parisons are shown in Fig. 1. We made also direct comparisons of our calculations with experimental observations of p-p scattering at 0.67, 0.97, and 1.4 GeV and found qualitative agreement. Examples are in Fig. 2. In the calculations of the observed quantities, real phase shifts for states with $l>2$ are approximated by those from the first-order calculation of the OBE contribution corrected for unitarity by the damping relation.[1] As for the $^3S_1$, $^3D_{1,2,3}$, $^1P_1$, and $^1F_3$ states, comparison of our calculation with the Kantor amplitudes is shown in Fig. 3 and satisfactory agreement is found. However, the comparisons in the left-hand-cut integral terms for the transitions between the $^3D_1$ states and between the $^3S_1$ and $^3D_1$ states suffer from considerable uncertainty of the deuteron-pole contributions, since the contributions to the transitions between the $^3D_1$ states and between the $^3S_1$ and $^3D_1$ states depend on asymptotic D- to S-wave ratio of the deuteron wave functions quadratically and linearly, respectively, and this ratio has considerable uncertainty at present.

Thus we summarize our results: The OBEM involving the π, ω, ρ, η_V, σ, and f with a nucleon-meson form factor and a high-energy cutoff describes satisfactorily all the N-N elastic data and the p-p scattering at 0.67, 0.97, and 1.4 GeV, and also is not inconsistent with the data at 2 and 2.85 GeV. It is noted that the f does not destroy the repulsive interaction in the $^1S_0$ state at high energies. More details will be published elsewhere.

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Axial-Current Divergences and the Reactions $\gamma+\gamma\to$ Pions*

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A symmetry argument is presented that gives a relation between the small-momentum limits of the reactions $\gamma+\gamma\to$ odd number of pions, as calculated explicitly from a single nucleon (or quark) loop in a theory with a formally conserved axial-vector current. These limits are nonvanishing as a consequence of the Adler anomalies in the divergence of the axial-vector current. It is argued that the relationship derived should be true in any chiral model. The cross section for $\gamma+\gamma\to\pi^++\pi^-+\pi^0$ is calculated in terms of the measured $\pi^0\to2\gamma$ decay rate.

The assumptions of current algebra and partial conservation of axial-vector current (PCAC) lead to a number of predictions about emission of soft pions in hadronic processes.[1] However in certain electromagnetic processes, notably $\pi^0\to2\gamma$, there have been found both theoretical and experimental deviations from the predictions of PCAC and current algebra applied in an uncritical way.[2,3] On the experimental side the $\pi^0\to2\gamma$ rate is comparable to that estimated on the basis of coupling constants and phase space alone, and not suppressed as predicted by a naive application of current algebra. On the theoretical side Adler and others have argued that the PCAC equation requires modification in the presence of electromagnetism, in order to take into account the contributions of some baryon-loop graphs.[2,4] We shall hereafter refer to such terms as the anomalous terms.

The actual coefficient of the anomalous term responsible for $\pi^0 \to 2\gamma$ has been found to be quite model dependent; thus there is no reliable quantitative explanation of the decay rate.[2] The present work is concerned with reactions such as $\gamma + \gamma \to 3\pi$ which also should be suppressed according to naive PCAC (actually forbidden to the lowest order of the pion momenta), but for which one can expect an anomalous contribution similar to that for $\pi^0 \to 2\gamma$.

Although a baryon-loop calculation of this amplitude would be model dependent, as is the calculation for $\pi^0 \to 2\gamma$, we find that the ratio between the two amplitudes can be determined in a model-independent way, as long as the dynamical framework is one in which the Adler anomaly has a definite meaning.[5] We thus can predict the cross sections for the reaction $\gamma+\gamma \to \pi^+ + \pi^- + \pi^0$ for production at fairly low invariant masses [say, for $M(3\pi) < 5\mu$].

Brodsky, Kinoshita, and Terazawa have recently pointed out the possibility of measuring cross sections such as $\gamma+\gamma \to 3\pi$ in electron colliding-beam experiments[6]; we therefore regard this prediction as providing a conceivable (and perhaps unique) experimental test of the Adler mechanism.

Our results are based on the consideration of the vacuum-to-vacuum S-matrix element in external, neutral, c-number pion fields π_3 and in external electromagnetic fields $F_{\mu\nu}$. Both π_3 and $F_{\mu\nu}$ will be taken as constant in space and in time.[7] Temporarily (and only for expository reasons) we shall add an external scalar field σ. This field will be required to transform with π_3 as a two-dimensional vector under the restricted chiral symmetry preserved by the electromagnetic coupling, in a theory with charged Dirac particles.[8]

Now we consider a contribution to the vacuum S-matrix element arising from a closed loop, with arbitrary radiative corrections. We demand only (a) that the graphs be constructed in a theory with the restricted chiral symmetry, with the electromagnetic current an invariant under the $Q_3^{(5)}$ transformation, and with π_3 and σ transforming as stated above; and that (b) the graphs cannot be cut in two pieces by cutting only π_3, σ, or photon lines. The quantity $\langle 0|S-1|0\rangle$ determined from the sum of such graphs, and considered now as a function of the external fields, can be thought of as giving rise to an effective interaction Lagrangian density[9] $\mathcal{L}_I$ for reactions involving only π's, σ's, and γ's.

The conclusion of previous authors is that $\mathcal{L}_I$ itself is too divergent a quantity for the argument that $\mathcal{L}_I$ is a chiral invariant to be justified. We circumvent this problem by considering instead the derivatives of $\mathcal{L}_I$ with respect to external fields, $\partial\mathcal{L}_I/\partial\sigma$ and $\partial\mathcal{L}_I/\partial\pi_3$. These are the quantities which are most directly computed in a specific model[10] and which, in a proton-loop calculation,[11] turn out to be sufficiently convergent to exhibit the chiral symmetry of the theory. We conclude that the pair $(\partial\mathcal{L}_I/\partial\pi, \partial\mathcal{L}_I/\partial\sigma)$ should transform as a two-vector under the $Q_3^{(5)}$ rotation [under which (π_3, σ) rotates in the same way as (x,y) does under a rotation around the z axis].[11]

At first sight this would seem to imply that $\mathcal{L}_I$ is invariant under the restricted chiral transformation.[12] However this is not the case, as we see from the following considerations: The pair $(\sigma, -\pi_3)$ is a vector which transforms exactly as does (π_3, σ). Thus for the derivatives of $\mathcal{L}_I$ we have the covariant structure

$$\partial\mathcal{L}_I/\partial\pi_3 = \pi_3 A(\pi_3^2+\sigma^2, F_{\mu\nu}) + \sigma B(\pi_3^2+\sigma^2, F_{\mu\nu}),$$
$$\partial\mathcal{L}_I/\partial\sigma = \sigma A(\pi_3^2+\sigma^2, F_{\mu\nu}) - \pi_3 B(\pi_3^2+\sigma^2, F_{\mu\nu}). \quad (1)$$

The direct calculation of the nucleon loop confirms the fact that the functional B is not generally zero.

As a condition of the integrability of Eq. (1), we find immediately that

$$B = R(F_{\mu\nu})/[\pi_3^2+\sigma^2] \quad (2)$$

and

$$\mathcal{L}_I = \tfrac{1}{2}\int_0^{\pi_3^2+\sigma^2} dx\, A(x, F_{\mu\nu}) + R(F_{\mu\nu}) \tan^{-1}\left(\frac{\pi_3}{\sigma}\right). \quad (3)$$

In this expression for $\mathcal{L}_I$ the second term on the right-hand side does not display the chiral symmetry of the underlying Lagrangian. This noninvariant part of $\mathcal{L}_I$ is precisely the Adler anomaly. In a model with a single nucleon or quark loop we have verified the form of this anomalous term (in its dependence on π_3 and σ) explicitly; only the coefficient R depends upon which model is employed. From the above we see that the possibility for such an anomaly stems from the fact that in two dimensions a vector function of x and y may be the gradient of a function which is not rotationally invariant.

To obtain the effective Lagrangian for $\pi^0 \to \gamma+\gamma$ we expand the $\tan^{-1}$ function in (3) to first order in the pion field. As will be made clear below, for single-pion emission σ should be replaced by $f \cong M_p/G_\pi$. In the proton-loop example we find $R(F_{\mu\nu}) = (-1/\pi)(\vec{E}\cdot\vec{B})\alpha$. Thus we obtain

$$L_{\text{eff}} = (-\alpha/\pi)(G_\pi/M_p)\pi_3(\vec{E}\cdot\vec{B}), \tag{4}$$

in agreement with Schwinger.[13]

Next we repeat the above considerations with σ eliminated as a field. We consider the vacuum S matrix as a function of external pion and electromagnetic fields alone. The models to be considered are those with a conserved axial current, $J_{\mu 3}{}^{(5)}$, and with a massless pion.[14] This is the most general class of models relevant to the Adler anomaly, that is, in which a nonvanishing $\partial_\mu J_{\mu 3}{}^{(5)}$ would be classed as anomalous. It is also the framework which best explains the successes of current algebra.

The pion transformation in such models is nonlinear.[15] We choose the simplest nonlinear transformation for π_3 under the restricted chiral rotation,

$$\delta\pi_3 = (f^2 - \pi_3{}^2)^{1/2}\delta\omega. \tag{5}$$

In principle f^2 here should be replaced by $f^2 - \pi_1{}^2 - \pi_2{}^2$, where π_1 and π_2 are the real components of the π_+ and π_- fields, invariant under the restricted chiral rotation. However, to calculate effective Lagrangians for π^0 emission only, we may set the external π_1 and π_2 fields equal to zero from the beginning.

Under this transformation we can form two two-vector functions of π_3:

$$\pi_3,\ (f^2 - \pi_3{}^2)^{1/2}$$

and

$$(f^2 - \pi_3{}^2)^{1/2},\ -\pi_3.$$

We can also write a differential form which transforms as a two-vector:

$$D_x = (f^2 - \pi_3{}^2)d/d\pi_3,$$

$$D_y = -\pi_3(f^2 - \pi_3{}^2)^{1/2}d/d\pi_3. \tag{6}$$

We then consider the pair $(D_x\mathfrak{L}_I, D_y\mathfrak{L}_I)$ and, as previously, demand that it transform like a two-vector:

$$D_x\mathfrak{L}_I = \pi_3 A(F_{\mu\nu}) + (f^2 - \pi_3{}^2)^{1/2}B(F_{\mu\nu}),$$

$$D_y\mathfrak{L}_I = (f^2 - \pi_3{}^2)^{1/2}A(F_{\mu\nu}) - \pi_3 B(F_{\mu\nu}). \tag{7}$$

In this construction we eliminate the π_3 dependence of A and B by noting that there exists no invariant function of π_3 alone, except for a constant. The integral for $\mathfrak{L}_I$ is

$$\mathfrak{L}_I = [\sin^{-1}(\pi_3/f)]B(F_{\mu\nu}) + C(F_{\mu\nu}). \tag{8}$$

In Eq. (8) we now take $B(F_{\mu\nu}) = a(\vec{E}\cdot\vec{B})$, where a is chosen to fit the $\pi^0 \to \gamma+\gamma$ lifetime.[16] Equation (8) can be immediately generalized to include the possibility of the process $\gamma+\gamma \to \pi^+ + \pi^- + \pi^0$ by changing $(\pi_3)^3$ on the right-hand side to $(\vec{\pi}\cdot\vec{\pi})\pi_3$. Here we have used the complete orbital symmetry of the 3π state in question, plus the $\Delta I < 3$ selection rule of second-order electromagnetism, to conclude that the three-pion state must have $I = 1$.[17]

To calculate $\gamma+\gamma \to 3\pi^0$ or $\pi^+ + \pi^- + \pi^0$ we must add to the direct contribution from (8) a term which comes from $\gamma+\gamma \to \pi^0$, followed by $\pi^0 \to 3\pi$. The $\pi^0 \to 3\pi$ vertex comes from the kinematical term of the total Lagrangian required to give invariance under the particular nonlinear transformation law chosen; in this case,

$$\mathfrak{L}_0 = \tfrac{1}{2}(\partial_\mu\vec{\pi})^2 + \tfrac{1}{2}[\partial_\mu(f^2 - \vec{\pi}\cdot\vec{\pi})^{1/2}]^2. \tag{9}$$

Our final result, combining both terms, is independent of the particular nonlinear transformation law chosen.

From the two terms together we get matrix elements (for all pions massless):

$$\mathfrak{M}(\gamma\gamma \to 3\pi^0) = 0,$$

$$\mathfrak{M}(\gamma\gamma \to \pi^+\pi^-\pi^0)$$

$$= \left(\frac{4\pi\Gamma^0}{\mu}\right)^{1/2}\left(\frac{4}{\mu f^2}\right)\left(\frac{1}{3} - \frac{(k_+ + k_-)^2}{(k_+ + k_- + k_0)^2}\right)(\vec{E}\cdot\vec{B}). \tag{10}$$

Here $f = (\sqrt{2}\mu^2)^{-1}f_\pi$, where f_π is defined as in Ref. 1 and has the numerical value $0.96\mu^3$; k_+, k_-, and k_0 are the pion four-momenta; and Γ^0 is the $\pi^0 \to \gamma+\gamma$ width. Thus we find a contribution from an Adler anomaly to the process $\gamma+\gamma \to \pi^+ + \pi^- + \pi^0$.[18]

Although for the purposes of calculating a number it is tempting to insert a pion mass at least into the denominator of Eq. (10), the effects of pion mass on the matrix element are in fact completely model dependent. For, say, $s = 4\mu^2$, we estimate an uncertainty from this source of up to 20%. For the phase-space integral we have taken into account the pion mass. As an example, we find the cross section for $\gamma+\gamma \to \pi^+ + \pi^- + \pi^0$ at $s = 4\mu^2$ to be 0.6×10^{-35} cm^2.

Our main predictions are thus the cross section implied by (10), and the prediction that $\gamma+\gamma \to 3\pi_0$ is greatly suppressed compared to $\gamma+\gamma \to \pi^+ + \pi^- + \pi^0$. Although the $\gamma+\gamma \to \pi^+ + \pi^- + \pi^0$ cross section predicted by (10) is very small, we estimate the background, coming from terms which have the normal (rather than anomalous) behavior as the pion momenta approach zero, to be smaller by a factor of $(\mu/M)^4$ or $(|k_\pi|/M)^4$, where M is some inverse range of interaction.

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[5]That is to say, in all models for which the axial current is conserved in the absence of anomalous terms, taking here the conserved axial current, zero-mass pion view of PCAC which we shall henceforth adopt.

[6]S. Brodsky, T. Kinoshita, and H. Terazawa, to be published.

[7]Meaning that we will work only to first order in photon frequencies and to zeroth order in pion frequencies.

[8]This is the rotation generated by the third component of the axial charge $Q_3^{(5)}$. π and σ will transform as $\delta\pi = \sigma\delta\omega$, $\delta\sigma = -\pi\delta\omega$. If we have isospin-$\frac{1}{2}$ Dirac particles, they transform according to $\delta\psi = \frac{1}{2}\tau_3\gamma_5\delta\omega\psi \cdots$, etc. This kind of model was first applied to $\pi^0 \rightarrow \gamma+\gamma$ by Bell and Jackiw, Ref. 3.

[9]$-i(S-1)$ is the interaction Lagrangian density integrated over all space-time. We remove the integral to obtain the Lagrangian density.

[10]For example $\partial\mathfrak{L}_I/\partial\pi_3$ in a γ_5 coupling theory, for the baryon-loop graphs, is $G_\pi \operatorname{Tr}\gamma_5 G(x,x)$ where $G(x,x)$ is the baryon propagator in all external fields.

[11]The calculation was performed using Schwinger's gauge-invariant Green's function for the baryon, modified to include external π_3 and σ fields. The results will be published elsewhere. See J. Schwinger, Phys. Rev. 82, 664 (1951).

[12]For a detailed model embodying the restricted chiral rotation see Bell and Jackiw, Ref. 3.

[13]Schwinger, Ref. 11, Eq. (5.12).

[14]If the symmetry is broken by a pion-mass term, then we get a PCAC theory; and all of our results would be obtainable in an extrapolation to zero pion four-momenta.

[15]For a review of the nonlinear representation method and its applications, see S. Gasiorowicz and D. Geffen, Rev. Mod. Phys. 41, 531 (1969).

[16]That is, we pick out the part quadratic in $F_{\mu\nu}$ which has to do with the 2γ processes. A peculiarity of the baryon-loop calculation is that processes with more than two photons do not appear in the anomalous terms. In our approach, this follows from Eq. (8) and dimensional arguments only. An additional general conclusion we can draw from Eq. (8) is the absence of anomalous terms for even numbers of pions emitted, as already noted by Adler, Ref. 2.

[17]See, for example, K. C. Wali, Phys. Rev. Lett. 9, 120 (1962).

[18]Precisely, this means the following: In a theory with massless pions and conserved axial current, the matrix element for $\gamma+\gamma \rightarrow \pi^+ + \pi^- + \pi^0$ does not vanish for vanishing π_0 momentum, as demanded by a formal application of current algebra.

Is SU(2)⊗SU(2) a Better Symmetry than SU(3)?

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Using the πN phase shifts and fixed-t dispersion relations we have calculated (to first order in symmetry breaking) the nucleon matrix element of the current algebra "sigma" term, and found a value of about 110 MeV. This is an order of magnitude larger than the prediction of the $(\underline{3}, \underline{3}^*) \oplus (\underline{3}^*, \underline{3})$ model for chiral symmetry breaking and it indicates that SU(2) ⊗ SU(2) breaking is comparable to SU(3) breaking.

The basic idea of chiral symmetry is that the strong Hamiltonian (density) can be meaningfully written as the sum of a SU(3)⊗SU(3)-invariant piece H_0, plus a correction (small in some sense) H'. It has become popular to look at H' itself as the sum $H' = H_1 + H_2$. H_1 breaks both SU(3)⊗SU(3) and SU(3) but conserves SU(2)⊗SU(2); H_2 then breaks SU(2)⊗SU(2) down to SU(2). It seems safe to assume that SU(2)⊗SU(2) is at least as good a symmetry as SU(3). Thus there are two interesting cases: (i) SU(2)⊗SU(2) and SU(3) breakings are comparable in magnitude, i.e., $H_1 \sim H_2$; and (ii) SU(2)⊗SU(2) is a much better symmetry than SU(3), i.e., $H_1 \gg H_2$. Case (ii) is suggested but not required[1] by the smallness of the pion mass.