New Regge Phenomenology of Inclusive Reactions*

Chan Hong-Mo,† C. S. Hsue, C. Quigg, and Jiunn-Ming Wang
The Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11790
(Received 5 February 1971)

Inclusive reactions are studied in terms of familiar techniques of Regge phenomenology. Distributions are predicted to approach limiting behavior as $A+Bs^{-1/2}$. The constraints of duality and factorization lead to many interesting relations between different single-particle spectra, all of which can be tested at energies below 20 GeV.

In spite of its inherent weaknesses, the Regge-pole model has proved to be a most useful tool for a qualitative description of the energy dependence of total cross sections for energies up to 30 GeV.[1] Its application to σ_T is based on unitarity which relates σ_T to the imaginary part of the forward elastic amplitude, as illustrated in Fig. 1(a). The Regge behavior of the elastic amplitude then implies the same for the total cross section. Similar reasoning,[2] when applied to single-particle distributions in inclusive reactions

$$a+b\rightarrow c+\text{anything}, \tag{1}$$

leads to a relation between these and forward elastic three-body amplitudes

$$a+b+\bar{c}\rightarrow a+b+\bar{c} \tag{2}$$

as depicted in Fig. 1(b). Unfortunately, the amplitude in this case is not the physical amplitude for the process (2) but only an analytic continuation of the completely connected part of it. Nonetheless, as proposed by Mueller,[2] if one assumes that this amplitude is dominated by the same Regge singularities as the physical three-body amplitude, one is led to extremely interesting results. Our purpose here is to investigate the phenomenological consequences of this hypothesis, and to clarify the experimental conditions under which they can be tested.

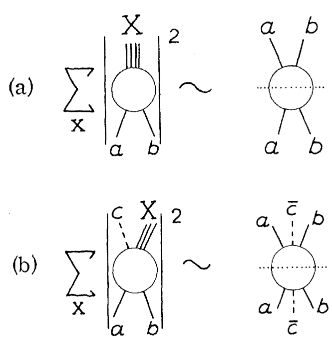

FIG. 1. (a) The familiar optical theorem, which relates total cross sections to the two-body forward elastic amplitude. (b) Mueller's optical theorem analog for single-particle spectra.

One can distinguish two different Regge limits of the amplitude (2) corresponding to different configurations of the final particle c in (1). The so-called "double-Regge limit" is hard to reach experimentally since the incoming energy $s=(p_a+p_b)^2$ has to be shared between the two two-body subsystems $(a\bar{c})$ and $(b\bar{c})$ to Reggeize both of them. For this reason, we shall restrict ourselves only to the case in which the mass of one of the two-body subsystems, say $(\bar{c}b)$, is held fixed while $s\rightarrow\infty$. This is the so-called "single-Regge limit" of the amplitude (2).

Assume now that the amplitude (2) is dominated by the usual Regge singularities, (i) the Pomeranchuk trajectory with $\alpha_P(0)=1$ and (ii) the approximately exchange-degenerate meson trajectories (ρ, $P'=f$, ω, A_2) with $\alpha_M(0)\simeq 0.5$. The preceding observations then imply that the invariant cross section corresponding to single-particle spectra, namely,

$$d\sigma/(d^3p/\omega)\equiv f(s;p_\parallel{}^b,p_\perp{}^2), \tag{3}$$

should have the Regge behavior

$$f(s;p_\parallel{}^b,p_\perp{}^2)=A(p_\parallel{}^b,p_\perp{}^2)+B(p_\parallel{}^b,p_\perp{}^2)s^{-1/2}. \tag{4}$$

In (3) and (4), $p_\parallel{}^b$ is the longitudinal momentum of particle c [in Reaction (1)] measured in the rest frame of b, corresponding to a fixed mass of the system $(b\bar{c})$ as specified above. The function f may thus conveniently be interpreted as the distribution of particles c due to the fragmentation of the target b. Obviously, the same statement can be made about the fragmentation products of the projectile a.

In spite of its simple appearance, Eq. (4) is an extremely powerful statement. It yields not only the limiting fragmentation hypothesis,[3] but also a definite prediction for the manner in which this limit is approached. Moreover, by relating single-particle spectra to Regge-behaved amplitudes, it allows the full armory of Regge phenomenology to be applied here also. There being a full distribution to study, instead of only one point at each energy as in the case of total cross sections, it can mean a whole new branch of Regge phenomenology at least as rich as two-body collisions.

To illustrate our point we cite as examples the following predictions derived from various principles.

(a) *Charge conjugation.*—Consider the reactions

$$\pi^+ + p \to c + \text{anything}, \quad (5)$$

$$\pi^- + p \to c + \text{anything}, \quad (6)$$

where c may be π, K, N, or $\overline{N}$. The distribution of c from the fragmentation of the target proton has the form (4) with

$$B = B_{P'} + B_\rho, \quad (7)$$

corresponding respectively to P' and ρ exchange. The coupling functions A and B for Reactions (5) and (6) are related to one another by charge conjugation at the pion vertex, so that

$$A^+ = A^-, \quad B_{P'}{}^+ = B_{P'}{}^-, \quad B_\rho{}^+ = -B_\rho{}^-. \quad (8)$$

As a result, for the distributions f^+ and f^- corresponding respectively to (5) and (6), we have

$$f^+(s; p_\parallel{}^{\text{lab}}, p_\perp{}^2) - f^-(s; p_\parallel{}^{\text{lab}}, p_\perp{}^2) = 2B_\rho{}^+(p_\parallel{}^{\text{lab}}, p_\perp{}^2) s^{-1/2}. \quad (9)$$

In other words, one expects the distributions from Reactions (5) and (6) to approach each other as $s^{-1/2}$. Similar conclusions can be drawn for the pairs of reactions initiated by (i) K^+ and K^- or (ii) p and $\bar{p}$ as projectiles.

(b) *Duality.*—For reactions in which the direct channel has exotic quantum numbers, duality implies, through exchange degeneracy, that the imaginary parts of meson exchanges (ρ, $P' = f$, ω, A_2) should cancel in the crossed channel. The imaginary part of the forward amplitude is then due entirely to Pomeranchuk exchange, which would in turn imply that cross sections for such processes are virtually independent of energy. A familiar example of this effect is the total cross section for K^+p scattering which is practically constant above 1.5 GeV/c. Apply now the same arguments to the reaction

$$K^+ + p \to \pi^\pm + \text{anything}. \quad (10)$$

This is related by means of Fig. 1(b) to the three-body reaction

$$K^+ + p + \pi^\mp \to K^+ + p + \pi^\mp, \quad (11)$$

which is exotic in the direct channel. The preceding observations then suggest that the invariant distributions f of pions from both projectile and target in (10) are practically constant as functions of the incoming energy. In contrast, the distributions of pions from the reaction

$$K^- + p \to \pi^\pm + \text{anything} \quad (12)$$

are expected to be quite strongly energy dependent. For reference, we list 72 such predictions in Table I. The reader may easily supply the corresponding predictions for the double-Regge limit by drawing six-point duality diagrams.[4]

(c) *Factorization.*—If the Pomeranchuk singularity is a simple pole, factorization of its residues implies that the limiting distributions of the fragmentation products of the target should be independent of the projectile, apart from the normalization.[5] To test this experimentally, one needs in general high incoming energies (say >30 GeV) to guarantee that the limit has indeed been reached. However, according to the duality arguments given in (b), such high energies are unnecessary for those reactions with exotic direct channels. If the K^+p total cross section can be accepted as a reliable guide, Pomeranchuk exchange should already dominate at beam momenta as low as 1.5 GeV/c. Thus, for example, for the reactions

$$K^+ + p \to \pi^+ + \text{anything}, \quad (13)$$

$$p + p \to \pi^+ + \text{anything}, \quad (14)$$

both of which are exotic in the direct channel, we expect that pion fragments from the target proton should have similar distributions already at moderate energies, for which data are immediately available. Other examples can easily be constructed from Table I. To the extent that limiting behavior is reached, it is unnecessary that Reactions (13) and (14) be compared at the same energy s.

Table I. Reactions (of the type, projectile + target → secondary + anything) to which the meson trajectories contribute in the single-Regge limit are denoted by the entries p or n, for proton or neutron target. Absence of an entry indicates an exotic channel, for which the cross section is expected to be energy independent.

Projectile \ Secondary	π^+	π^-	K^+	K^-	p	$\bar{p}$
π^+	p,n	n	p,n		p,n	
π^-	p	p,n	p		p	
K^+			p,n		p,n	
K^-	p	p,n	p	p,n	p	
p					p,n	
$\bar{p}$	p	p,n	p	p,n	p,n	p,n

To test the factorization of residues for secondary trajectories such as ρ, one may either make an overall fit as Barger and Phillips did for total cross sections[1] or attempt to isolate the contribution of a single trajectory by forming judicious combinations of distributions from several reactions. For example, the ρ-exchange contribution to Reactions (5) and (6) can be isolated as in (9). Similarly, from Kp and $\bar{K}p$ collisions, one can isolate the ρ-exchange contribution by forming the linear combination

$$\{f(K^+p)-f(K^-p)\}+\{f(K^0p)-f(\bar{K}^0p)\}, \qquad (15)$$

where in deriving (15) one has used charge conjugation and isospin. Further, one may replace the last two distributions in (15) by their charge-symmetric partners which are more accessible experimentally. One then has the relation

$$\{f(\pi^+ +p\to c+\text{anything})-f(\pi^- +p\to c+\text{anything})\}=Z\{[f(K^+ +p\to c+\text{anything})-f(K^- +p\to c+\text{anything})] + f(K^+ +n\to \tilde{c}+\text{anything})-f(K^- +n\to \tilde{c}+\text{anything})]\} \qquad (16)$$

between the distributions of the target fragments. The constant of proportionality Z is predicted to be unity by SU(3), and $\tilde{c}$ is the charge-symmetry partner of c.

We conclude with a few miscellaneous remarks:

(i) In all previous discussion we have restricted ourselves to single-particle distributions. It is obvious, however, that virtually everything we have said applies also to inclusive reactions where more than one particle is measured in the final state [i.e., to $\rho_n(p_1,\cdots,p_n)$ in the notation of Benecke *et al.*, Ref. 3].

(ii) Strictly speaking, the region of validity for our predictions depends on $p_\parallel^{\text{lab}}$ and $p_\perp^2$, subject to the criterion $s_{ab}\gg s_{b\bar{c}}$ in Reaction (2). However, for investigations of secondary trajectories, it is important to push measurements down in energy even further than the criterion implies. Experience with duality in two-body collisions suggests that the predictions may still hold at energies so low that the essentially asymptotic justification is absent.

(iii) In Eq. (4) it is in general not possible to know the relative strengths of A and B. As a guide for the design of experiments, it seems reasonable to assume that this is about the same as for total cross sections.

(iv) Except for the predicted relation between (13) and (14), all the other predictions relate the *normalized* distributions at different energies or from different reactions. Having examined data[6] on $p+p\to p+\text{anything}$ at incident momenta of 12, 19, and 30 GeV/c in search of the energy dependence suggested by (4), our experience is that existing data are insufficiently accurate to test the predictions.[7] One needs, in general, distributions which are correct in relative normalization to about 2%, which we believe can be achieved in counter experiments. In contrast, the relation between (13) and (14) and other tests for factorization of the Pomeranchuk pole in exotic channels can very likely be tested already with present bubble-chamber data.

(v) Limited statistical precision should not preclude testing our predictions. Since all of our relations are linear, they remain valid when integrated over any region in $p_\parallel^{\text{lab}}$ and $p_\perp^2$, subject only to the proviso that the criterion discussed in (ii) be satisfied.

We thank Professor G. F. Chew for first acquainting us with Mueller's work, and Professor H. R. Blieden and Professor C. N. Yang for useful discussions. In addition, one of us (C.H.-M.) is grateful to Professor C. N. Yang and Professor M. Dresden for their hospitality at Stony Brook.

*Work supported in part by the U. S. Atomic Energy Commission under Contract No. AT(30-1)-3368B.

†On leave from CERN, Geneva, Switzerland.

[1]V. Barger and R. J. N. Phillips, Phys. Rev. 187, 2210 (1969).

[2]A. H. Mueller, Phys. Rev. D 2, 2963 (1970).

[3]J. Benecke, T. T. Chou, C. N. Yang, and E. Yen, Phys. Rev. 188, 2159 (1969); R. P. Feynman, in *High Energy Collisions*, edited by C. N. Yang *et al.* (Gordon and Breach, New York, 1970), p. 237. Whether the "limit" is a mathematical limit or only a regime of weak (say, logarithmic) energy dependence is of no concern for our low-energy considerations.

[4]H. Harari, Phys. Rev. Lett. 22, 562 (1969); J. L. Rosner, Phys. Rev. Lett. 22, 689 (1969).

[5]The relative normalization is predicted to be in the ratio of the K^+p and pp total cross sections, but the shape prediction can be tested even with data of dubious normalization.

[6]At 12.4 GeV/c: J. L. Day *et al.*, Phys. Rev. Lett. 23, 1055 (1969); at 18.8 GeV/c: D. Dekkers *et al.*, Phys. Rev. 137, B962 (1965); at 19.2 GeV/c: J. Allaby *et al.*, CERN Report No. 70-12, 1970 (unpublished); at 30 GeV/c: E. W. Anderson *et al.*, Phys. Rev. Lett. 19, 198 (1967).

[7]The measurements by R. W. Anthony *et al.*, Phys. Rev. Lett. 26, 38 (1971), of $\pi^- + p\to \pi^\pm +\text{anything}$ from 2 to 6 GeV/c are likewise consistent with our prediction but are too coarse to test it severely.