visible photons have been considered.[7,8] The study of such nonlinearities will provide a valuable probe of material properties.

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Observation of Direct Nonlinear Coupling of Electromagnetic Waves and Electrostatic Waves in a Plasma*

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Experiments demonstrating resonant nonlinear electromagnetic wave excitation of electron-plasma and ion-acoustic waves in a plasma column are described. Direct verification of the wave-number selection rule and indirect measurements of the mode power levels are shown.

We report the observation of direct second-order nonlinear coupling among electromagnetic (EM) and electrostatic (ES) waves in a potassium plasma column in a Q machine.[1] By the term "direct" we mean that the exciting waves and the interacting waves are the same. The coupling takes place in the volume of the column and not in the boundaries or in a locally resonant layer. The measurements are made under conditions that enable the unambiguous determination of the wavelength, the frequency, the polarization, the field configuration, and the power levels of the interacting waves. Earlier observations of wave-wave interactions in a plasma have been made in a nonuniform volume and have involved electrostatic waves exclusively. The original work of Stern and Tzoar[2] used the Tonks-Dattner resonances, externally driven by EM waves. Later work on Bernstein modes contains a verification of the wave-number selection rules for waves propagating across a plasma column.[3] Plasma-like interactions between electron-stream and slow waveguide waves have also been observed.[4]

It is well known that the lowest- or second-order nonlinearity in the charged-particle response resonantly couples plasma waves, if their fields and currents are not orthogonal, when the vector selection rules

$$\omega_3=\omega_1\pm\omega_2, \quad \vec{k}_3=\vec{k}_1\pm\vec{k}_2, \tag{1}$$

are satisfied in the plasma volume.[5,6] The kinematic character of the interaction depends upon the energy densities and group velocities.[7]

As shown schematically in Fig. 1, the Q machine produces a plasma column with a typical density of 10^{10} cm^{-3} in an axial magnetic field of 5 kg. A cylindrical microwave cavity, 35 cm long by 10 cm in diameter, which can be excited in two resonant modes simultaneously, surrounds the 2-cm-diam plasma column. Axially movable coupling probes detect electron and ion waves and make possible the determination of wavelength. The interferometer system used to detect the interactions is similar, with the addition of a nonlinearly generated reference, to that described in greater detail elsewhere.[8] The phase coherence of the detected signal insures the observation of the desired nonlinear output at the difference frequency.

The electromagnetic and electron-plasma body waves which propagate in this configuration are the normal modes[9] of a cylindrical waveguide, partially filled with a plasma, in a strong magnetic field. The axial-field variation of the TM_{lmn} transverse-magnetic mode is given by

$$(E_z)_{lmn}=\hat{E}_0 J_l(T_m r)\exp[i(\omega_n t-k_n z-l\theta)], \tag{2}$$

in the standard waveguide notation.[10] Most of the experiments were performed with a large-amplitude TM-wave "pump," thereby insuring that the

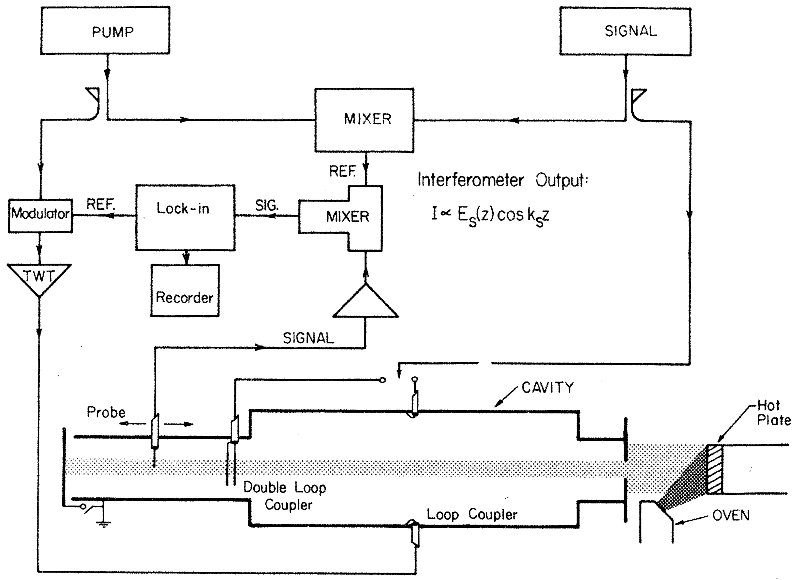


FIG. 1. The experimental configuration used for the study of nonlinear interactions of electromagnetic waves with electrostatic column waves.

dominant interaction is driven by the current density excited by axial electric fields. The coupling is weak in the sense that the wave energy in the plasma is less than the thermal level.

The nonlinear interactions possible in this system are identified by means of dispersion diagrams, one set of which is shown in Fig. 2. The parallelogram in the figure is a convenient way of establishing the resonance conditions, Eqs. (1). The nonlinear spatial/temporal harmonic,[7] shown as a dashed line in Fig. 2, is a construction that is especially useful for visualizing the interactions predicted by the relations of Eqs.

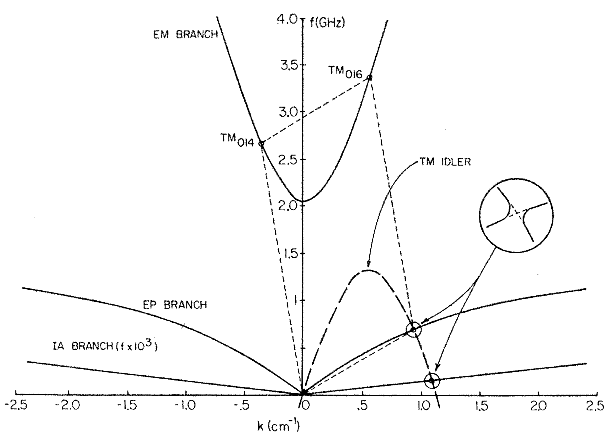
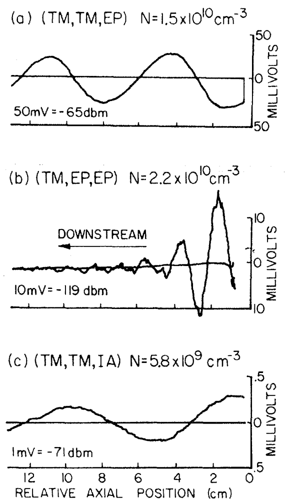


FIG. 2. Linear-dispersion diagram for the waves studied, with the ion-acoustic branch exaggerated in frequency by a factor of 10^3. The dashed-line constructions are used to identify nonlinear wave interactions, as described in the text.

(1). The nonlinear, or "idler," mode is required by a generalization of the Floquet-Bloch theorem, and represents excitation stimulated by the finite-amplitude, propagating pump wave. The kinematic character of the two weak interactions of the waveguide-mode idler with the electron-plasma and (frequency-exaggerated) ion-acoustic modes is shown in the circle. The spatially resonant mixing indicated can, when the coupling is stronger than the dissipation, exhibit absolute instability[4] (now called "decay"[11]).

A typical measurement consists of exciting the cavity with the finite-amplitude pump (ω_3) and with the low-level input signal (ω_1) and then, for a plasma wave, detecting the resulting output mode at ω_2 with the movable probe. The wave amplitude and phase, as referred to the coherent nonlinearly generated reference, are plotted on a recorder as a function of the axial position of the probe. The power levels in the modes are determined by auxiliary linear measurements of the probe coupling coefficients. Three representative recorder outputs, taken at the interaction resonances of Eqs. (1) and illustrative of the basic coupling processes that we have studied, are shown in Fig. 3. The measured power levels and propagation characteristics of each mode are given in the legend. In particular, the accuracy of the $\vec{k}$ selection rule can be seen.

The first trace shows the electron-plasma (EP) wave produced by two TM waves in the upper interaction shown in Fig. 2. This coupling is very strong and allows the excitation and detection



	MODE	f(GHz)	k(cm^{-1})	P(dbm)
PUMP	TM_{016}	3.28	.57	37
SIGNAL	TM_{014}	2.66	.39	13
RESULT	EP_{01}	.63	.99	-68

PUMP	TM_{012}	2.23	.24	30
SIGNAL	EP_{01}	1.14	3.12	-19
RESULT	EP'_{01}	1.09	2.88	-115

PUMP	TM_{014}	2.59	.39	23
SIGNAL	TM'_{014}	2.59^-	$.39^-$	3
RESULT	IA	4×10^{-5}	.70	-78

FIG. 3. Chart records of interferometer measurements on the nonlinearly generated waves. Dispersion and power data are given in the legends.

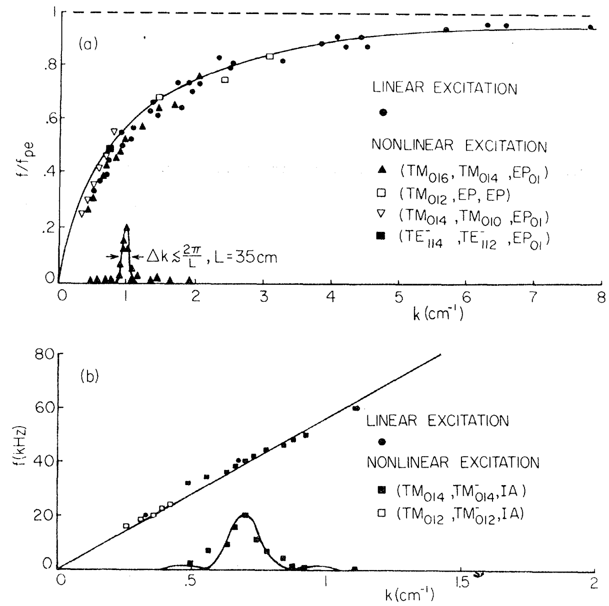


FIG. 4. Comparison of dispersion data with theory. The data were obtained from (a) interferometer measurements of both linear and nonlinear excitation of electron-plasma waves, and (b) ion-acoustic waves including drift. The power-coupling profile measurements are described in the text [Eq. (3)], and are indicative of the finite length of the interaction volume.

of electron waves in the absence of immersed probes. The relatively high TM-mode amplitudes do not disturb the plasma since only a small fraction of the energy storage is in plasma motion. We have also observed the circularly polarized TE-mode version of this interaction. In this case a further relation, $l_3 = l_1 \pm l_2$ [l as defined in Eq. (2), for example], which is equivalent to the conservation of angular momentum, was verified.

The second recording of Fig. 3 shows the electron-wave output due to the coupling between an oppositely moving EP wave and a waveguide mode near cutoff. This interaction is related to its complement (EP + EP → TM, which we have also observed) which allows suprathermal electrostatic energy to radiate from high-temperature plasmas and is the basic mechanism for radiation from type-II and type-III solar radio bursts.[5,12]

The final trace of Fig. 3 demonstrates the ion-acoustic (IA) wave output of a TM-TM coupling of the kind shown by the lower circle of Fig. 2. The oppositely moving TM waves, one of which is audio modulated, are both excited within one cavity resonance which is wider than the ion-wave spectrum. The pickup probe for the ion waves is a thin wire extending through the plasma column. The signal is then detected in phase coherence with the original modulation. The maximum output, at the proper wave-number difference, comes at a modulation frequency agreeing with the IA dispersion. The diagnostic capabilities of this interaction have, also, been demonstrated by our observation of Raman scattering from an unstable ion-fluctuation spectrum when the cavity is excited externally at one of the TM resonances.

Dispersion data from these and other nonlinearly generated EP and IA waves are compared with linear measurements in Figs. 4(a) and 4(b). The plasma frequency and ion-acoustic velocity used for the (ω, k) curves are taken from the linear-dispersion measurements and thus include the effects of radial density variations and drifts.

The energy line-shape data shown at the lower edges of Figs. 4(a) and 4(b) are obtained by varying the wave number of one of the interacting modes (by changing the plasma density) while keeping k constant for the other two. The output power in the changing mode is then plotted versus the departure in k from the exact δ-function synchronism described by Eq. (1). For a finite-length interaction ($L \simeq 35$ cm in this case), the spatial Fourier analysis, which results in the wave-number representation, produces a power-coupling profile with the form

$$P_0(\Delta k) = \left[\frac{\sin\frac{1}{2}\Delta k L}{\frac{1}{2}\Delta k L}\right]^2, \tag{3}$$

where $\Delta k = k_3 \mp k_2 - k_1$ is the uncertainty in Eq. (1). This function is plotted (lower curves) in Figs.

4(a) and 4(b), and the agreement with the data points demonstrates that the coupling takes place over the volume excited by the resonant TM mode.

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Magnetization Cooling and Magnetosolidification of ^{3}He

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A spin-wave calculation of the thermodynamic properties of antiferromagnetically ordered solid ^{3}He in a magnetic field has been carried out. A cooling effect with isentropic magnetization is predicted for fields near the $(T=0)$ critical value $H_c = 2zJI/\gamma_n\hbar$ ($\cong$74 kG). In a second calculation a field $H \sim H_c$ is found to displace the melting curve $P(T)$ to significantly lower pressures; it follows that a two-phase mixture of ^{3}He can be isentropically solidified (and thus cooled) by application of a field from starting temperatures $T \sim 5$ mdeg K as a useful adjunct to the Pomeranchuk cooling method.

Some time ago experiments were reported in which the Pomeranchuk[1] or adiabatic-compression (AC) cooling effect was employed to bring specimens of solid ^{3}He down to the millidegree temperature region.[2,3] It is of interest to examine the effect of an applied magnetic field on this cooling process. This was discussed by Goldstein in a recent Letter[4] on the basis of a molecular-field calculation of what is, in effect, an Ising model[5]; it was concluded that an applied field of correct magnitude would, in principle, disorder the ^{3}He nuclei in such a way as to reduce the theoretical lower limit of AC cooling to 0°K. We have re-examined this question on the realistic assmuption of isotropic exchange coupling[6] in both the molecular-field and spin-wave approximations. Our findings are as follows: (a) There is now no disordering effect with field H in the molecular-field approximation[5] such as that reported in Ref. 4, i.e., $(\partial S/\partial H)|_T \le 0$ where S is the entropy. (b) However, viewed in terms of the spin-wave theory of antiferromagnetically ordered solid ^{3}He, it appears that a disordering effect does exist for applied fields H in the vicinity of the $(T=0)$ critical value $H_c = 2zIJ/\gamma_n\hbar$, where z is the coordination number, I the spin quantum number, γ_n the gyromagnetic ratio, and J the isotropic exchange coupling, where the coupling of nearest-neighbor nuclei is taken to be $J\vec{I}_i\cdot\vec{I}_j$. In particular at $H=H_c$ the intersection temperature of the liquid- and solid-^{3}He entropy curves, which constitutes the theoretical lower limit of AC cooling, is found to be reduced by two orders of magnitude from its value at $H=0$. (c) In a second calculation we have examined the effect of a magnetic field $H \sim H_c$ on the melting curve $P(T)$. The field is found to displace $P(T)$ to lower pressures, suggesting that a two-phase mixture can be solidified from a starting temperature $T \lesssim 5$ mdeg K by applying a field at constant pressure. This may lead to a substantial reduction of the frictional losses which occur at the low-temperature end of AC cooling.

In examining spin-wave effects in bcc solid ^{3}He we assume isotropic, nearest-neighbor (only) exchange coupling.[7] In zero applied field the anti-