Research under Contract No. N0001467-A-0230-0003.

[1]G. V. Chester, Phys. Rev. A 2, 256 (1970).
[2]A. F. Andreev and I. M. Lifshitz, Zh. Eksp. Teor. Fiz. 56, 2057 (1969) [Sov. Phys. JETP 29, 1107 (1969)].
[3]A. J. Leggett, Phys. Rev. Lett. 25, 1543 (1970).
[4]W. J. Mullin, to be published.
[5]Similar cell-model Hamiltonians have been used by T. Matsubara and H. Matsuda, Progr. Theor. Phys. 16, 416, 569 (1956), and 17, 19 (1957); R. Whitlock and P. Zilsel, Phys. Rev. 131, 2409 (1963); P. R. Zilsel, Phys. Rev. Lett. 15, 476 (1965); H. A. Gersch and J. M. Tanner, Phys. Rev. 139 A1769 (1965); R. A. Guyer and L. I. Zane, Phys. Rev. Lett. 24, 660 (1970), and to be published; R. Balakrishnan and R. V. Lange, Phys. Rev. A 3, 496 (1971).
[6]See the first two sets of papers of Ref. 5. Also see M. E. Fisher, Rep. Progr. Phys. 30, 615 (1967), Sect. 4.
[7]P. W. Anderson, Rev. Mod. Phys. 38, 298 (1966).
[8]R. A. Guyer, Phys. Rev. Lett. 26, 174 (1971).

Anomalous Skin Effect of Microwaves Incident on Magnetoplasmas

Kazuo Minami and Susumu Takeda
Department of Electrical Engineering, Nagoya University, Nagoya, Japan
(Received 9 February 1971)

The anomalous skin effect of microwaves is experimentally investigated for a high-density gaseous plasma near the electron cyclotron resonance and is compared with a theoretical prediction.

The increase of the penetration depth of the electromagnetic field in a gaseous plasma due to the thermal electron motion is called the anomalous skin effect[1,2] just as in metals.[3] Shafranov[4] analyzed the penetration of a wave into a collision-free, magnetized plasma uniformly distributed in a half-space. Platzman and Buchsbaum[5] extended the analysis, showing some numerical results. Weibel[2] also strictly analyzed the anomalous skin effect in a uniform, nonmagnetized plasma, referring to a previous experiment.[6] Drummond[7] extended Weibel's theory to a plasma with a magnetic field. The experimental works[1,6,8] were limited to plasmas where the collision frequency ν was higher than the applied frequency ω.

The classical penetration depth of the electromagnetic field into high-density plasmas in a cutoff region d defined by $1/\mathrm{Im}\,(k)$ is derived from the dielectric constant of the plasma, $1-(\omega_p/\omega)^2 \times(1+i\nu/\omega)^{-1}$, as

$$d \sim (2\nu/\omega)^{1/2} c/\omega_p \text{ for } \omega \ll \nu \tag{1}$$

and

$$d \sim c/\omega_p \text{ for } \omega \gg \nu, \tag{2}$$

where the thermal motion of electrons is neglected for plasmas and the plasma frequency ω_p is much higher than ω. This case is called the cold-plasma theory. Equation (1) is the same as the well-known classical skin depth in metals. For Eq. (2), the plasma behaves like a dielectric material with respect to the electromagnetic field.

When d is smaller than v_{th}/ν for $\omega \ll \nu$, the thermal velocity of electrons v_{th} should be taken into account for the calculation of the skin depth. For $\omega \gg \nu$, v_{th}/ν should be replaced by v_{th}/ω. These results are expected from the theoretical analysis.[2,7] The effect of a magnetic field is quite different for the case $\omega \gg \nu$ from that for the case $\omega \ll \nu$. In the latter, the magnetic field is not too sensitive to the skin depth because the plasma is collision dominated. On the other hand, for $\omega \gg \nu$, the characteristic of the plasma medium is generally expected to be strongly affected by the magnetic field, particularly near the electron cyclotron resonance.

As the gas pressure in the positive column, where $\omega \ll \nu$, decreases, the skin depth is found[1,8] to become larger than the classical value because of the increased v_{th}/ν. However, no experiments have been reported on a plasma with a magnetic field for $\omega \gg \nu$. Thus, the detection of the anomalous skin effect for $\omega \gg \nu$ by using a microwave reflection technique[9] is tried here.

A right-hand circularly polarized wave (R wave) is used, since the anomalous skin effect is expected[7] to be dominant near the resonance even for relatively low electron temperatures. When the R wave is normally incident on a plasma boundary with perpendicular magnetic field, the wave reflection coefficient is calculated from the electric field expressed in Eq. (14) of Ref. 2. From the coefficient, we obtain a refractive index n, as

$$n=\frac{1}{2}\left\{\sum_{j=1,2}\left[\frac{d\epsilon_j(n)}{dn}\right]^{-1}_{n=n_j}+\frac{(\omega_p/\omega)^2}{(\pi)^{1/2}v_{\rm th}/c}\int_0^\infty \exp(-\tau^{-2})\left[\prod_{j=1,2}\epsilon_j\left(\frac{1-\omega_c/\omega+i\nu/\omega}{v_{\rm th}/c}\tau\right)\right]^{-1}\frac{d\tau}{\tau}\right\}^{-1}, \tag{3}$$

where

$$\epsilon_1(t)=t^2-1-\frac{(\omega_p/\omega)^2}{tv_{\rm th}/c}Z\left(\frac{1-\omega_c/\omega+i\nu/\omega}{tv_{\rm th}/c}\right)$$

and

$$\epsilon_2(t)=t^2-1+\frac{(\omega_p/\omega)^2}{tv_{\rm th}/c}Z\left(-\frac{1-\omega_c/\omega+i\nu/\omega}{tv_{\rm th}/c}\right);$$

Z is the plasma dispersion function tabulated by Fried and Conte[10] and ω_c is the electron cyclotron frequency. The plasma electrons, which are distributed uniformly in the half-space, are assumed to reflect specularly when they are incident on the boundary. The second term in the curly braces of Eq. (3) is due to this effect. Then the electromagnetic field in the plasma does not decay exponentially from the boundary.

If the effect of the thermal electrons is neglected, n is expressed by the cold-plasma theory as

$$n=1-\frac{(\omega_p/\omega)^2(1-\omega_c/\omega)^{-1}}{1+i\nu(1-\omega_c/\omega)^{-1}} \tag{4}$$

for an R wave. By equating Eqs. (3) and (4), the apparent plasma and collision frequencies ω_{pa} and ν_a are defined instead of ω_p and ν. The parameter $(\omega_{pa}/\omega)^2/(\omega_p/\omega)^2$ thus calculated is shown in Fig. 1, for different temperatures near the resonance. The Coulomb collision frequency of electrons with ions, given by Spitzer,[11] is used in the calculation. The apparent normalized plasma density $(\omega_{pa}/\omega)^2$ is smaller than the true normalized density $(\omega_p/\omega)^2$, since the penetration of microwaves increases as a result of the anomalous skin effect.

The value of ω_{pa} becomes smaller than that shown in Fig. 1 if the anomalous skin effect is simply calculated from the inverse value of Im(k). Here k is the wave number obtained from the dispersion equation $\epsilon_z(n)=0$, in an infinite plasma where the thermal motion of electrons is taken into account.

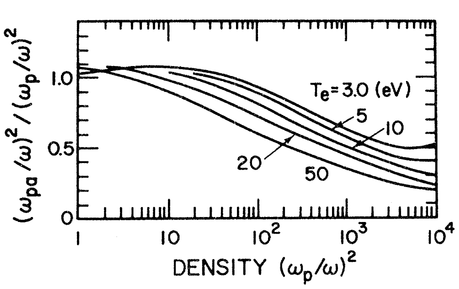

FIG. 1. Calculation of $(\omega_{pa}/\omega)^2/(\omega_p/\omega)^2$ at $\omega_c/\omega=0.95$ for various values of T_e (eV).

The experimental apparatus and procedure are the same as those previously reported.[12] The measurements were performed during the decaying period of a plasma in a 9-GHz circular waveguide. The electron density changes from 10^{15} cm^{-3} to 10^{13} cm^{-3} within the time of 50 μsec. The electron temperature is around 1 eV throughout this decay time. The reflection coefficients were independently measured for the R wave and the L wave (left-hand circularly polarized wave). The electron density determined from the microwave reflection of the R wave corresponds to ω_{pa}. On the other hand, the plasma density determined by the L wave is considered to be exact, since the correction by the anomalous skin effect is not necessary for the L wave because of the absence of the resonance. Figure 2(a) shows the density decay for off resonance. The result for the R wave agrees very well with that for the L wave. In Figs. 2(b) and 2(c) the density is plotted for two different values of ω_c/ω. The discrepancy between two lines of each figure increases as the resonance condition is approached. This results from the fact that ω_{pa}, measured by the reflection of the R wave, should be smaller than the real value ω_p, as is shown in Fig. 1. How-

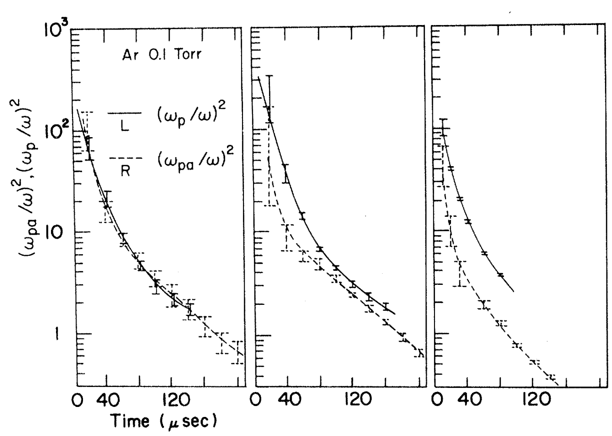

FIG. 2. Electron density decay measured by the R and L waves; (a) $\omega_c/\omega=0.5$, (b) $\omega_c/\omega=0.8$, and (c) $\omega_c/\omega=0.95$.

ever, the quantitative comparison cannot entirely explain the discrepancy. Somewhat surprisingly, ω_{pa} simply calculated from Im(k) is closer to the experimental result than the rigorous theoretical results shown in Fig. (1).

The density, which might not be uniformly distributed near the boundary, may deviate somewhat from measured density. The distribution is governed by the diffusion and recombination loss processes during the decaying period. The exact estimation of the recombination coefficient, which determines the possible nonuniformity of the plasma, and an improved theoretical treatment would be necessary to explain quantitatively the experimental results under the present condition.

Helpful discussions by Dr. Hirshfield and Dr. Hooper are appreciated. Also, the authors are indebted to Dr. Collins for reading the manuscript.

[1]M. J. Kofoid and J. M. Dawson, Phys. Rev. Lett. 17, 1086 (1966); M. J. Kofoid, Phys. Fluids 12, 1290 (1969).

[2]E. S. Weibel, Phys. Fluids 10, 741 (1967).

[3]A. B. Pippard, Proc. Roy. Soc., Ser. A 191, 385 (1947).

[4]V. D. Shafranov, Zh. Eksp. Teor. Fiz. 34, 1475 (1958) [Sov. Phys. JETP 7, 1019 (1958)].

[5]P. M. Platzman and S. J. Buchsbaum, Phys. Rev. 128, 1004 (1962).

[6]R. A. Demirkhanov, I. Ya. Kadysh, and Yu. S. Khodyrev, Zh. Eksp. Teor. Fiz. 46, 1169 (1964) [Sov. Phys. JETP 19, 791 (1964)].

[7]J. E. Drummond, Phys. Fluids 11, 1196 (1968).

[8]J. A. Reynolds, H. A. Blevin, and P. C. Thonemann, Phys. Rev. Lett. 22, 762 (1969).

[9]S. Takeda and M. Roux, J. Phys. Soc. Jap. 16, 95 (1961); S. Takeda and T. Tsukishima, *Microwave Reflection Techniques for Dense Plasma Diagnostics*, U. S. National Bureau of Standards Technical Note No. 256 (U.S.G.P.O., Washington, D. C., 1965).

[10]B. D. Fried and S. D. Conte, *Plasma Dispersion Function* (Academic, New York, 1961).

[11]L. Spitzer, *Physics of Fully Ionized Gases* (Interscience, New York, 1962), Eq. (5-31).

[12]K. Minami and S. Takeda, Phys. Fluids 12, 1089 (1969).

Critical β for Toroidal Equilibrium

H. R. Strauss
Center for Plasma Physics and Thermonuclear Research, The University of Texas at Austin, Austin, Texas 78712
(Received 1 February 1971)

Toroidal, ideal magnetohydrodynamic equilibrium, with magnetic surfaces enclosing a single magnetic axis, is possibly for arbitrary β (plasma pressure/poloidal magnetic pressure). When β exceeds about one-half the aspect ratio, the toroidal current reverses on the inner edge of the plasma. The poloidal current balances the pressure gradient.

We study a class of high-β, axisymmetric, toroidal, ideal magnetohydrodynamic (MHD) equilibria, to see if there is a "critical β" at which the equilibrium either ceases to exist or becomes bad for confinement. (By β we mean the ratio of the plasma pressure to the pressure of the poloidal magnetic field.) We find that an equilibrium exists for all β, with magnetic surfaces enclosing a single magnetic axis. The surfaces are displaced outwards and change shape as they displace. When β exceeds about one-half the aspect ratio, the toroidal current reverses on the inner edge of the plasma. The pressure gradient is balanced by the poloidal current and the toroidal magnetic field.

We first consider the scaling of the equation governing equilibrium. We can remove β, ϵ, and all other free parameters from the equation by changes of scale. All the properties of the solutions can then be seen from a single graph.

The equations of ideal MHD equilibrium are

$$\vec{j}\times\vec{B}=c\nabla p,\quad \nabla\cdot\vec{B}=\nabla\cdot\vec{j}=0,\quad \nabla\times\vec{B}=4\pi\vec{j}/c. \tag{1}$$

Assuming symmetry in the φ direction of a cylindrical coordinate system (R, z, φ), it has been shown[1] that

$$\vec{B}=(2I/cR)\vec{\varphi}+(\nabla\psi\times\vec{\varphi})/2\pi R, \tag{2}$$

$$\vec{j}=j_\varphi\vec{\varphi}+(\nabla I\times\vec{\varphi})/2\pi R, \tag{3}$$

$$\mathcal{L}\psi=-\frac{8\pi^2}{c}\left(2\pi cR^2\frac{dp}{d\psi}+\frac{1}{c}\frac{dI^2}{d\psi}\right)=-\frac{8\pi^2}{c}Rj_\varphi \tag{4}$$

where

$$\mathcal{L}\equiv R\frac{\partial}{\partial R}\frac{1}{R}\frac{\partial}{\partial R}+\frac{\partial^2}{\partial z^2} \tag{5}$$