

Repulsive Effects Due to the Absorption in Exotic Atoms*

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An attractive K^- -nuclear strong interaction as favored by a recent measurement weakens the atomic binding. The repulsion is due to the absorption in the nucleus and starts to dominate over the strong attraction. It is also against normal perturbation calculational experience that a smaller absorptive potential may produce a higher absorption rate. These highly nonlinear phenomena are displayed. The gradient potential has little influence for kaonic atoms.

We may obtain numerically in two ways the widths Γ_n and shifts \mathcal{E}_n which are produced by the complex nuclear potential in the x-ray lines (shifted from the pure Coulomb case) of exotic atoms: either by perturbation calculation, or by solving directly the eigenvalue problem of a wave equation with a complex potential. A recent experiment by the CERN group¹ favors the result of the latter method which has been used in a previous analysis of pionic atoms.² With the earlier measurements by Wiegand³ of yields of radiative transitions in kaonic atoms alone, one could not yet decide whether a complete solution is really necessary. In fact the analyses by Ericson and Scheck⁴ and Bethe and Siemens⁵ explained these measurements on a perturbation-calculational basis. The measurement¹ of the exact energy and shape of the kaonic $4f$ - $3d$ transition in sulfur, however, gives full information about the broadening ($\Gamma_{3d} = 2.2 \pm 0.6$ keV) and the shift ($\mathcal{E}_{3d} = -0.36 \pm 0.25$ keV) of the relatively low-lying $3d$ state. The calculation on the data indicates also that the assumption of repulsive real K^- -nuclear interaction as supposed from the negative real part of the K, N scattering length is probably not fulfilled.⁶ The discussion will be certainly extended very soon to the newly discovered antiprotonic atoms.⁷

The following result is of general importance: If a nuclear potential is both attractive and absorptive to a particle, the repulsive effect due to the absorption may dominate and lead to a net repulsion when a certain probability of finding the particle in the nucleus is reached. For exotic atoms such a potential leads to positive (attractive) shifts in the weakly distorted higher states, but to negative (repulsive) shifts in the strongly distorted lower states.⁸ Moreover, the absorption rate or the level broadening Γ_n shows a tendency to saturate. This last observation cannot be ruled out completely because of the limiting condition for the applicability of the complex po-

tential model ($\Gamma_n < \Delta E$, level broadening smaller than level spacing). Both shifts and widths show a strongly oscillating behavior on variation of the strength of either the attractive real potential or the imaginary part. Thus, making the imaginary potential smaller may nevertheless increase the absorption rate. We explain these features with the help of calculations on K -atomic sulfur, for which the experimental result is known.¹

We assume for sulfur ($A = 2Z = 32$) the nuclear density

$$\rho(r) = N \{1 + \exp[4 \ln 3(r-c)/t]\}^{-1}. \quad (1)$$

The normalization is given by

$$4\pi \int_0^\infty \rho(r) r^2 dr = A.$$

The half-density radius $c = 3.20$ fm and the surface thickness $t = 2.60$ fm are taken to be equal to those of the charge density.⁹ The low-energy behavior of the K, N interaction¹⁰ defines the structure of the K^- -nuclear potential⁴ (with $\hbar = c = 1$):

$$2\mu V = -4\pi p \{ \bar{A}^0 \rho(\vec{r}) - 3p^{-2} [\nabla \bar{A}^1 \rho(\vec{r}) \nabla] \}. \quad (2)$$

Here μ is the reduced mass of the kaon and the nucleus and p represents the kinematical factor $1 + m_K/m_N$, where m_K and m_N are the kaon and the nucleon mass, respectively. \bar{A}^0 and \bar{A}^1 are the average effective and complex K, N s -wave and p -wave scattering lengths, respectively.

This complex nuclear potential has basically the same form as that for pions though one does not expect a noticeable contribution from the gradient term.⁴ It has to be treated in a Klein-Gordon-type equation with the electromagnetic potential V_c from the charge distribution proportional to expression (1). Thus

$$\{ \nabla^2 + [(\tilde{E} - V_c)^2 - \mu^2] \} \psi = 2\mu V \psi. \quad (3)$$

Here \tilde{E} is the relativistic energy, $\tilde{E} = E + \mu$. The numerical calculation can be done with exactly the same methods as given previously in the pionic atom case.^{11,2}

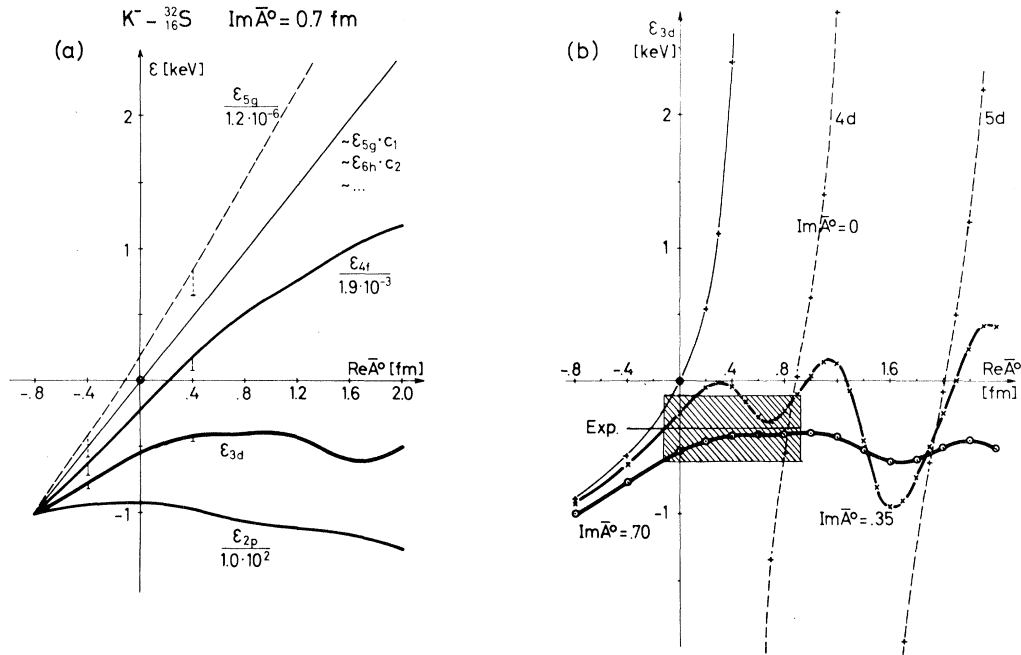


FIG. 1. Energy shifts \mathcal{E} as a function of the real part of the local nuclear potential. (a) Reduced representation shows influence of nuclear absorption. Error bars indicate the change due to twice the predicted gradient term. (b) The oscillating behavior is resolved for the $3d$ shift by decreasing absorption. "Exp." marks the experimental value for \mathcal{E}_{3d} with uncertainty indicated by the hatched area.

We see in Fig. 1 the shift \mathcal{E} from the binding energies $-E^0$ of a hypothetical point nucleus when a complex nuclear potential is introduced. The point-nuclear energies are, in keV, $-E_{5g}^0 = 132.488$, $-E_{4f}^0 = 207.040$, $-E_{3d}^0 = 368.200$, and $-E_{2p}^0 = 829.536$. The electromagnetic finite-size effect is included in the shifts. The shifts are shown as a function of the strength of the real part of the local nuclear potential. Figure 1(a) shows the behavior at a fixed $\text{Im}\bar{A}^0 = 0.7$ fm—a reasonable assumption according to Refs. 10, 4, and 5—for various circular-orbit states. The shifts (except for $3d$) are divided by such a factor that they all go through the same point at $\text{Re}\bar{A}^0 = -0.8$ fm in order to allow a better comparison of the slopes of the curves. A first-order perturbation theory gives curves going through the origin. The higher states ($5g$ and above) follow closely such a behavior, but the lower the state the greater the deviation from the perturbative description. For the $3d$ state and for the lower states we find, accepting the value $\text{Im}\bar{A}^0 = 0.7$ fm, that the shift remains negative for increasing real nuclear attraction. The curves are calculated with $\bar{A}^1 = 0$. However, for the points $\text{Re}\bar{A}^0 = -0.4$ and $+0.4$ fm, we also give in the form of error bars the changes produced by assuming $\text{Re}\bar{A}^1 = -0.1$ fm³ (twice the predicted value). It is

seen that the influence of the gradient term is small. The dotted thin line indicates the numerical error due to a finite step length for integration¹³ and computer "noise." For these tiny shifts (of the order of 10^{-6} keV compared with $-E_{5g}^0 = 132$ keV), the perturbation calculation turns out to be more accurate.

Because of a certain oscillating structure of the curves, we display two other calculations on the $3d$ shifts in Fig. 1(b). The medium-thick line is obtained with $\text{Im}\bar{A}^0 = 0.35$ fm and the thin line without absorption at all. We have explicitly marked the points at which calculation has been done in order to demonstrate the continuity of the curves. The oscillations appear just at those values of the strength of attraction where, in the nonabsorptive case, the binding energies of the higher principal quantum-number ($n > 3$) states with the same orbital angular momentum number ($l = 2$) reach the binding energy of the point-nuclear $3d$ state. Compared to this no-absorption case the stabilizing effect of (even a smaller) absorption is impressive. The figure also contains the experimental result of the CERN group.¹ The region where the measurement is still compatible with the calculation is indicated by a hatched area [see also Fig. 2(b)]. We note that, contrary to former predictions, an attractive potential of a

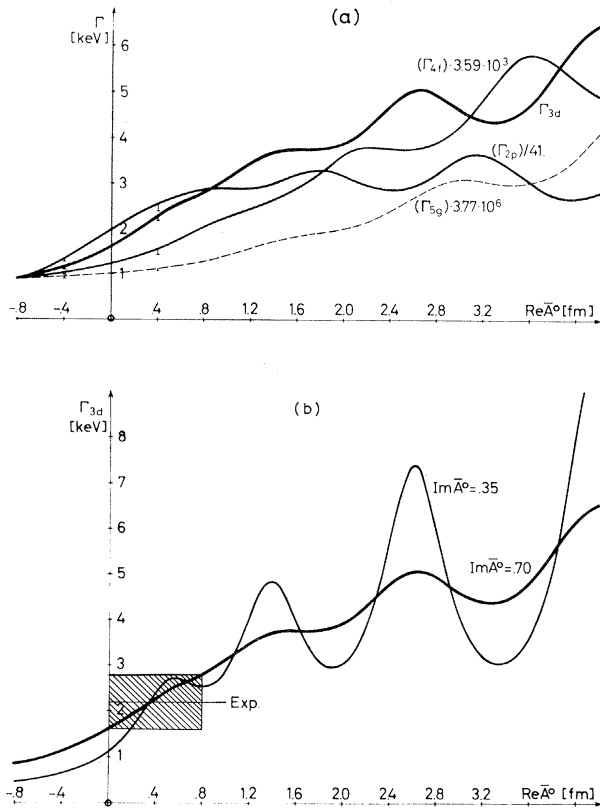


FIG. 2. Broadenings Γ corresponding to states of Fig. 1 as function of $\text{Re}\bar{A}^0$. (a) Curves calculated with $\text{Im}\bar{A}^0 = 0.7$ fm show in reduced representation the influence of absorption along with changing the strength of the real potential. Error bars are for twice the predicted gradient term. (b) Smaller absorption yields stronger oscillating curves in the region of attraction. Experimental result marked by hatched area.

strength $\text{Re}\bar{A}^0 \approx 0.4$ fm is favored.¹²

A peculiarity is indicated by the thin line in Fig. 1(b). We learn that already for small attraction a perturbation calculation, producing the widths Γ by wave functions which include the real interaction, may yield worse results than a simple perturbation calculation with point-nuclear wave functions.

In Fig. 2(a) the widths for the same states as in Fig. 1(a) are drawn for $\text{Im}\bar{A}^0 = 0.7$ fm and for variable real interaction. Again a reduced scale is used to make the slopes of the curves explicit. The lower states show a kind of "oscillating saturation." Error bars, indicating the change which the inclusion of twice the predicted gradient term produces, show again the insignificance of the $K, N p$ -wave interaction.

Figure 2(b) shows the situation for the $3d$ state in comparison with a calculation for an absorptive potential of half the predicted strength. The

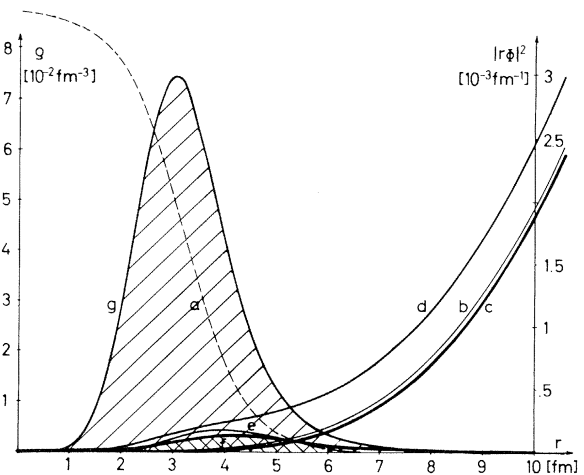


FIG. 3. Densities $|r\Phi|^2$ of the K -atomic $3d$ state in sulfur are shown by curves $b, c,$ and d . Curve a represents ρ , the nuclear density; $e, f,$ and g are for the overlap functions $\rho|r\Phi|^2$. Curve b with e is for "no," and c with f for "full" nuclear potential; curve d with g is for pure attractive potential.

striking feature is the strong oscillation of Γ as a function of $\text{Re}\bar{A}^0$ for smaller $\text{Im}\bar{A}^0$. With the above mentioned special perturbation calculation, we obtain for any absorptive potential a curve Γ which behaves proportional to the thin line in Fig. 1(b). Only for very small values of $\text{Im}\bar{A}^0$ or repulsive $\text{Re}\bar{A}^0$ does this perturbation calculation yield an approximate description in the lower states. The measured value¹ is again marked by a hatched area. One finds, in agreement with the result given in Fig. 1(b), that $\text{Re}\bar{A}^0 \approx 0.4$ fm.¹²

Finally, in Fig. 3 are presented the densities of the nucleus (ρ , curve a) and of the probability of finding the kaon, multiplied by r^2 (i.e., $|r\Phi|^2$, curves $b, c,$ and d), and the overlap function (i.e., $\rho|r\Phi|^2$, curves $e, f,$ and g), for various assumptions of the complex potential strength \bar{A}^0 . This figure refers to the $3d$ state in sulfur. Curve b and the corresponding overlap, shown in curve e , result from the unperturbed case ($\text{Re}\bar{A}^0 = \text{Im}\bar{A}^0 = 0$). The kaon-state density c and the overlap function f are found for the complete nuclear potential $\text{Re}\bar{A}^0 = 0.4$, $\text{Im}\bar{A}^0 = 0.7$ fm, which yields the level shift $\mathcal{E} = -0.43$ keV and the broadening $\Gamma = 2.27$ keV. We see clearly that the attraction is already overcompensated by the repulsion due to the absorption. A simple perturbation calculation on Γ dealing with the area below e instead of f could still be acceptable. The situation for attraction alone ($\text{Re}\bar{A}^0 = 0.4$ fm, $\text{Im}\bar{A}^0 = 0$), however, yields quite different $|r\Phi|^2$ (curve d) and $\rho|r\Phi|^2$ (curve g).

We realize why the changes are so extreme for exotic atoms: The absorption of the particle takes place on the border of the nucleus where, because of the strong change in the nuclear density, a small shift in the wave function may yield a big change in the overlap between particle and nuclear density and thus in \mathcal{E} and Γ .

From Heisenberg's uncertainty relation for the product of momentum and position it follows that the attractive $\text{Re}\bar{A}^0$ has to be increased beyond the scale given in the figures here, in order to make possible a kaon bound to the nuclear dimensions of sulfur. The measurement of Ref. 1, and this and other studies,^{4,5} exclude this possibility. If not, the observation of an electromagnetic-dipole transition from the atomic to the rather separated nuclear spectrum would not be unthinkable: The $(\Delta E)^3$ law may operate like a compensator to the matrix element.¹⁴ Further discussions on this aspect, especially interesting for the heavier particles, would be too speculative with the present experimental information. In any case, the facts described in this paper have to be taken into account especially for hypernuclei.

It is difficult to give a simple and yet reliable explanation for these effects. In optics, the situation is much simpler, and repulsive effects due to absorption have been well known for some time. The complex refractive index,

$$\hat{n} = \alpha + i\beta \sim [E/(E-V)]^{1/2},$$

yields a reflectivity

$$R = \frac{|\hat{n}-1|^2}{|\hat{n}+1|^2} = \frac{(\alpha-1)^2 + \beta^2}{(\alpha+1)^2 + \beta^2}.$$

Especially for attractive potentials ($\alpha < 1$), the absorption contributes considerably to the reflectivity ($R \rightarrow 1$ with $\alpha \rightarrow 0$). On the other hand, "A perfect conductor (absorber) . . . would not permit the penetration of an electromagnetic wave to any depth at all and would reflect all the incident light."¹⁵

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⁸The change from negative shifts (for $l=0$) to positive shifts (for $l>0$) of the pionic atom states has a different origin. It is due to the repulsive local and attractive gradient potential. But it is of interest to look at effects which are produced by a small additional contribution of absorption by a one-nucleon density.

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