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Observations of Nonlinear Landau Damping*

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Nonlinear Landau damping, the nonlinear interaction between two waves in which the beat disturbance is resonant with the particle thermal velocities, has been observed for electron plasma waves. The results confirm the predictions of weak-turbulence theory.

Although plasma turbulence is known to be an important and sometimes dominant process in many experimental devices, there is only one well-developed quantitative theory for such processes: weak-turbulence theory.[1-3] Being a perturbation theory treating the strength of the turbulence as the expansion parameter, it is a direct extension of linear theory and requires that the energy in turbulence be much less than the plasma thermal energy. However, the qualitative features of the theory are often used to describe turbulence even when this restriction is violated because they are the only concepts we have developed. Considering the widespread use of weak-turbulence theory, experimental verification of its various predictions is worthwhile.

Weak-turbulence theory analyzes plasma turbulence by developing equations for the interaction of the waves that constitute the turbulent spectrum. The theory can be tested experimentally by examining either the interaction of two waves or narrow spectra and comparing the interaction characteristics with the predictions. Two types of interaction appear in the theory.

If the beat disturbance of the two waves, $\Delta\omega = \omega_1 - \omega_2$ and $\Delta k = k_1 - k_2$, lies on the dispersion curve of a natural mode of the plasma, resonant wave-wave interaction takes place, producing this third wave. Resonant wave-wave interactions have been observed and positively identified.[4-6] Unfortunately, it has proved difficult to measure the strength of the interaction for comparison with the theory.

If the beat disturbance does not lie on the dispersion curve, as interaction is still possible if its phase velocity $v_b = \Delta\omega/\Delta k$ resonates with the particles. Since this is equivalent to the Landau damping of a single wave that resonates with particles, it is called nonlinear Landau damping. The effect is unique to weak-turbulence theory. The effect plays a vital role in the theory, for it is the primary mechanism for transferring wave energy to the particles; it is the heating and "collision" mechanism in weak-turbulence theory. (An analogous effect is possible if the beat disturbance is subject to cyclotron damping instead of Landau damping, i.e., $\Delta\omega = \Omega_c$. This ef-

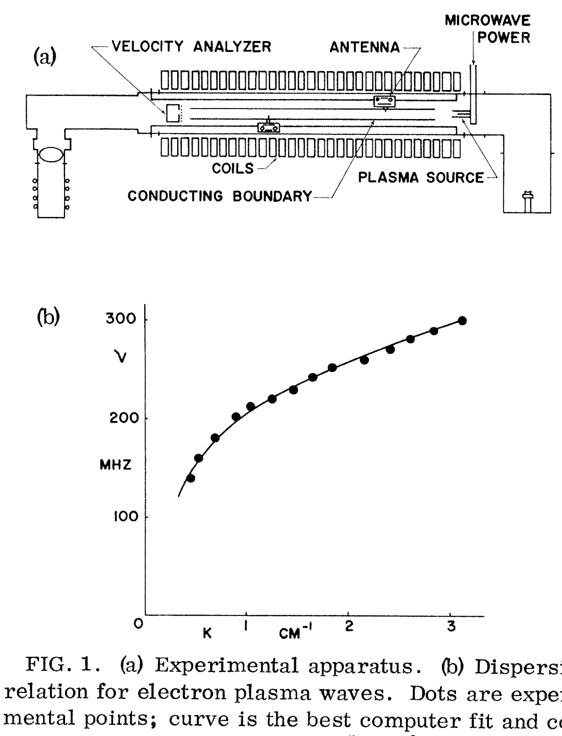

FIG. 1. (a) Experimental apparatus. (b) Dispersion relation for electron plasma waves. Dots are experimental points; curve is the best computer fit and corresponds to a density of 1.1×10^9 cm^{-3} and $T_e = 14$ eV.

fect has recently been observed by Chang and Porkolab.[7])

In this Letter, we report an observation of the nonlinear Landau damping of electron plasma waves through interaction with the electrons. The apparatus is shown in Fig. 1(a). Hydrogen plasma from a coaxial microwave source drifts freely down a uniform magnetic field. The plasma is terminated on a plate that is biased to reflect electrons. The plasma is bounded by a metal cylinder of 10-cm diam with slits down its 2.5-m length through which antenna probes may be moved. The cylinder is a waveguide beyond the cutoff at the frequencies used and so prevents direct electromagnetic coupling between antennas. The magnetic field is of order 1 kG, which is effectively infinite for the plasma densities used. The background hydrogen pressure in the apparatus is less than 10^{-5} Torr, giving an electron-neutral mean free path in excess of 40 m as the shortest mean free path for electrons. The plasma can be correctly described by the one-dimensional Vlasov equation.

Under these conditions, the equation for the cylindrically symmetric, electrostatic, wave potential of the electron plasma wave becomes[8]

$$\frac{d^2\Phi(r)}{dr^2}+\frac{1}{r}\frac{d\Phi(r)}{dr}-k^2\left[1-\frac{\omega_p^2(r)}{k^2v_e^2}W\left(\frac{-\omega}{kv_e}\right)\right]\Phi(r)=0, \tag{1}$$

where Φ is the wave potential, k and ω are the complex wave number and angular frequency of the wave, $\omega_p^2(r)$ is the radially dependent plasma frequency, v_e is the mean thermal velocity of the plasma electrons, and W is related to the plasma dispersion function by $W(x)=\frac{1}{2}Z'(-x/\sqrt{2})$. This is an eigenvalue problem which may be solved for each ω to obtain k and $\Phi(r)$. Only the lowest eigenmode need be considered; the higher ones are strongly Landau damped except at low frequencies.

The experimental dispersion curve for the waves is determined from measurements of wavelength as a function of the frequency of the plasma wave. The temperature and density of the plasma are inferred by computer using a program that solves Eq. (1) to obtain the best least-squares fit to the experimental points. The result for a typical plasma is shown in Fig. 1(b). (On the upper part of the dispersion curve, the wave group velocity is almost constant and equal to $1.6v_e$.)

The nonlinear Landau-damping interaction between two electron plasma waves for an infinite one-dimensional plasma has been calculated by Aamodt and Drummond[9] for the initial-value problem. For the spatial interaction in finite geometry the damping length of wave 1 caused by the presence of wave 2 can be written

$$\alpha=\frac{(e/m)^2}{v_{g1}}\frac{2\pi\nu_1}{(v_1-v_b)^4}\lambda_D^2\left(\frac{\lambda_1}{\lambda_2}\right)^2\frac{W_1}{W_2}\operatorname{Re}\left(\frac{i}{W_b}\right)\left\{2\frac{W_{1-2}}{W_1}\left[\frac{\lambda_2-\lambda_1W_2/W_1}{\lambda_2-\lambda_1}-\frac{W_{1-2}}{2W_1}\right]-\left[\frac{\lambda_2-\lambda_1W_2/W_1}{\lambda_2-\lambda_1}\right]^2\right\}I_{000}\left(\frac{8\pi P_2}{Av_{g2}}\right), \tag{2}$$

where ν, λ, and λ_D have their usual meaning; v_g is a group velocity; v is a phase velocity; P is the total power in the wave; A is the cross-sectional area of the plasma; and the W functions are evaluated at arguments indicated by the subscripts: W_b being $W(v_b/v_e)$ and W_{1-2} being $W(v_{1-2}/v_e)$, where v_{1-2} is the phase velocity of a wave on the dispersion curve with $k=k_1-k_2$, etc. This expression follows directly from the result for the temporal case with two modifications. The first term in the denominator, v_{g1}, is the group velocity to convert temporal effects to spatial. The I_{000} term represents the effects of finite geometry. It is an overlap integral of the radial eigenfunctions and is nearly unity in practice. The final term is the analog of $|E_2|^2$, the energy density in wave 2.

Equation (2) embodies simplifications made possible by the smallness of $k\lambda_D$ for all waves involved. This permits extraction of the factor $\operatorname{Re}(i/W_b)$, which is roughly proportional to the derivative of the distribution function evaluated at v_b. The electrons traveling with the phase velocity of the beat disturbance are the ones most affected by the interaction. A special characteristic of this interaction is that the sign depends upon the sign of $\nu_1-\nu_2$. Second waves at frequencies below ν_1 cause the wave to damp, whereas waves at higher frequencies cause the wave to grow.

The damping length for wave 1 caused by another wave 2 can be written directly in terms of observable quantities with conventional units:

$$\alpha=\frac{4.4\times10^{50}\nu_1P_2\lambda_1^2W_1T}{v_{g1}(v_1-v_b)^4v_{g2}\lambda_2^2W_2NA}I_{000}\operatorname{Re}\left(\frac{i}{W_b}\right)\left\{2\frac{W_{1-2}}{W_1}\left[\frac{\lambda_2-\lambda_1W_1/W_2}{\lambda_2-\lambda_1}-\frac{W_{1-2}}{2W_1}\right]-\left[\frac{\lambda_2-\lambda_1W_2/W_1}{\lambda_2-\lambda_1}\right]\right\}, \tag{3}$$

where T is electron temperature in volts, and N is the central plasma density. All lengths are in centimeters; P_2 is in watts.

Equation (3) has a rather complicated appearance, but all the terms in the equation except P_2 can be obtained from the experimental dispersion relation. To determine the power in the wave, we must know the absolute coupling constant of the transmitting probe. No direct method is available, but a reasonably accurate value may be obtained from the principle of reciprocity which states that the coupling of an antenna to a particular mode is independent of the direction of power flow. The coupling of a probe to a plasma wave is therefore the same whether the probe is used as a transmitter or receiver. Using a pair of probes spaced so that only the undamped wave of interest couples them, the sum of the coupling constants is obtained directly from the total observed coupling between transmitter and receiver. With three probes in the machine, three transmitter-receiver pairs are possible. This gives three equations involving the three unknown single-probe coupling constants and determines the constants with an accuracy of 1 to 2 dB. This method was developed for a device similar to the one used here by Malmberg and Wharton[10], who have found the result in agreement with other methods of inferring the coupling coefficient in several experiments.[10-12] With this information and the dispersion curve, all the terms in Eq. (3) can be evaluated to obtain the theoretical prediction of the nonlinear damping rate.

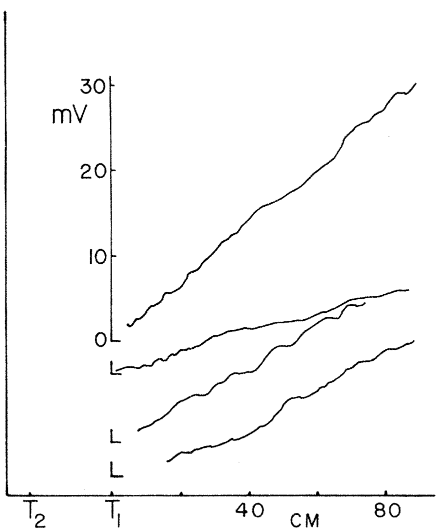

FIG. 2. Development of nonlinear interactions: Representative traces of modulation in the signal received at ν_1 when the second wave is modulated, taken as the receiver is moved away from the transmitters. The upper two traces correspond to the points at 5.8 and 4.0 mW on the upper line of Fig. 3; the lower two traces are the 3- and 1.5-mW points on the lower line. In no case does the interaction change the amplitude of the test wave by more than 10%. The vertical scale is changed for each trace.

Nonlinear damping is measured directly by transmitting a test wave of sufficiently high phase velocity such that no linear Landau damping is observed in the machine. The receiver is tuned to this wave, and the output is fed to a synchronous detector. With a second transmitting probe, the second wave (at high power) is coupled into the plasma. This wave is modulated at 1000 Hz, which serves as the reference for the sychronous detector. The increase in output from the detector as the receiver moves away from the transmitter indicates a nonlinear interaction occuring as the waves propagate, and the sign determines whether it is growth or damping. Typical traces for the development of the nonlinear interaction are shown in Fig. 2. By measuring the change in wave 1 caused by wave 2 as a function of position, the effects of possible nonlinear interactions in the probe sheaths are removed. Although the second wave is injected at moderately high powers, the levels are well within the limits of weak-turbulence theory. The ratio of wave energy to thermal energy, $|E_2|^2/8\pi nkT$, is always less than 10^{-3}. Furthermore, the actual potential applied to the transmitting probe is kept below kT to preclude the generation of electron "beams" at the antenna.[13] To insure that there was no coupling to ion waves, $|\nu_1-\nu_2|$ was always at least 20 MHz, well above the ion plasma and cyclotron frequencies. For dispersion curves as in Fig. 1, it is clearly easy to choose a pair of waves for which v_b is near v_e, giving as strong an interaction as possible for this effect.

Two tests of Eq. (3) have been made. First,

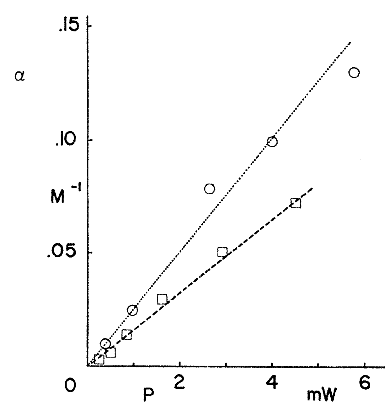

FIG. 3. Dependence of damping coefficient on P_2. The circles are for $\nu_1=270$ MHz, $\nu_2=230$ MHz; the squares are for $\nu_1=250$ MHz, $\nu_2=220$ MHz.

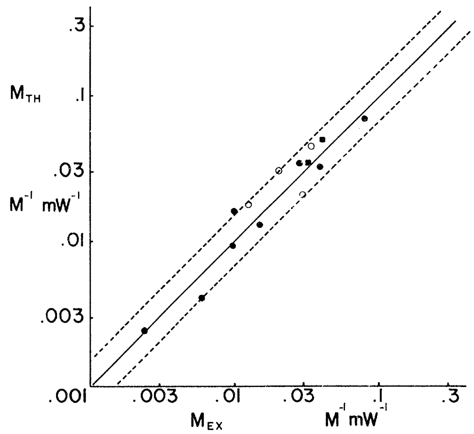

FIG. 4. Observations of the matrix element. The experimental value is obtained from the observed nonlinear interaction and power; the theoretical value is computed from Eq. (3) using results of the linear dispersion curve. Solid circles are wave damping, open circles are wave growth, and solid squares are wave damping when the second wave is replaced by a narrowband noise spectrum. Dashed lines are ±2 dB, showing limits of uncertainty in coupling measurements.

the dependence of the damping coefficient on P_2 has been measured. The damping coefficient as a function of P_2 for two different wave pairs is shown in Fig. 3. This establishes that the effect seen varies as $|E_2|^2$, the expected dependence.

The most significant test is to measure the matrix element for the coupling, defined by $\alpha = MP_2$. The matrix element has been measured for several pairs of waves. The plasma conditions are typically those for which the dispersion relation of Fig. 1(b) is appropriate, although temperatures and densities differing by 50% have been used. The results are summarized in Fig. 4, showing the experimental matrix element plotted against the value calculated from Eq. (3). The dashed lines are the 2-dB limits set by the coupling constant of the probes, which is the major uncertainty. The solid points represent cases where $\nu_1 > \nu_2$ and the interaction causes damping; the open circles represent cases where $\nu_1 < \nu_2$ and the interaction produces growth. The sign of the interaction does change with the sign of $\nu_1 - \nu_2$, a special feature of nonlinear Landau damping. For the solid squares, the wave at ν_2 was replaced by a narrow noise spectrum centered at ν_2. Weak-turbulence theory treats the interaction among elements of a broad spectrum. Although it should also apply to single waves of the amplitudes used here, the demonstrated irrelevance of whether P_2 is present as either a single wave or a spectrum confirms this and establishes that the observed effect is not a peculiar interaction of a test wave with a large-amplitude wave.

The observations verify all the characteristics of nonlinear Landau damping (within the experimental error): The sign of the interaction changes with the sign of $\nu_1 - \nu_2$, the effect varies linearly with the power in the second wave, and the parametric dependences and absolute magnitude agree with the theory.

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