_1$ close to one or more plane boundaries.) In zero field, the amplitudes $D^{(n)}$ vanish identically unless $\hat{A}$ contains products of m single-spin operators, σ^z, with $m+n$ even: Thus, in particular, $D_{\mathcal{E}}^{(1)} \equiv 0$ so that the behavior of $G_{\mathcal{E}\mathcal{E}}$ for $H=0$ is determined by the second term in (3). All available exact information confirms (3). (For $d=1$, all amplitudes vanish for $n \geq 2$.)

These results have been derived in detail for spin-$\frac{1}{2}$ ferromagnetic Ising models on d-dimensional hypercubic lattices with arbitrary short-range interactions $J(\vec{r})$ in the $(d-1)$-dimensional "layers" and with nearest-neighbor interactions J' between spins in adjacent layers.[9] Our calculations are based on the transfer matrix (or, for a general classical system, integral kernel) which adds a layer to the system.[10-13] Accordingly, we use the decomposition

$$\vec{R} = \vec{r} + Z\vec{e}_z, \quad Z = R_z = \vec{R}\cdot\vec{e}_z, \quad (4)$$

where $\vec{e}_z$ is a unit vector perpendicular to the layers. The formula (3) is established for general H and J' when

$$K = J/k_B T \ll 1, \quad (5)$$

although, with normal interactions, we expect it to remain valid for all fixed $T > T_c$ and sufficiently small H. We have used a more or less straightforward perturbation procedure for the eigenvalues λ_j and eigenvectors $|j\rangle$ of the transfer matrix $\overleftrightarrow{K}$ based upon a decomposition of $\overleftrightarrow{K}$ corresponding to an unperturbed system[13] consisting of N uncoupled linear chains. The expansion parameter K couples the chains together into layers of N spins.

If the eigenvalues are written $\lambda_j = \exp(-aE_j)$ where a is the lattice spacing, we find that the E_j can be interpreted as the energy levels of a quantal many-body lattice system (or discrete field theory) of N sites in $d' = d-1$ dimensions. The corresponding "particles" have infinitely repulsive hard cores which gives the many-particle wave functions a fermionlike character, even though the field operators $\Psi(\vec{r})$ and $\Psi^\dagger(\vec{r})$ (which effectively flip the eigenstates of the single chains) commute on different sites. The vacuum state $|0\rangle$ of energy $E_0(H,T)$ determines the largest eigenvalue λ_0 and hence all the thermodynamics.[10-13] In zero field $|0\rangle$ is even under the operator $\mathcal{S}$, corresponding to total spin inversion.

Immediately above the vacuum lies a band of N single-particle states with excitation energy[8]

$$\omega_1(\vec{q}; H, T) = \kappa(H,T) + \epsilon(H,T) q^2 + O(q^4), \quad (6)$$

where the d'-dimensional wave vector $\vec{q}$ runs over the Brillouin zone of the layer lattice. When $H=0$ the corresponding eigenvectors $|1;\vec{q}\rangle$ are odd under $\mathcal{S}$. The "energy gap" has the zero-field expansion

$$a\kappa(0,T) = \ln\coth K' - 2d'K - 2d'(d-2)(\cosh 2K')K^2 + O(K^3) \quad (7)$$

for the nearest-neighbor model, while for general H,

$$a\kappa(H,T) = \ln(\mu_+/\mu_-) - 2d'[\Sigma_{++}(\Sigma_{--} - \Sigma_{++}) + |\Sigma_{+-}|^2]K + O(K^2), \quad (8)$$

where $\mu_\pm(H,K')$ are the larger and smaller eigenvalues for a single (uncoupled) linear chain and

$$\Sigma_{++}, \Sigma_{--} = O(H)$$

and

$$\Sigma_{+-}, \Sigma_{-+} = 1 + O(H^2) \tag{9}$$

are the corresponding single-spin matrix elements $\langle \pm|\sigma^z|\pm\rangle$. The "stiffness" parameter is

$$\epsilon(H,T) = a|\Sigma_{+-}|^2 K + O(K^2) = aK + 2d'aK^2\cosh 2K' + O(K^3, H^2). \tag{10}$$

Above the single-particle band lies a two-particle band with $\frac{1}{2}N(N-1)$ states.[8] The higher levels are grouped into n-particle bands ($n = 2, 3, \cdots$) of $\binom{N}{n}$ states with excitation energy

$$\omega_n(\vec{q}_1, \cdots, \vec{q}_n) = \sum_{i=1}^{n} \omega_1(\vec{q}_i) + O(K^2) = n\kappa(H,T) + \epsilon(H,T)\sum_{i=1}^{n} q_i^2 + \cdots. \tag{11}$$

When $H = 0$ the eigenvectors $|n\{q\}\rangle$ have parity $(-)^n$ under $\mathcal{S}$.

The standard transfer-matrix expressions[10-12] for G_{AB} in terms of the matrix elements and the powers $(\lambda_j/\lambda_0)^{|Z|/a}$ now reduces straightforwardly to a sum over contributions from successive bands $n \geq 1$, namely,

$$G_{AB}^{(n)}(\vec{R}_1, \vec{R}) = [(a/2\pi)^{nd'}/n!] \int d\vec{q}_1 \cdots \int d\vec{q}_n M_A^{(n)}(\vec{r}_1; \{\vec{q}\}) M_B^{\dagger(n)}(\vec{r}_1 + \vec{r}; \{\vec{q}\}) \exp[-|Z|\omega_n(\vec{q}_1 \cdots \vec{q}_n)], \tag{12}$$

where we use the decomposition (4), $Z_1 \approx \infty$, and

$$M_A^{(n)}(\vec{r}; \{\vec{q}\}) = \lim_{N\to\infty} N^{n/2}\langle 0|\tilde{A}(\vec{r})|n\{\vec{q}\}\rangle, \tag{13}$$

in which $\tilde{A}(\vec{r})$ denotes the layer operator representing the original bulk operator $\hat{A}(\vec{R})$, while the dagger in (12) implies the conjugate matrix element. It is clear from (11) and (12) that $G_{AB}^{(n)}$ is of order $e^{-n\kappa|Z|}$ as $|Z|$ and $R \to \infty$. This identifies $\kappa(H,T)$ as the universal inverse correlation length (for the z direction). The formulas (7) for κ may thus be checked against Onsager's exact result[11,12] for $d = 2$ and the exact expansion[12] for $d = 3$. The symmetry properties under spin inversion imply the $H = 0$ "selection rules" for $M_A^{(n)}$ (and, hence, $D_A^{(n)}$) stated after (3): Specifically we find $M_\Psi^{(1)} \propto \Sigma_{+-}$ and $M_{\mathcal{E}}^{(1)} \propto \Sigma_{++}\Sigma_{+-} = O(H)$ [see (9)].

In order to evaluate the integrals in (12) asymptotically for large Z it is crucial to know the behavior of the $M^{(n)}$ for small q: We find generally that $M_A^{(1)}(\vec{q}) \to m_A^{(1)} \neq 0$ for $q \to 0$. Hence as $R \to \infty$ in the simplest case $\vec{r} = 0$, $Z = R$, we have

$$G_{AB}^{(1)}(\vec{R}_1, \vec{R}) \approx m_A^{(1)} m_B^{(2)} (a/2\pi)^{d'} e^{-\kappa R} \int d\vec{q}\, e^{-\epsilon R q^2} \approx m_A^{(1)} m_B^{(1)} (a^2/4\pi\epsilon)^{d'/2} e^{-\kappa R}/R^{(d-1)/2}. \tag{14}$$

This confirms the OZ behavior of the first term in (3). For $d = 2$ and $H = 0$ the amplitude factor checks against the exact Ising results to the appropriate order in k. When $\vec{r} \neq 0$ the matrix element introduces a factor $e^{i\vec{q}\cdot\vec{r}}$ which then yields the directional dependence of κ in leading order for near-axis directions.

For the second band matrix elements we find

$$M^{(2)}(\vec{q}_1, \vec{q}_2) \approx \dot{m} \sum_{\vec{\delta}} [\exp(i\vec{q}_1 \cdot \vec{\delta}) - \exp(i\vec{q}_2 \cdot \vec{\delta})], \tag{15}$$

where $\vec{\delta}$ runs over the nearest-neighbor layer vectors, which for small $\vec{q}$ introduces a factor $(\vec{q}_1 - \vec{q}_2)^2$ into the integrals. This crucial feature arises directly from the hard-core or fermionlike character of the particles interacting in the $n = 2$ band. We hence have (as $\vec{R} = Z\vec{e}_z \to \infty$)

$$G_{AB}^{(2)}(\vec{R}_1, \vec{R}) \approx \tfrac{1}{2}\dot{m}_A^{(2)}\dot{m}_B^{(2)}(a/2\pi)^{2d'} e^{-2\kappa R} I_2(R),$$

$$I_2(R) = \int d\vec{q}_1 \int d\vec{q}_2 (\vec{q}_1 - \vec{q}_2)^2 \exp[-\epsilon R(\vec{q}_1^{\,2} + \vec{q}_2^{\,2})] = \pi^{d'}(d-1)\epsilon^{-d}/R^d, \tag{16}$$

which completes the derivation of (3). Higher band contributions could likewise be computed but will be asymptotically relevant only for those special operators and loci for which <u>all</u> the lower-order amplitudes vanish.

We note that without the factor $(\vec{q}_1 - \vec{q}_2)^2$ in (16) we would obtain merely

$$G^{(2)}(\vec{R})_{\text{RPA}} \propto [G^{(1)}(\vec{R})]^2 \sim e^{-2\kappa R}/R^{d-1}, \tag{17}$$

which is the normal prediction of the random phase and decoupling treatments. This approximation can, in turn, be re-expressed in Fourier space as

$$\hat{G}^{(2)}(\vec{k})_{\rm RPA} = (2\pi)^{-d} \int d\vec{q}\, W(\vec{k},\vec{q})\hat{G}^{(1)}(\vec{q})\hat{G}^{(1)}(\vec{k}-\vec{q}), \tag{18}$$

if it is assumed that $W(\vec{k},\vec{q})$ is constant or effectively so. Polyakov,[14] noticing that (18) has a divergent singularity at $k=\pm 2i\kappa$, performed a resummation of bubbles which leads essentially to the screened singularity

$$\hat{G}^{(2)}(\vec{k})_{\rm Pol} \approx {\rm const} + 1/\hat{G}^{(2)}(\vec{k})_{\rm RPA}. \tag{19}$$

On inverting this for $d=3$ he obtained

$$G^{(2)}(\vec{R})_{\rm Pol} \sim e^{-2\kappa R}/R^2(\ln R)^2 \tag{20}$$

which disagrees with our result. For $d=2$, one obtains this way the correct Ising form, but we believe the amplitudes will be incorrect. For $d\geqslant 4$ one finds $\hat{G}^{(2)}(\vec{k})_{\rm RPA}$ is finite at the nearest singularity and no screening occurs, so Polyakov's argument would still yield the RPA form (17). On the other hand, these results can be corrected (without need for further resummation) simply by taking

$$W(\vec{k},\vec{q}) \propto (2\vec{k}-\vec{q})^2 \tag{21}$$

which reflects the $(\vec{q}_1-\vec{q}_2)^2$ behavior of (15). This yields

$$G_W^{(2)}(\vec{R}) \propto -[G^{(1)}]^2\nabla^2 \ln G^{(1)} \propto (d-1)\kappa e^{-2\kappa R}/R^d \tag{22}$$

which agrees with our result. At present, however, the replacement (21) has not been justified although a sufficiently careful diagrammatic analysis might do so.

Finally we remark that if the form (16) holds for the zero-field energy-energy correlation function up to and at the critical point where $\kappa \sim (T-T_c)^\nu$ it would imply a logarithmic specific-heat anomaly for all d. However there are excellent reasons[2,3] for believing this is wrong; even in $d=2$ (where the specific heat is logarithmic) the amplitude for $R\to\infty$ with $T>T_c$ does not match that for $R\to\infty$ at $T=T_c$. More generally, however, near a critical point in a system such as gas/liquid, which has no exact symmetry about the critical field ζ_c, the most appropriate continuation of the vapor-pressure curve $\zeta_\sigma(T)$ to $T>T_c$ might be the "fluctuation" locus defined by $D_{\mathcal{E}}^{(1)}(\zeta,T)\equiv 0$. [In default of a more basic choice the critical isochore $\Psi(\zeta,T)=\Psi_c$ is often used.] On this fluctuation locus the order and energy fluctuations would be uncoupled in leading order as they clearly are in the symmetric systems on $\zeta=\zeta_c$. Only on this locus would the energy fluctuations diverge "weakly," that is, less strongly than the order fluctuations.

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Electromagnetic Instability in Counterstreaming Plasmas*

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It is shown that in a system of two counterstreaming plasmas, the growth rates of the ordinary-mode electromagnetic instability can be several times the plasma frequency even for nonrelativistic velocities. The electromagnetic instability therefore grows faster than the coexisting electrostatic instabilities of the two-stream type, contrary to the common belief that electromagnetic instabilities are only of interest if electrostatic ones are absent.

It is often stated in the plasma physics literature that for nonrelativistic plasmas, electromagnetic instabilities are much more slowly growing than the electrostatic ones and are therefore only of interest if the latter are absent.[1-4] In particular, for the well-known system of two plasmas counterstreaming along an external magnetic field, current theories show that the familiar electrostatic two-stream instability, which has maximum growth rate of the order of the plasma frequency ω_p,[5] is much more important than the recently discussed ordinary-mode electromagnetic instability,[6] which according to cold-plasma theory has maximum growth rate of the order of $(u/c)\omega_p$, where $2u$ is the relative streaming velocity and c is the velocity of light. The purpose of this Letter is to show that when the Vlasov equation is used to analyze the ordinary-mode electromagnetic instability in a system of two colliding plasma streams, in each of which the electrons and ions are streaming at the same velocity, it yields growth rates which can be several times the plasma frequency even for nonrelativistic velocities. This work therefore offers an example that an electromagnetic instability can be the dominant instability even though there are electrostatic instabilities coexisting in the plasma.

The system under study consists of two colliding plasma streams of infinite extent, each with density $N/2$. The electrons and ions of one plasma are streaming with velocity u along the direction of a static and uniform magnetic field B_0, while those of the second plasma are streaming with equal velocity in the opposite direction. The equilibrium velocity distribution functions are of the form

$$F_{0e} = \frac{N}{2}\frac{\exp(-v_\perp^2/V_{\perp e}^2)}{\pi^{3/2}V_{\perp e}^2 V_{\parallel e}}\left\{\exp\left[-\frac{(v_\parallel - u)^2}{V_{\parallel e}^2}\right] + \exp\left[-\frac{(v_\parallel + u)^2}{V_{\parallel e}^2}\right]\right\},$$
$$F_{0i} = \frac{N}{2}\frac{\exp(-v_\perp^2/V_{\perp i}^2)}{\pi^{3/2}V_{\perp i}^2 V_{\parallel i}}\left\{\exp\left[-\frac{(v_\parallel - u)^2}{V_{\parallel i}^2}\right] + \exp\left[-\frac{(v_\parallel + u)^2}{V_{\parallel i}^2}\right]\right\}, \quad (1)$$

where subscripts e and i denote electrons and ions, respectively; u is the directional velocity and $V_\perp = (2T_\perp/m)^{1/2}$, $V_\parallel = (2T_\parallel/m)^{1/2}$ are thermal velocities perpendicular and parallel to B_0. ($T_\perp$ and $T_\parallel$ are nonisotropic temperatures.)

Colliding plasma streams described by Eq. (1) have been studied by Stringer[7] with regard to electrostatic instabilities propagating along the direction of streaming, and by Parker[8] in the cold plasma limit. As noted by Parker,[8] plasmas of this sort are encountered in naturally occurring phenomena