predicted by (10) is very small, we estimate the background, coming from terms which have the normal (rather than anomalous) behavior as the pion momenta approach zero, to be smaller by a factor of $(\mu/M)^4$ or $(|k_\pi|/M)^4$, where M is some inverse range of interaction.

We wish to thank Dr. E. Abers for stimulating our interest in this subject and Dr. Steven Auerbach for several discussions.

*Work supported by the National Science Foundation.

[1]For a summary, see S. L. Adler and R. F. Dashen, *Current Algebras and Application to Particle Physics* (Benjamin, New York, 1968).

[2]S. L. Adler, Phys. Rev. 177, 2426 (1969).

[3]J. S. Bell and R. Jackiw, Nuovo Cimento 60, 47 (1969).

[4]C. R. Hagen, Phys. Rev. 177, 2622 (1969); R. Jackiw and K. Johnson, Phys. Rev. 182, 1459 (1969); S. L. Adler and W. A. Bardeen, Phys. Rev. 182, 1517 (1969); W. A. Bardeen, Phys. Rev. 184, 1848 (1969).

[5]That is to say, in all models for which the axial current is conserved in the absence of anomalous terms, taking here the conserved axial current, zero-mass pion view of PCAC which we shall henceforth adopt.

[6]S. Brodsky, T. Kinoshita, and H. Terazawa, to be published.

[7]Meaning that we will work only to first order in photon frequencies and to zeroth order in pion frequencies.

[8]This is the rotation generated by the third component of the axial charge $Q_3^{(5)}$. π and σ will transform as $\delta\pi = \sigma\delta\omega$, $\delta\sigma = -\pi\delta\omega$. If we have isospin-$\frac{1}{2}$ Dirac particles, they transform according to $\delta\psi = \frac{1}{2}\tau_3\gamma_5\delta\omega\psi \cdots$, etc. This kind of model was first applied to $\pi^0 \to \gamma+\gamma$ by Bell and Jackiw, Ref. 3.

[9]$-i(S-1)$ is the interaction Lagrangian density integrated over all space-time. We remove the integral to obtain the Lagrangian density.

[10]For example $\partial\mathcal{L}_I/\partial\pi_3$ in a γ_5 coupling theory, for the baryon-loop graphs, is $G_\pi \mathrm{Tr}\gamma_5 G(x,x)$ where $G(x,x)$ is the baryon propagator in all external fields.

[11]The calculation was performed using Schwinger's gauge-invariant Green's function for the baryon, modified to include external π_3 and σ fields. The results will be published elsewhere. See J. Schwinger, Phys. Rev. 82, 664 (1951).

[12]For a detailed model embodying the restricted chiral rotation see Bell and Jackiw, Ref. 3.

[13]Schwinger, Ref. 11, Eq. (5.12).

[14]If the symmetry is broken by a pion-mass term, then we get a PCAC theory; and all of our results would be obtainable in an extrapolation to zero pion four-momenta.

[15]For a review of the nonlinear representation method and its applications, see S. Gasiorowicz and D. Geffen, Rev. Mod. Phys. 41, 531 (1969).

[16]That is, we pick out the part quadratic in $F_{\mu\nu}$ which has to do with the 2γ processes. A peculiarity of the baryon-loop calculation is that processes with more than two photons do not appear in the anomalous terms. In our approach, this follows from Eq. (8) and dimensional arguments only. An additional general conclusion we can draw from Eq. (8) is the absence of anomalous terms for even numbers of pions emitted, as already noted by Adler, Ref. 2.

[17]See, for example, K. C. Wali, Phys. Rev. Lett. 9, 120 (1962).

[18]Precisely, this means the following: In a theory with massless pions and conserved axial current, the matrix element for $\gamma+\gamma \to \pi^+ + \pi^- + \pi^0$ does not vanish for vanishing π_0 momentum, as demanded by a formal application of current algebra.

Is SU(2)⊗SU(2) a Better Symmetry than SU(3)?

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Using the πN phase shifts and fixed-t dispersion relations we have calculated (to first order in symmetry breaking) the nucleon matrix element of the current algebra "sigma" term, and found a value of about 110 MeV. This is an order of magnitude larger than the prediction of the $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ model for chiral symmetry breaking and it indicates that SU(2) ⊗ SU(2) breaking is comparable to SU(3) breaking.

The basic idea of chiral symmetry is that the strong Hamiltonian (density) can be meaningfully written as the sum of a SU(3)⊗SU(3)-invariant piece H_0, plus a correction (small in some sense) H'. It has become popular to look at H' itself as the sum $H' = H_1 + H_2$. H_1 breaks both SU(3)⊗SU(3) and SU(3) but conserves SU(2)⊗SU(2); H_2 then breaks SU(2)⊗SU(2) down to SU(2). It seems safe to assume that SU(2)⊗SU(2) is at least as good a symmetry as SU(3). Thus there are two interesting cases: (i) SU(2)⊗SU(2) and SU(3) breakings are comparable in magnitude, i.e., $H_1 \sim H_2$; and (ii) SU(2)⊗SU(2) is a much better symmetry than SU(3), i.e., $H_1 \gg H_2$. Case (ii) is suggested but not required[1] by the smallness of the pion mass.

In the $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ model of Gell-Mann, Oakes, and Renner[2] and Glashow and Weinberg,[3] we have $H'=u_0+cu_8$ and $H_2=\frac{1}{3}(\sqrt{2}+c)(\sqrt{2}u_0+u_8)$. Fitting the pseudoscalar meson masses gives $c\approx-1.25$ or $\frac{1}{3}(\sqrt{2}+c)\approx 0.05$. We see that the $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ model definitely falls into case (ii). In particular, this model predicts that a typical matrix element of H_2 should be about 10 MeV, i.e., 0.05 times SU(3) breaking. It can be shown[1] that other schemes, for example with H' belonging to $(\underline{8},\underline{8})$, will generally correspond to the opposing case (i). They have typical matrix elements of H_2 with values around 100 to 200 MeV, the size of SU(3) breaking.

Unfortunately, objects like the matrix element of the density H_2 between nucleon states,[4] $\langle N|H_2(0)|N\rangle$, cannot be measured directly. However the closely related quantity,[5]

$$\sigma_{NN}\equiv\tfrac{1}{3}\sum_{a=1}^{3}\langle N|[Q_a^5,[Q_a^5,H_2(0)]]|N\rangle \tag{1}$$

(the nucleon matrix element of the "sigma" commutator, Q_a^5 being axial-vector charges), can be obtained from on-shell πN scattering amplitudes,[6] provided that effects of second order in H_2 can be neglected.[7] A simplified derivation of this connection goes as follows. Consider the process $\pi(q)+N(p)\rightarrow\pi(q')+N(p')$. The Adler consistency conditions (with two- and one-pion reductions) are[8,9]

$$T(\nu=0,\ \nu_B=0,\ q^2=0,\ q'^2=0)=-4f_\pi^2\sigma_{NN}, \tag{2}$$

$$T(0,0,\mu^2,0)=T(0,0,0,\mu^2)=0, \tag{3}$$

where T is related to the conventional πN invariant amplitudes[10] A and B by $T=A+\nu B$ and is isospin even; $\nu=(p+p')\cdot(q+q')/4M$; $\nu_B=-q\cdot q'/2M=(t-2\mu^2)/4M$; μ and M are the pion and nucleon masses, respectively; and f_π is the pion decay constant ($\approx 0.74\mu^{-1}$). The amplitudes with either one or two pions on shell may be expanded in powers of μ^2 (which is of order H_2):

$$\begin{aligned}T(0,0,\mu^2,0)&=T(0,0,0,0)+(\partial/\partial q^2)T(0,0,0,0)\mu^2+O(\mu^4),\\ T(0,0,0,\mu^2)&=T(0,0,0,0)+(\partial/\partial q'^2)T(0,0,0,0)\mu^2+O(\mu^4)\\ T(0,0,\mu^2,\mu^2)&=T(0,0,0,0)+(\partial/\partial q^2)T(0,0,0,0)\mu^2+(\partial/\partial q'^2)T(0,0,0,0)\mu^2+O(\mu^4).\end{aligned} \tag{4}$$

Using Eqs. (2), (3), and (4), one can clearly obtain an expression $T(0,0,\mu^2,\mu^2)$ accurate to order μ^4, in which the off-shell derivatives do not appear; it is

$$T(0,0)=4f_\pi^2\sigma_{NN}+O(H_2^2), \tag{5}$$

where we no longer display the q^2, q'^2 dependence of the on-shell amplitude $T(\nu,\nu_B)$.

To compare Eq. (5) with experiment, we first note that the point $\nu=\nu_B=0$ is clearly outside the physical region. It can, however, be reached by a fixed-t dispersion relation. Using existing results of πN phase-shift analyses, we have made a thorough evaluation of the dispersion integral. Our result is that $T(0,0)$ is roughly $1.7\mu^{-1}$, which by Eq. (5) gives about 110 MeV for σ_{NN}. This is what one would expect in case (i) and would appear to be in serious disagreement with the $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ model.

Our computational method is as follows. Employing the usual broad-area subtraction method, we define a new function,

$$F(\nu)=\frac{T(\nu,0)\nu_1^{2\beta}\nu_2^{2(1-\beta)}}{(\nu_1^2-\nu^2)^\beta(\nu_2^2-\nu^2)^{1-\beta}}, \tag{6}$$

which is equal to $T(0,0)$ when $\nu=0$, which is real analytic and satisfies an unsubtracted dispersion relation. This dispersion integral for $F(0)$, which converges fairly rapidly,[11] is then evaluated using phase-shift analyses. The denominator in F introduces a cut on the real axis (from ν_1 to ν_2) in the ν plane, and the discontinuity of F across this artificial cut is determined by the imaginary and real parts of T. Thus it has the effect of smearing the needed subtraction for T over a region so that our results will not be overly sensitive to errors in the phase shifts at any one point. Furthermore, the presence of these three parameters ν_1, ν_2, and β provides us built-in checks on the compatibility of the phase-shift solutions used with respect to dispersion relations.

Since it is difficult to estimate the errors in each one set of phase-shift analyses, we have made our computation using all the different solutions included in the Berkeley Particle Data Group compilation.[12] The variation in the outputs should give us some idea of the uncertainties in the final result. Details of these calculations will be published elsewhere; here we can illustrate the general nature of the calculation by listing in Table I some numbers obtained by using (A) the CERN experimental phase shifts,[12] and (B) the 0- to 350-MeV solution of Roper, Wright, and Feld,[13] supplimented by the Berkeley[12] or Saclay[12] phase

Table I. $T(0,0)$ computed in units of μ^{-1} with parameters ν_1, ν_2 (in units of μ), and β. Phase-shift sets A and B are as discussed in the text.

ν_1, ν_2 / β	1.52, 2.85		1.18, 1.96		1.74, 2.29		2.07, 3.18	
	A	B	A	B	A	B	A	B
0.9	1.7	1.7	1.6	1.9	2.0	1.6	1.9	1.6
0.8	1.8	1.7	1.6	1.9	2.0	1.6	1.8	1.7
0.7	1.8	1.7	1.6	1.9	2.0	1.6	1.7	1.7
0.6	1.8	1.7	1.6	1.8	2.0	1.6	1.7	1.7
0.5	1.8	1.7	1.6	1.8	2.1	1.6	1.6	1.8
0.4	1.8	1.7	1.7	1.8	2.1	1.6	1.5	1.9
0.3	1.7	1.7	1.8	1.7	2.2	1.6	1.4	2.0
0.2	1.6	1.8	2.0	1.7	2.2	1.6	1.3	2.1
0.1	1.5	1.8	2.1	1.7	2.3	1.6	1.2	2.2

shifts at higher energies.[14]

For the parameters ν_1 and ν_2 the values[15] $\nu_1 = 1.52\mu$ and $\nu_2 = 2.85\mu$ are close to optimal. This allows us to sample ReT over a large area while avoiding the low-energy region $E_\pi \lesssim 70$ MeV where there is very little experimental information, and the region around $E_\pi \approx 300$ MeV where the high- and low-energy phase-shift solutions have to be joined. As β varies from 0 to 1, different segments of the interval $\nu_1 < \nu < \nu_2$ are emphasized. The last three groups of numbers in the table list the values of $T(0,0)$ computed with three more sets of values for the parameters ν_1 and ν_2. One would expect more variations here. The choice $\nu_1 = 1.18\mu$ and $\nu_2 = 1.96\mu$ makes the dispersion integral sensitive to the real part of T in the region $E_\pi \lesssim 70$ MeV, especially when β is near 1. The second choice, $\nu_1 = 1.74\mu$ and $\nu_2 = 2.29\mu$, emphasizes a small region part way up the 3-3 resonance and presumably makes the integral sensitive to the exact shape of the resonance. Finally, with the choice $\nu_1 = 2.07\mu$ and $\nu_2 = 3.18\mu$ the region $E_\pi \gtrsim 300$ MeV begins to make dominant contributions, particularly when β is near zero. Clearly a disadvantage of placing the subtraction cut too high up on the ν axis is that the D, F, G, $\cdots$ phase shifts, which are not so well determined, become important.

Besides performing the computation with all the different phase-shift solutions (and obtaining numbers that are in general agreement with the above-quoted result), we have made further consistency checks. One was to evaluate the dispersion integral using a threshold subtraction which we determined from the S- and P-wave scattering lengths of Hamilton and Woolcock.[10] Another was to replace Roper's low-energy phase shifts by an older ("model-independent") S- and P-wave solution of McKinley,[16] which is adequate if ν_1 and ν_2 are low enough so that the effects of D waves and the Roper resonance are small. Both of these calculations gave results consistent with those discussed above. Also, we should note that our values for $T(0,0)$ lie within the errors quoted by Adler[8] in his original evaluation of the amplitude $A^{(+)}(0,0)$.

Recently Bugg *et al.*[17] have measured the total and differential cross sections of πN scatterings in the range 70 to 290 MeV with an improvement of one order of magnitude in its accuracy over the previous data. By fitting the π^+p total cross section, with the CERN phase shifts for the small waves as background, they have obtained a set of new P_{33} phase shifts with significantly lower values for the resonance mass and width. However the effect of these changes on $T(0,0)$ was found to be unimportant. Thus we anticipate that the qualitative nature of our conclusion should be fairly stable with respect to future changes in the πN phase shifts.

The known smallness of the isospin-even scattering length[10] $a_1 + 2a_3$ is often associated with the assumed smallness of σ_{NN}.[8] In fact the scattering length gives a value of the amplitude at the physical threshold, $T(\mu, -\mu^2/2M)$, which is at least an order of magnitude smaller than would be suggested by naive extrapolation of our value for $T(0,0)$. The way this appears to come about is most interesting. Near the point $\nu = \nu_B = 0$, T can be approximated by[9]

$$T \approx \frac{g_r^2}{M}\frac{\nu_B^2}{\nu_B^2 - \nu^2} + T(0,0)$$

$$= \frac{[g_r^2/M + T(0,0)]\nu_B^2 - T(0,0)\nu^2}{\nu_B^2 - \nu^2}, \qquad (7)$$

where g_r is the pion-nucleon coupling constant. Taking into account the fact that $T(0,0)$ is posi-

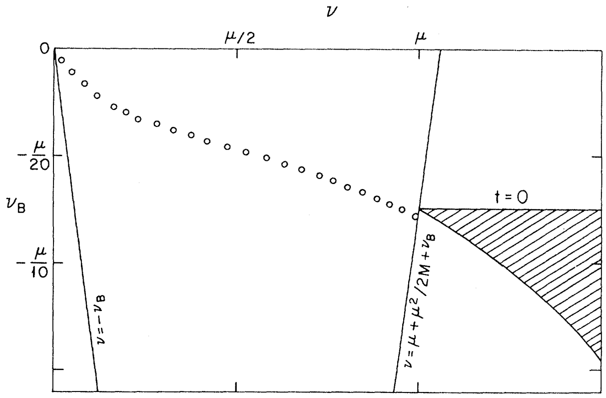

FIG. 1. Line of zeros of the amplitude (open circles) in the Euclidean region. The two straight lines correspond to positions of poles and threshold of ν for fixed ν_B. Shaded area is the physical region.

tive and small compared to g_r^2/M, one easily sees that near $\nu=\nu_B=0$, T is zero along $\nu \approx \pm g_r \times [MT(0,0)]^{-1/2}\nu_B$. This must be a segment of a line of zeros of T. Such a line cannot leave the real ν-ν_B plane in a region where T is analytic. The zeros start out in the general direction of the threshold, but if the line were a straight one with the above slope it would miss the threshold by a considerable margin. We have followed the zeros numerically. As shown in Fig. 1, the string of zeros leaves $\nu=\nu_B=0$ with the expected slope, then curves a bit and heads for a point very close to the threshold. Needless to say, this makes T very small there. Thus we see that the present data seem to lead to a completely consistent, if somewhat surprising, picture.

Our numbers for σ_{NN} are in disagreement with a number of previous estimates, notably those of Kim and von Hippel.[18] We do not understand the reason for this and would prefer not to speculate.

After this Letter was written, F. von Hippel informed one of us (R.D.) that he has begun some similar calculations.

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[1]R. Dashen, to be published.

[2]M. Gell-Mann, R. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).

[3]S. Glashow and S. Weinberg, Phys. Rev. Lett. 20, 224 (1968).

[4]Our normalization is such that $\langle N|H_2(0)|N\rangle$ is equal to the contribution of H_2 to the nucleon mass.

[5]In the $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ model σ_{NN} happens to be exactly equal to $\langle N|H_2(0)|N\rangle$. In general σ_{NN} should be of the same order of magnitude as $\langle N|H_2(0)|N\rangle$.

[6]R. Dashen and M. Weinstein, Phys. Rev. 188, 2330 (1969). In this paper it is incorrectly stated that σ_{NN} is related to the spin-averaged amplitude which differs slightly from T defined below.

[7]In case (ii) where SU(2) ⊗ SU(2) is much better than SU(3), the neglected second-order effect should be particularly harmless.

[8]See, for example, S. Adler and R. Dashen, *Current Algebra and Applications to Particle Physics* (Benjamin, New York, 1968).

[9]We always approach the point $\nu=\nu_B=0$ by first setting $\nu_B=0$ and then $\nu=0$. The nucleon pole vanishes in this limit.

[10]See, for example, R. G. Moorhouse, Annu. Rev. Nucl. Sci. 19, 301 (1969).

[11]At high energies ($E_\pi \gtrsim 2$ GeV) where phase shifts are not available, we assume that the diffractive form $\mathrm{Im}T=e^{at}\,\mathrm{Im}T|_{t=0}$, which works well for negative t, can be extrapolated to $\nu_B=0$ or $t=2\mu^2$. Since this high-energy tail typically contributes about $-0.2\mu^{-1}$ to $T(0,0)$, errors introduced by this assumption should be negligible.

[12]Except for L. Roper, R. Wright, and B. Feld, Phys. Rev. 138, B190 (1965), we have taken all our phase shifts from the Berkeley compilation: D. J. Herndon, A. Barbaro-Galtieri, and A. H. Rosenfeld, UCRL Report No. UCRL-20030, 1970 (unpublished). We are grateful to these authors for sending us a tape containing the solutions.

[13]Roper, Wright, and Feld, Ref. 12.

[14]In case (B), the calculated values of $T(0,0)$ are essentially independent of whether we use Berkeley Path 1, Berkeley Path 2, or Saclay solutions. When listing numbers we will not distinguish among these high-energy solutions.

[15]We note that for $\nu_B=0$, the threshold of ν is at 1.07μ; the peak of (3, 3) resonance is at 2.44μ; and the high- and low-energy solutions are usually joined at $\nu=3.3\mu$.

[16]J. McKinley, Rev. Mod. Phys. 35, 788 (1963).

[17]D. V. Bugg, P. J. Bussey, A. A. Carter, D. R. Dance, and J. R. Williams, unpublished. We are grateful to Dr. R. Plano for making available to us this unpublished manuscript.

[18]F. von Hippel and J. Kim, Phys. Rev. D 1, 151 (1970).