due to adiabatic deceleration in a uniform solar wind given by

$$dT/dt = -T/\tau_{ad}, \tag{6}$$

where $\tau_{ad} = 3r/4V$ and r is the distance from the sun to the point of observation, i.e., 1 A.U. for the present observation.

Evaluating τ_{ad} during this event, using $V = 400 \pm 20$ km/sec as measured by the Vela satellites,[5] we find that

$$\tau_{ad} = 78 \pm 4 \text{ h}, \tag{7}$$

which is significantly smaller than the observed time constant

$$\tau_E = 210 \pm 10 \text{ h}.$$

We interpret this discrepancy as an indication that either the effects of adiabatic deceleration are not adequately described by Eq. (6), or that there is a competing acceleration process. If such an acceleration process could be described by an exponential time constant τ_A, then the observed time constant would be given by

$$\tau_E^{-1} = \tau_{ad}^{-1} - \tau_A^{-1}. \tag{8}$$

For this event we find that

$$\tau_A = 125 \pm 10 \text{ h}.$$

Jokipii[6] has recently shown that Fermi acceleration in the solar wind is an attractive possibility for such an acceleration mechanism. However, more observations must be analyzed before a complete evaluation of interplanetary acceleration and deceleration processes is possible.

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[1]E. N. Parker, Planet. Space Sci. 13, 9 (1965).

[2]L. A. Fisk and W. I. Axford, J. Geophys. Res. 73, 4396 (1968).

[3]M. A. Forman, J. Geophys. Res. 75, 3147 (1970).

[4]W. E. Althouse, E. C. Stone, R. E. Vogt, and T. H. Harrington, IEEE Trans. Nucl. Sci. 15, 229 (1967).

[5]Sol.-Geophys. Data No. 299, Part I (1969), and No. 300, Part I (1969), and No. 304, Part II (1969), and No. 305, Part II (1970).

[6]J. R. Jokipii, following Letter [Phys. Rev. Lett. 26, 666 (1971)].

Deceleration and Acceleration of Cosmic Rays in the Solar Wind*

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Recent observations of the deceleration of low-energy solar cosmic rays in the solar wind are discussed in the context of cosmic-ray transport theory. It is concluded that the rate of deceleration is much slower than would be produced by adiabatic energy change due to the observed expansion of the plasma, so that a competing acceleration process must be operative. Second-order Fermi acceleration by hydromagnetic waves, acting together with adiabatic energy change, is shown to provide a natural interpretation of the observations.

Recent observations[1] have clearly isolated an energy-loss process occurring in the transport of solar cosmic rays in the solar wind. The rate of energy change of ~3-MeV protons at Earth, obtained by following a characteristic feature of the energy spectrum as a function of time, is given by[1]

$$dT/dt = -T/(210 \pm 10 \text{ h}), \tag{1}$$

where T is the kinetic energy. Interpretation of the result in terms of processes not involving energy change appears to be quite unlikely.[1]

It is the purpose of this Letter to consider the interpretation of the above result in the context of cosmic-ray transport theory. The observation is found to fit very well into the standard theory, but only if one admits that Fermi acceleration is important for protons with energies of the order of 1-10 MeV.

Theoretical studies of cosmic-ray transport in the solar wind generally proceed from the Fokker-Planck equations[2,3] first derived by Parker. If $U(\vec{r}, t, T)$ is the cosmic-ray density as a function of position, time, and kinetic energy; if κ_{ij}

is the diffusion tensor; and if $\vec{V}$ is the solar-wind velocity, then the equation reads

$$\frac{\partial U}{\partial t} = \nabla \cdot (\vec{\kappa} \nabla U) - \nabla \cdot (\vec{V} U) - \frac{\partial}{\partial T}\left(\frac{dT}{dt} U\right), \quad (2)$$

with an associated, diffusion-related anisotropy which is not of interest here. Although in Parker's original discussion of Eq. (2) Fermi acceleration was suggested as a possible contributor to the energy change dT/dt, it was not considered further, and subsequent work has concentrated on the effect of large-scale compression or expansion of the solar-wind plasma. This produces adiabatic heating or cooling of the cosmic-ray gas at a rate

$$\left(\frac{dT}{dt}\right)_{\rm ad} = -\frac{\alpha}{3} \nabla \cdot \vec{V} T, \quad (3)$$

where $\alpha(T) \equiv (2m_0c^2 + T)/(m_0c^2 + T)$. It has been assumed in all detailed applications of Eq. (2) that the rate of energy change is given by Eq. (3).[2,3] This adiabatic term has the added virtue of being much simpler than Fermi acceleration.

Only a few special solutions to Eq. (2) are at present available and they do not apply here, but some general considerations indicate that the *observed* rate of energy change represented by Eq. (1) should be at least roughly equal to the local value of dT/dt appearing in Eq. (2).

First, note that the characteristic feature of the energy spectrum observed by Murray *et al.*[1] (their Fig. 2) does not appear to spread out noticeably as it moves to lower energies. If dT/dt varies with position and the particles *diffuse* over regions of different dT/dt, one would expect a *spreading* of any characteristic feature in addition to the deceleration because different particles will have experienced different energy changes. The reader is referred, for example, to discussions of galactic particles,[2,4] where this spreading is clearly important. From this it is concluded that diffusion is relatively slow during the period of the present observations, so that the particles observed at any point at a given time have experienced nearly the same energy change.

If diffusion is relatively unimportant during the period of observation, Eq. (2) can be integrated. Consider the case where $(dT/dt)_{\rm ad}$ is the only contributor to energy change and in which V is constant and radial from the sun. One finds that for nonrelativistic particles,[5]

$$U(\vec{r}, T, t) = a(t, t_0) \times U(r - V(t-t_0), T[a(t,t_0)]^{-2}, t_0), \quad (4)$$

where r is the distance from the sun and

$$a(t, t_0) = \exp\left\{-\int_{t_0}^{t} \frac{2V dt'}{3[r - V(t-t')]}\right\}.$$

Equation (4) maps the density spectrum at one radius and time to a different radius at a later time. Note that for any given event, Eq. (4) will only hold during an intermediate time period when the gradients are small. Examination of Eq. (4) shows that at a given radius, a characteristic feature of the spectrum will be observed to move to lower energies at a rate somewhat greater than $(dT/dt)_{\rm ad}$ at that point. In view of these approximate considerations it appears that is appropriate to compare the observed dT/dt from Eq. (1) with that in Eq. (2).

In view of the extensive use of adiabatic energy change as given by Eq. (3), it is of interest to determine whether it alone can account for the observed rate of energy change. Following the above discussion, we compare $(dT/dt)_{\rm ad}$ with the observed rate of energy change. The available solar-wind data at 1 A.U. during the time of the cosmic-ray observations (9-14 June 1969) consist of daily averages of the radial wind velocity observed by the Vela satellites.[6] These are tabulated below:

Date, June	9	10	11	12	13	14
V_r (km/sec)	377	455	380	···	···	455

Simultaneous measurements on Pioneer VI give similar results. Further, it is in general true that the perpendicular, or nonradial, components of the wind velocity are small, being usually less that 10% of the radial component. We may safely assume that their contribution to $\nabla \cdot \vec{V}$ is negligible. Thus, Eq. (3) becomes

$$\left(\frac{dT}{dt}\right)_{\rm ad} \simeq -\frac{\alpha}{3}\left(\frac{2V_r}{r} + \frac{\partial V_r}{\partial r}\right) T. \quad (4)$$

Consider first the term $2V_r/r$ alone. The observed mean velocity of ~400 km/sec produces a deceleration of nonrelativistic particles with a characteristic time of 78 h at 1 A.U., which is much too fast.[1] It is next a simple matter to show that the value of $\partial V_r/\partial_r$ deduced from the observations is too small to change this time of 78 h appreciably. Since the fluctuations are frozen into the plasma, we have as a lower bound[7] that

$$\partial V_r/\partial r \simeq -V_r^{-1} \partial V_r/\partial t \simeq -4 \times 10^{-7}\ \rm{sec}^{-1}, \quad (5)$$

noting that the observed V_r tabulated above increases from roughly 380 to 450 km/sec over the five-day interval. This would change the 78-h time deduced above by less than 10%. Similarly, if one argued that because of the convection by the solar wind, the observed deceleration reflects processes occurring closer to the sun, the adiabatic deceleration would be even faster (see previous paragraph). The observed rate of deceleration corresponds to adiabatic deceleration occurring at some 3 A.U., and it is highly unlikely that the observations reflect processes occurring at that distance. It is therefore concluded that the observed rate of energy change cannot be a consequence of adiabatic processes alone. The only apparent uncertainty in this conclusion concerns the highly unlikely possibility that the wind from higher ecliptic latitudes converged very strongly toward the ecliptic during this period, to substantially reduce $\nabla\cdot\vec{V}$ from that deduced above. This possibility is remote and is therefore neglected. Thus, *there must be a further, competing acceleration process which reduces the effective energy loss rate below that produced by adiabatic cooling of the cosmic-ray gas.*

In the remainder of this paper it is demonstrated that (second order) Fermi acceleration is a promising candidate for this competing process. The words "Fermi acceleration" are used here in analogy with Fermi's original discussion,[8] and imply stochastic acceleration by the moving magnetic irregularities which scatter the particles. They are not intended to imply the existence of magnetic clouds.

The *rate* of Fermi acceleration can be estimated in various approximations. Perhaps the most straightforward is to consider scattering by randomly moving scattering centers. The rms velocity of the centers relative to the plasma will be taken to be the Alfvén velocity $\vec{V}_A$, since the irregularities doing the scattering are presumably hydromagnetic waves. It is a simple matter to show[9] that the *average* rate of energy change for an isotropic pitch-angle distribution of nonrelativistic particles is

$$\left(\frac{dT}{dt}\right)_F \simeq (m_0c^2+T)\frac{4V_A^2}{c^2}\frac{1}{\tau_c} \simeq \frac{8V_A^2}{3\kappa_\parallel}T, \tag{6}$$

where $\tau_c = 3\kappa_\parallel/w^2$ is the mean scattering time, $\kappa_\parallel$ is the parallel diffusion coefficient, and w is the particle speed. Clearly $(dT/dt)_F$ increases toward lower energies as $\kappa_\parallel$ decreases.

Alternatively, one could proceed by considering a stochastic ensemble of small-amplitude Alfvén waves and integrate the equations of motion. This approach may be shown to lead to a result similar to Eq. (6), where $\kappa_\parallel$ is related to the wave power spectrum as in the ordinary theory of cosmic-ray diffusion.[10] Actually, the latter approach turns out to lead to somewhat larger rates of acceleration, so that using Eq. (6) will give a reasonable lower estimate of the rate of Fermi acceleration. The actual rate may be somewhat higher. Considerations of the *spread* in energy produced by the stochastic aspect of the energy gain may be shown not to affect the results appreciably.

Consider, then, Eq. (6). Recent evidence[11] indicates that $\kappa_\parallel$, for protons with an energy of a few MeV, is typically of the order of 10^{20} cm^2/sec, with an uncertainty of perhaps a factor of 3. The value of V_A^2 depends on the ambient magnetic-field intensity and plasma density. The available data[6] indicate that the plasma density was quite low during this period and was approximately 2 to 7 protons cm^{-3}. The magnetic field at Earth was approximately 7×10^{-5} G.[12] Substituting these values into Eq. (6), one obtains for the rate of Fermi acceleration,

$$\tau_F \simeq \left[\frac{1}{T}\left(\frac{dT}{dt}\right)_F\right]^{-1} \simeq 120 \text{ h}, \tag{7}$$

with an uncertainty of the order of a factor of 3.

The point is that at low energies *Fermi acceleration may well be fast enough to offset a sizable fraction of the adiabatic deceleration given in Eq. (3).* The effective net time scale for deceleration is

$$\tau_{\text{net}} \simeq (\tau_{\text{ad}}^{-1} - \tau_F^{-1})^{-1}. \tag{8}$$

It is readily shown that $\tau_{\text{net}} = 200$ h is consistent with Eq. (8). Because the diffusion coefficient increases rapidly as a function of particle energy, Fermi acceleration is less important at energies higher than the ~3 MeV considered here.

The above discussion has established the need for an acceleration mechanism and has shown that second-order Fermi acceleration is a natural candidate. It does not appear that electric fields associated with other wave modes can be important, although they cannot be completely excluded. Since the energy change would probably be produced by a random walk in energy, the mean energy change is zero and the above would only produce an energy *spread*. The only alternative to the Fermi mechanism would appear to be some coherent electric field, but we have been unable to devise a plausible model. It is therefore concluded that the observations of Murray *et al.*,[1] in addition to establishing the existence of energy change in the solar wind, also lead to the conclusion that Fermi acceleration by hydromagnetic waves may well be an important effect in the transport of low-energy cosmic rays.

Further observations are necessary to establish the present picture definitely. One would expect the efficiency of Fermi acceleration to be greater during periods when $\kappa_{\parallel}$ is small and V_A is large and hence the observed deceleration would be less during such periods. Observations to test the expected correlation between energy change and plasma parameters in different events are necessary before the presence of Fermi acceleration can definitely be established. A future publication will consider in detail the problem of Fermi acceleration in the solar wind and its effect on the transport of low-energy cosmic rays.

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[1]S. Murray, E. C. Stone, and R. Vogt, preceding Letter [Phys. Rev. Lett. 26, 663 (1971)].

[2]E. N. Parker, Planet. Space Sci. 13, 9 (1965).

[3]E. N. Parker, Space Sci. Rev. 9, 325 (1969); L. J. Gleeson and W. I. Axford, Astrophys. J. 154, 1011 (1968); J. R. Jokipii and E. N. Parker, Astrophys. J. 160, 735 (1970); J. R. Jokipii, to be published.

[4]M. L. Goldstein, L. A. Fisk, and R. Ramaty, Phys. Rev. Lett. 25, 832 (1970).

[5]This solution is similar to a slightly less general one presented by M. A. Forman [J. Geophys. Res. 75, 3147 (1970)]. After this was completed the author received a preprint by L. J. Gleeson which discusses a similar result.

[6]Sol.-Geophys. Data No. 300, Part I (1969).

[7]The observed variation may be a manifestation of an azimuthal velocity dependence being rotated past the spacecraft. In this case $|\partial V_r/\partial r|$ may actually be less than the value computed.

[8]E. Fermi, Phys. Rev. 75, 1169 (1949).

[9]This result is most easily obtained by following the approach given by R. B. Leighton, *Principles of Modern Physics* (McGraw-Hill, New York, 1959), p. 705. Leighton neglects the pitch-angle distribution, but this does not appreciably affect the result.

[10]The derivation is in preparation and will be published later. A somewhat similar approach has been presented by K. Hasselman and G. Wibberentz, Z. Geophys. 34, 353 (1968). They also pointed out that Fermi acceleration could be important.

[11]See, e.g., J. R. Jokipii and P. J. Coleman, J. Geophys. Res. 73, 5495 (1968); L. Fisk, to be published; L. Gleeson, S. Krimigis, and W. I. Axford, to be published; S. Murray, thesis, California Institute of Technology, 1970 (unpublished).

[12]This magnetic-field data was kindly furnished through the courtesy of Dr. D. S. Coburn and Dr. C. P. Sonett.

Exotic Resonances from the Multiperipheral Model

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Using fundamental theorems about integral equations, we show that the multiperipheral model for forward pion-pion elastic scattering requires the existence of a Regge trajectory with isospin two.

The possible existence of exotic resonances has been a subject of considerable interest in recent years. Experimentally there is mounting evidence for the existence of the Z^* resonance,[1] first seen by Cool *et al.*,[2] as well as for exotic exchanges.[3] Theoretically it has long been recognized that duality in its usual sense requires exotic resonances.[4] Until now, however, there have been no dynamical models that required the existence of exotic resonances or trajectories. In this Letter we would like to point out that the multiperipheral model necessarily generates an $I = 2$ trajectory, which very likely has an intercept $\alpha(0)$ which is greater than zero.

The multiperipheral model produces poles or resonances through unitarity; in fact, unitarity is the underlying principle that requires the existence of certain poles. Under certain conditions unitarity forces the kernel of the multiperipheral integral equation to be a positive operator; that is, the kernel $K_n(x, y)$ is greater than zero for all x and y. There exist several powerful theorems for such kernels,