4(a) and 4(b), and the agreement with the data points demonstrates that the coupling takes place over the volume excited by the resonant TM mode.

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Magnetization Cooling and Magnetosolidification of ^{3}He

R. E. Walstedt, L. R. Walker, and C. M. Varma
Bell Telephone Laboratories, Murray Hill, New Jersey 07974
(Received 3 February 1971)

A spin-wave calculation of the thermodynamic properties of antiferromagnetically ordered solid ^{3}He in a magnetic field has been carried out. A cooling effect with isentropic magnetization is predicted for fields near the $(T=0)$ critical value $H_c = 2zJI/\gamma_n\hbar$ ($\cong$74 kG). In a second calculation a field $H \sim H_c$ is found to displace the melting curve $P(T)$ to significantly lower pressures; it follows that a two-phase mixture of ^{3}He can be isentropically solidified (and thus cooled) by application of a field from starting temperatures $T \sim 5$ mdeg K as a useful adjunct to the Pomeranchuk cooling method.

Some time ago experiments were reported in which the Pomeranchuk[1] or adiabatic-compression (AC) cooling effect was employed to bring specimens of solid ^{3}He down to the millidegree temperature region.[2,3] It is of interest to examine the effect of an applied magnetic field on this cooling process. This was discussed by Goldstein in a recent Letter[4] on the basis of a molecular-field calculation of what is, in effect, an Ising model[5]; it was concluded that an applied field of correct magnitude would, in principle, disorder the ^{3}He nuclei in such a way as to reduce the theoretical lower limit of AC cooling to 0°K. We have re-examined this question on the realistic assmuption of isotropic exchange coupling[6] in both the molecular-field and spin-wave approximations. Our findings are as follows: (a) There is now no disordering effect with field H in the molecular-field approximation[5] such as that reported in Ref. 4, i.e., $(\partial S/\partial H)|_T \leq 0$ where S is the entropy. (b) However, viewed in terms of the spin-wave theory of antiferromagnetically ordered solid ^{3}He, it appears that a disordering effect does exist for applied fields H in the vicinity of the $(T=0)$ critical value $H_c = 2zIJ/\gamma_n\hbar$, where z is the coordination number, I the spin quantum number, γ_n the gyromagnetic ratio, and J the isotropic exchange coupling, where the coupling of nearest-neighbor nuclei is taken to be $J\vec{I}_i\cdot\vec{I}_j$. In particular at $H=H_c$ the intersection temperature of the liquid- and solid-^{3}He entropy curves, which constitutes the theoretical lower limit of AC cooling, is found to be reduced by two orders of magnitude from its value at $H=0$. (c) In a second calculation we have examined the effect of a magnetic field $H \sim H_c$ on the melting curve $P(T)$. The field is found to displace $P(T)$ to lower pressures, suggesting that a two-phase mixture can be solidified from a starting temperature $T \lesssim 5$ mdeg K by applying a field at constant pressure. This may lead to a substantial reduction of the frictional losses which occur at the low-temperature end of AC cooling.

In examining spin-wave effects in bcc solid ^{3}He we assume isotropic, nearest-neighbor (only) exchange coupling.[7] In zero applied field the anti-ferromagnetic spin-wave dispersion has the well-known form $\epsilon_k \propto k$ for $ka \ll 1$ (a = bcc lattice constant), yielding a leading term in the entropy[8] $S_{\rm anti}/R \cong 0.0548(k_{\rm B}T/J)^3$. This and subsequent spin-wave results are valid for $k_{\rm B}T \ll zJI$. The behavior of spin waves in the spin-flop configuration is discussed by Keffer.[9] As the field H is increased, one branch of spin waves is lost to thermal excitation for $\gamma_n \hbar H > k_{\rm B}T$. When H reaches the critical value $H_c(T)$ for the spin-flop to paramagnetic-phase transition,[10] the spin-wave dispersion becomes characteristically ferromagnetic with $\epsilon_k \propto k^2$ for $ka \ll 1$. The corresponding entropy expansion gives $S_{\rm ferro}/R = 0.1065(k_{\rm B}T/J)^{3/2}$, a much more rapidly increasing function of T than $S_{\rm anti}$ for $k_{\rm B}T/J \ll 1$. Consequently, the condition $S_{\rm ferro} > S_{\rm anti}$ holds over most of the ordered region.

In terms of these results we envision the following behavior of the temperature $T(H)_S$ on isentropic magnetization beginning at some initial temperature T_i. As H is increased from zero, T first increases by a factor $2^{3/2}$ owing to the loss of one branch of excitations. As $H \to H_c$, "magnetization cooling" occurs[11] with a temperature minimum T_f corresponding to $S_{\rm ferro}(T_f) = S_{\rm anti}(T_i)$. This condition yields

$$T_f = 0.642 T_i (k_{\rm B}T_i/J), \qquad (1)$$

where it is seen that the cooling effect increases on lowering T_i. Here it is worth noting that the specific heat, which is of considerable experimental importance, decreases by just a factor of 2 on reaching T_f [Eq. (1)] in contrast to the expected T^3 variation in zero field. As H is increased beyond H_c, a gap $\Delta\epsilon = \gamma_n \hbar (H - H_c)$ develops in the magnon energy, giving rise to a rapid increase in T again.

The temperature minimum at $H = H_c$ is quite sharp; and, moreover, the slope $(\partial T/\partial H)|_S$ is discontinuous at $H = H_c$, giving the minimum a cusplike quality. The latter point can be easily demonstrated using the magnetocaloric equation $(\partial T/\partial H)|_s = -(T/C_H)[\partial M/\partial T]_H$. Evaluation of $[\partial M/\partial T]_H$ for $H < H_c$ and $H > H_c$ in terms of spin-wave theory shows it to be >0 and <0, respectively, as $H \to H_c$ from either side. The width ΔH of the temperature minimum (on the low-field side) can be estimated to be $\Delta H \sim 0.05 H_c (k_{\rm B}T_f/J)^{1/2}$. Thus ΔH is on the order of 1% of H_c and varies quite slowly with T_f.

The spin-wave results are illustrated with reference to the AC cooling technique in the entropy-versus-temperature diagram of Fig. 1.

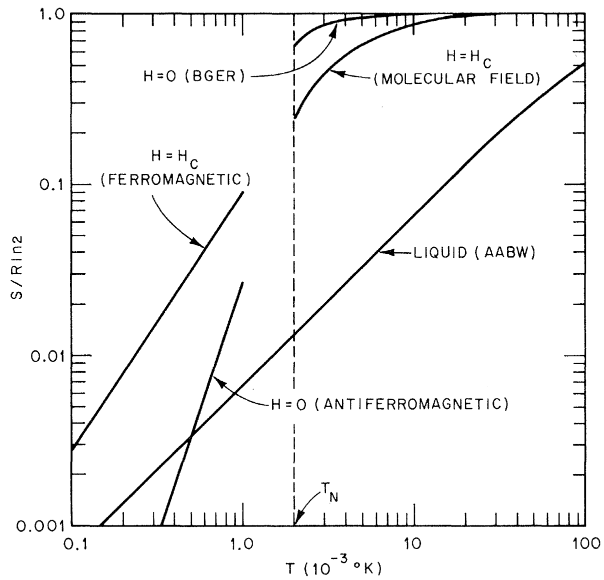

FIG. 1. Log-log plot of entropy versus temperature for liquid and solid ^{3}He. $S_{\rm liq}$ is taken from Ref. 13. For solid ^{3}He, $S_{\rm anti}$ ($H=0$) and $S_{\rm ferro}$ ($H=H_c$) from the text are plotted at $T \leq T_{\rm N}/2$. For $T \geq T_{\rm N}$ the series expansion results from Ref. 14 are plotted for $H=0$, the molecular-field theory results for $H=H_c$.

The solid-^{3}He entropy expressions $S_{\rm anti}(T)$ and $S_{\rm ferro}(T)$ given above have been plotted for $T \leq \frac{1}{2}T_N$ on a log-log scale, assuming the melting-curve exchange value $J/k_{\rm B} = 1.44$ mdeg K given by Panczyk and Adams.[12] The corresponding critical field value is $H_c = 74.0$ kG. Although the entropy curves in Fig. 1 are incomplete, it is evident that the cooling effect with field [Eq. (1)] is only appreciable below $S/R\ln 2 \sim 0.1$. Also plotted is $S_{\rm liq}/R \cong \gamma T$, where[13] $\gamma \cong 4.6$°K^{-1} is field independent for a Fermi liquid. For $T > T_{\rm N}$, curves of solid-^{3}He entropy are plotted for $H = 0$ using the high-temperature series-expansion results of Baker *et al.*[14] and for $H = H_c$ using molecular-field theory. The estimated Néel point is[14] $T_{\rm N} = 1.38 J/k_{\rm B} \cong 2.0$ mdeg K. The intersection point of $S_{\rm liq}(T)$ and $S_{\rm anti}(T)$ (i.e., for $H = 0$) is seen in Fig. 1 to occur at $T \cong 0.5$ mdeg K, giving the theoretical lower limit of AC cooling in zero field. The corresponding intersection of $S_{\rm ferro}(T)$ with $S_{\rm liq}(T)$ is about two orders of magnitude lower at $T \cong 5.6$ μdeg K.

Turning our attention to the ^{3}He-melting curve $P_H(T)$, we extrapolate the experimental data of Scribner, Panczyk, and Adams[15] to lower temperatures by integrating the Clausius-Clapeyron equation $dP_H/dT = -[S_{\rm sol}(H,T) - S_{\rm liq}(T)]/\Delta V_m$. For

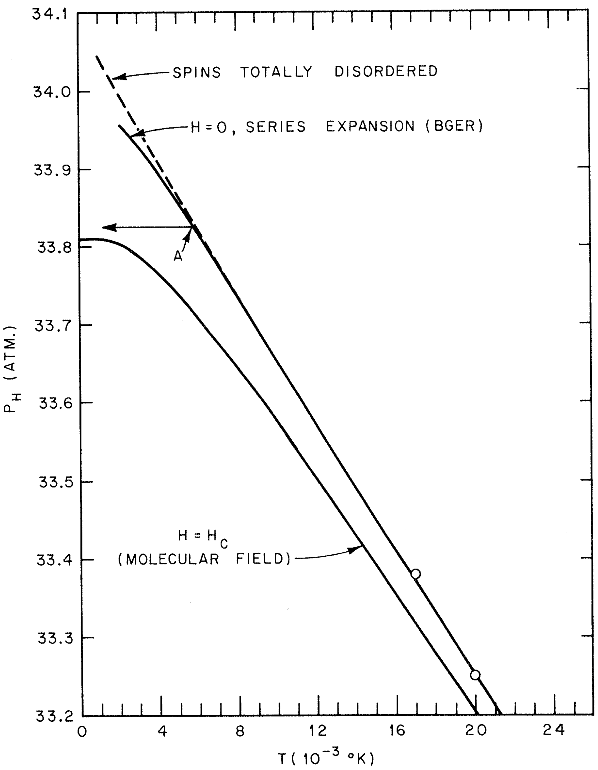

FIG. 2. Extrapolation of the melting-curve results of Scribner, Panczyk, and Adams (Ref. 15) to lower temperatures, arbitrarily fitting the $H=0$ curve to their 20-mdeg-K data point (their 17-mdeg-K point is also shown). The dashed curve represents the hypothetical condition that the spins remain totally disordered. The $H=0$ and $H=H_c$ values are calculated relative to the dashed curve according to Eq. (2) of the text.

the volume change on melting we take[15] $\Delta V_m = 1.272\ \text{cm}^3/\text{mole}$, neglecting its temperature dependence. The calculated curves are shown in Fig. 2 and are obtained as follows. The uppermost (dashed) curve corresponds to totally disordered nuclear spins and is obtained taking $S_{\text{sol}}(T) = R\ln 2$ and $S_{\text{liq}}(T) = R\gamma T$, with γ given above. The $H=0$ and $H=H_c$ curves are calculated with the equation

$$\Delta P_H(T) \cong -\Delta V_m^{-1} \int_T^\infty [R\ln 2 - S_{\text{sol}}(H,T)]dT, \quad (2)$$

giving the pressure difference $\Delta P_H(T)$ relative to the dashed curve. For $H=0$, $S_{\text{sol}}(0,T)$ is obtained from the high-temperature series expansion for the specific heat of an isotropic, spin-$\frac{1}{2}$ antiferromagnet given by Baker *et al.*[14] For $H=H_c$, $S_{\text{sol}}(H_c,T)$ is calculated from molecular-field theory.[16] Such a theory is known to be very approximate; however, we note that for $T=0$ the integral in Eq. (2) is simply the free energy of the ground state and is given *exactly* by molecular-field theory: $F(0) = -N[\gamma\hbar HI - \frac{1}{2}zJI^2]$. This formula is correct for $H \geq H_c$. The corresponding $T=0$ pressure decrease $\Delta P_{H_c}(0)$ is 0.279 atm and should be quite reliable. In addition, the quantity $\Delta P_{H_c}(T)$ has been calculated with the Green-function random-phase-approximation method, where the magnon dispersion is renormalized according to the magnetization. The latter results agree with the molecular-field curve of Fig. 2 for $T\to 0$ and $T \gg J/k_B$, and lie below the curve at intermediate temperatures by an amount less than 0.007 atm. Thus the agreement is excellent.

Figure 2 reveals an interesting possibility regarding the cooling of liquid-solid mixtures of ^{3}He, namely that the last few millidegrees of cooling, which are difficult to carry out isentropically by compression, can be accomplished *by applying a magnetic field.* Starting at point A with an equilibrium two-phase mixture in zero field, a field applied at constant pressure will cool the mixture until the liquid is exhausted, after which the discussion of $T(H)_S$ given above for the solid will apply. Such a scheme, when combined with the critical-field effect described above, may provide a feasible means of bringing ^{3}He into the submillidegree temperature region.

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Time-Resolved Light-Scattering Measurements of the Spectrum of Turbulence within a High-β Collisionless Shock Wave

M. Keilhacker and K.-H. Steuer
Max-Planck-Institut für Plasmaphysik—EURATOM Association, Garching bei München, Germany
(Received 9 February 1971)

Strong current-driven turbulence is observed in high-β collisionless shock waves under $T_e \sim T_i$ and $v_d \ll v_e$ conditions. The level and the frequency and wave-number spectra of this turbulence are measured by scattering light from the shock. The turbulence is probably due to an electron-cyclotron drift instability.

Previous experiments[1,2] on collisionless shock waves have established that the electron heating observed in low-Mach-number shocks ($M<M_{crit}$, resistive shocks) implies a resistivity in the shock front which is about two orders of magnitude larger than the "classical" value based on binary Coulomb collisions. It is generally presumed that this "anomalous" resistivity is due to scattering of the electrons by suprathermal electrostatic fluctuations arising from some microinstability.

This Letter deals with time-resolved measurements of the level[3] and spectrum[4] of suprathermal fluctuations in a collisionless shock wave. The quasistationary shock, which propagates perpendicular to a magnetic field B_1 with Mach number $M=2.5$, is produced by radial compression of an initial deuterium plasma by a fast θ pinch.[5] The initial plasma conditions[2] ($n_{e1}=4\times10^{14}$ cm^{-3}, $T_{e1}=4$ eV, $T_{i1}=18$ eV,[6] $B_1=700$ G) differ from other shock experiments insofar as the plasma β is high ($\beta_1\sim0.7$) and $T_{e1}/T_{i1}<1$. As a consequence of the latter the electron temperature T_e does not substantially exceed the ion temperature T_i during the shock-heating process ($T_{e2}=110$ eV from 90° laser scattering, $T_{i2}\sim70$ eV from Rankine-Hugoniot relations). This makes it unlikely that the microturbulence causing the observed collisionless electron heating[2] results from an ion acoustic instability as proposed for other experiments[7,8] in which $T_e/T_i\gg1$.

Laser scattering experiments.—In the laser scattering experiments, described in detail elsewhere,[3,9] the light pulse of a 500-MW ruby laser is timed to hit the shock wave while it traverses the beam. The pulse width (12 nsec) and divergence of the laser beam make it possible to resolve the structure of the shock wave.

The light scattered in the forward direction $\theta=2.5°$-$6°$ is detected by a photomultiplier, either directly or after spectral resolution. The geometry of incident and scattered-light paths is such that the scattering plasma waves have a wave vector $\vec{k}$—with $|\vec{k}|$ of order $1/D$ ($\alpha\sim1/|\vec{k}|D\geq1$), D being the Debye length—collinear with the azimuthal current in the shock front. Simultaneously with the forward-scattering measurements, the density and electron temperature in the shock are determined by 90° scattering ($\alpha\ll1$) using a multichannel detector arrangement.

Measured level of fluctuations.—Figure 1 shows the measured total level of density fluctuations n_eS as a function of time for a scattering angle $\theta=2.5°\pm0.5°$ ($|\vec{k}|=4\times10^3$ cm^{-1}). Time is measured from the beginning of the θ-pinch discharge. The experimental points denoted by open cir-