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Failure of the Bloch T^5 Law for the Low-Temperature Electrical Resistivity of Metals

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It is shown that the Bloch T^5 law does not describe the low-temperature electrical resistivity of any metal.

It is universally believed that the famous Bloch T^5 law[1] correctly describes the temperature dependence of the electrical resistivity $\rho(T)$ of simple metals for temperatures appreciably less than Θ, the Debye temperature. This belief persists in spite of the fact that there exist in the literature reports[2,3] of low-temperature measurements of $\rho(T)$ which display marked deviations from the T^5 law. We have carried out calculations which avoid approximations introduced in the Bloch theory, and we find that (i) the Bloch T^5 law never applies to any metal and that (ii) our calculations for $\rho(T)$ at low temperatures account perfectly for the hitherto unexplained experimental deviations from the Bloch T^5 law.

The Bloch law is conveniently expressed as the low-temperature limit of the Bloch-Grüneisen[1] formula

$$\rho_{\mathrm{Bloch}}(T) \simeq A(T/\Theta)^5[1-0.008(\Theta/T)^5 e^{-\Theta/T}], \quad T \ll \Theta. \tag{1}$$

We may determine the extent of the low-temperature regime by comparing the second term in the brackets with unity. For $T<\Theta/10$, the second term is less than 0.05, and therefore Eq. (1) yields $\rho_{\mathrm{Bloch}}(T) \propto T^5$, for $T<\Theta/10$. The metal which should fit the Bloch theory best is Na, because Na has an almost perfectly spherical Fermi surface totally within the first Brillouin zone. For Na, $\Theta \simeq 160°$K, implying that we may expect $\rho(T) \propto T^5$ for $T \lesssim 16°$K. The low-temperature meausrements of Woods[2] for $\rho(T)$ for Na give

$$\rho_{\mathrm{expt}}(T) \propto T^{5.0\pm0.1}, \quad 9°\mathrm{K} \lesssim T \lesssim 15°\mathrm{K} \tag{2}$$

which would seem to imply a perfect confirmation of the Bloch theory. However, for still lower temperatures, where the Bloch theory should be better, Woods[2] obtains

$$\rho_{\mathrm{expt}}(T) \propto T^{6.0\pm0.1}, \quad 4°\mathrm{K} \lesssim T \lesssim 9°\mathrm{K} \tag{3}$$

in striking disagreement with Eq. (1).

We have carried out complete calculations of $\rho(T)$ for Na (details to be published separately) and obtained complete agreement with the experimental results of *both* Eqs. (2) and (3). Not only is the temperature dependence in accord with experiment in each region (see Fig. 1), but in addition, we obtained the correct temperature ($\simeq 9°$K) at which the temperature-dependence changes. Moreover, our calculations show that

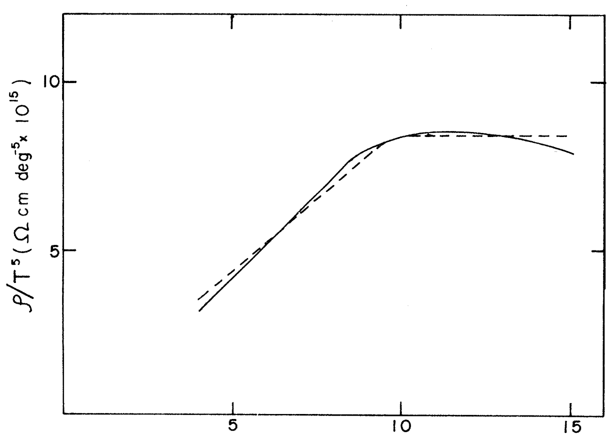

FIG. 1. Comparison between the smoothed experimental data (dashed line) and our calculations (solid line) for $\rho(T)$ for Na. The calculated values are within the scatter of the experimental data. A horizontal line indicates T^5 behavior and a straight line through the origin indicates T^6 behavior.

the T^5 behavior for $\rho(T)$ reported in Eq. (2) has nothing whatsoever to do with the predictions of the Bloch theory, but is merely a fortuitous numerical accident.

Our calculations are based on the weak-coupling theory[1] for describing the electron-phonon interaction and the Boltzmann equation[1] for describing the transport theory. The reason for the inadequacy of the Bloch theory is that Eq. (1) is based on a treatment that completely ignores both unklapp processes in the scattering of electrons by phonons and also the momentum dependence of the electron-phonon-interaction scattering amplitude. We have taken proper account of both of these very important effects and find complete agreement with experiment, Eqs. (2) and (3).

Only when one goes to extremely low temperatures ($T \lesssim 3°K$ for Na) is one justified in making the Bloch approximation of ignoring both umklapp processes and the momentum dependence of the electron-phonon scattering amplitude. However, for such extremely low temperature, phonon drag[1] is very important and a third approximation of the Bloch theory, namely, ignoring phonon drag, is no longer justified. We have computed the contribution of phonon drag to $\rho(T)$ throughout the entire temperature range, and we find that whereas phonon drag is indeed relatively unimportant[4] above 4°K, it makes an extremely improtant contribution to $\rho(T)$ below 3°K.

We therefore see that there exists no temperature regime for which it is valid to make simultaneously the three assumptions which are crucial to the derivation of the Bloch T^5 law. Either phonon drag is very important ($T \lesssim 3°K$ for Na) or else umklapp scattering and the momentum dependence of the electron-phonon scattering amplitude are very important ($T \gtrsim 3°K$ for Na). Thus, the Bloch T^5 law never applies at any temperature.

Although we have concentrated our discussion on Na, the failure of the T^5 law is not peculiar to Na, but occurs for all other metals as well. For example, our calculations of $\rho(T)$ for K show clearly that at $T \simeq 3°K$, the temperature dependence changes from T^6 behavior to T^5 behavior. As is the case for Na, the low-temperature measurements[3] for K yield precisely this result. Moreover, the calculations also show that the observed T^5 behavior is totally unrelated to the Bloch law. Finally, we have succeeded in proving for the polyvalent metals as well that the Bloch T^5 law never applies.

In summary, we have obtained the following results:

(i) At least one of the three basic approximations of the Bloch theory breaks down in every temperature regime, with the consequence that the Bloch T^5 law is never valid for any metal.

(ii) Our calculations account precisely for the experimental results for $\rho(T)$ for Na, Eqs. (2) and (3), and for K, Fig. 2 of Ref. 3.

(iii) Our calculations show that the agreement between the observed T^5 regime for $\rho(T)$ for Na and K and the Bloch T^5 law is purely fortuitous.

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